Special Class of G-Open Sets in Ideal $\mathcal{T}$-Topological Spaces

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Abstract:
In the present paper, we introduce and investigate the notions of several $\mathcal{T}$-open sets such as $\mathcal{T}^*_{\mathfrak{I}}$-open sets in generalized ideal topological spaces and investigate various relationships of these sets by making the use of some suitable and counter examples for the justification of results. Hence we have categorized a special class of generalized open sets in this paper. The present study will be useful in establishing the application in the different useful fields of science and technology especially in digital topology and circuit theory.

Keywords: *$\mathcal{T}_{\mathfrak{I}}$-open, $\mathcal{T}^*_{\mathfrak{I}}$-open, $\mathcal{T}^*_{\mathfrak{I}}$-semi-open, $\mathcal{T}^*_{\mathfrak{I}}$-pre-open, $\mathcal{T}^*_{\mathfrak{I}}$-$\alpha$-open, $\mathcal{T}^*_{\mathfrak{I}}$-$\beta$-open sets*

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1. Introduction

The idea of generalized topology was initiated by Csaszar [3-4]. Further the author has given thoughts of the semi-open set, pre-open set, α-open set and β-open set with regards to generalized topological spaces. These generalized open sets are known about g-semi-open sets, g-pre-open sets, g-α-open sets and g-β-open sets. Additionally the author presented the ideas of generalized closure and generalized interior in the classification of generalized topological spaces. Veera Kumar [22-23] studied the concepts of closed sets, g-closed sets and $g^\sharp$-closed sets. Further the authors [19] verified various results and provide certain characterization of closed sets in topological spaces.
The subject of ideals in topological spaces was studied by Kuratowski [12] almost half a century ago, which motivated the research in applying topological ideals to generalize the most basic properties in general topology. Jankovic and Hamlett [10-11] introduced the concept of I-open sets in ideal topological spaces that initialized the application of topological ideals in the generalization of most fundamental properties in general topology provide the concept of compactable extensions of ideals extend the concept of topologies from old via ideals. They introduced and studied several generalized closed and open sets in ideal topological spaces such as I-closed sets, pre-I-open sets, semi-I-open sets and alpha-I-open sets. Nithyanantha and Thangavelu [16] studied the concept of binary topology and investigated some of its basic properties.
In this paper, we developed the very useful concept of $\mathcal{T}^*_{\mathfrak{I}}$*-open sets* and established the relationship of these sets with some other generalized sets. The results have been shown by several counter examples and applications. Some require basic definitions, concepts of $\mathcal{T}$*-topological, ideal* $\mathcal{T}$*-topological spaces* and notations are discussed in Section 2. The demonstration of $\mathcal{T}^*_{\mathfrak{I}}$*-open sets* is given in Section 3. The applications with future scope of paper are discussed in Section 4. Finally, in Section 5 concluded the paper.

2. Preliminaries

Definition 2.1: If $\mathcal{Z}$ is a non-empty set and the $\mathcal{T}$ a collection of subsets of $\mathcal{Z}$ satisfying axioms i.e., $\emptyset$ and entire set $\mathcal{Z}$ are in $\mathcal{T}$ and the union of the elements of any sub collection of $\mathcal{T}$ is in $\mathcal{T}$. Then $\mathcal{T}$ is said to be GT on $\mathcal{Z}$ and set the $\mathcal{Z}$ together with the topology $\mathcal{T}$ on $\mathcal{Z}$ is known as GTS, and is denoted by $(\mathcal{Z},\mathcal{T})$. The elements of $\mathcal{T}$ are called $\mathcal{T}$-open sets and their complements are called as $\mathcal{T}$-closed sets.

Example 2.1: If $\mathcal{Z} = \{1,2,3\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1,2\}, \{1,3\}, Z\}$ is a GT on $\mathcal{Z}$.

Definition 2.2: If $(\mathcal{Z},\mathcal{T})$ is a GTS and $S \subseteq \mathcal{Z}$. Then the union of all $\mathcal{T}$-open sets in $\mathcal{Z}$ contained in S is called $\mathcal{T}$-interior of S and is denoted by $\mathfrak{I}_{\mathcal{T}}(S)$.

Definition 2.3: If $(\mathcal{Z},\mathcal{T})$ is a GTS and $S \subseteq \mathcal{Z}$. Then the intersection of all $\mathcal{T}$-closed sets in $\mathcal{Z}$ containing S is called $\mathcal{T}$-closure of S and is denoted by $\mathcal{C}_{\mathcal{T}}(S)$.
Definition 2.4: If $(\mathcal{Z},\mathcal{T})$ is a GTS and $S \subseteq \mathcal{Z}$. Then S is called
i) $\mathcal{T}$-semi open if $S \subseteq \mathcal{C}_{\mathcal{T}}(\mathfrak{T}_{\mathcal{T}}(S))$
ii) $\mathcal{T}$-pre open if $S \subseteq \mathfrak{T}_{\mathcal{T}}(\mathcal{C}_{\mathcal{T}}(S))$
iii) $\mathcal{T}$-α open if $S \subseteq \mathfrak{T}_{\mathcal{T}}(\mathcal{C}_{\mathcal{T}}(\mathfrak{T}_{\mathcal{T}}(S)))$
iv) $\mathcal{T}$-regular open if $S = \mathfrak{T}_{\mathcal{T}}(\mathcal{C}_{\mathcal{T}}(S))$

Proposition 2.1:
i) Each $\mathcal{T}$-open set is $\mathcal{T}$-α-open.
ii) Each $\mathcal{T}$-α open set is $\mathcal{T}$-semi-open.
iii) Each $\mathcal{T}$-α-open set is $\mathcal{T}$-pre-open.
iv) Each $\mathcal{T}$-regular open set is $\mathcal{T}$-open.

Remark 2.1: The converse of the Proposition 2.1 is not true in general which can be seen in Example 2.2, Example 2.3, Example 2.4 and Example 2.5.

Example 2.2: If $\mathcal{Z} = \{1, 2, 3, 4\}$ and $\mathcal{T} = \{\emptyset, \{2,3\}, \{1,3, 4\}, \mathcal{Z}\}$ is GTS. Then the set $\{1,2, 3\}$ is $\mathcal{T}$-α-open but not $\mathcal{T}$-open set.

Example 2.3: If $\mathcal{Z} = \{1, 2, 3, 4\}$ and $\mathcal{T} = \{\emptyset, \{2\}, \{2,3\}, \{3,4\}, \{2,3, 4\}, Z\}$ is GT. Then the set $\{1,2\}$ is $\mathcal{T}$-semi-open but not $\mathcal{T}$-α-open.

Example 2.4: If $\mathcal{Z} = \{1, 2, 3, 4\}$ and $\mathcal{T} = \{\emptyset, \{1,2\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{2,3, 4\}, \mathcal{Z}\}$ is GT. Then the set $\{1,3,4\}$ is $\mathcal{T}$-pre-open but not $\mathcal{T}$-α-open.

Example 2.5: If $\mathcal{Z} = \{1, 2, 3, 4\}$ and $\mathcal{T} = \{\emptyset, \{2\}, \{2,3\}, \{3,4\}, \{2,3, 4\}, \mathcal{Z}\}$ is GT. Then the set $\{2,3\}$ is $\mathcal{T}$-open but not $\mathcal{T}$-regular-open.

Definition 2.5: If $\mathcal{Z}$ is non-empty set. Then an ideal is a non-empty collection $\mathfrak{T}$ of subsets of $\mathcal{Z}$ satisfying the axioms i.e. if. $P \in \mathfrak{T}$ and $Q \subseteq P$ implies $Q \in \mathfrak{T}$ and If $P \in \mathfrak{T}$ and $Q \in \mathfrak{T}$ implies $(P \cup Q) \in \mathfrak{T}$. If $(\mathcal{Z},\mathcal{T})$ is GTS and $\mathfrak{T}$ ideal of subsets of $\mathcal{Z}$, then $(\mathcal{Z},\mathcal{T},\mathfrak{T})$ is GITS.

Example 2.6: If $(\mathcal{Z},\mathcal{T})$ is GTS. Then the collection $\mathfrak{T} = \{\emptyset\}$ and $\mathfrak{T} = \wp(\mathcal{Z})$ are also ideals on $\mathcal{Z}$.

Definition 2.6: If $(\mathcal{Z},\mathcal{T},\mathfrak{T})$ is GITS and $P \subseteq \mathcal{Z}$. Then the set $(P)^*(\mathfrak{T}) = \{ x \in \mathcal{Z}/(S \cap P) \notin \mathfrak{T}$ for each neighborhood S of x$\}$ is called the local function of P in respect of $\mathfrak{T}$ and write P^* instead of $(P)^*(\mathfrak{T})$ to avoid confusion.

3. $\mathcal{T}^*{}_{\mathfrak{T}}$-open Sets

Definition 3.1: If $(\mathcal{Z},\mathcal{T},\mathfrak{T})$ is GITS and $P \subseteq \mathcal{Z}$, then P is said to be **$\mathcal{T}^*{}_{\mathfrak{T}}$-open** if there is any $\mathcal{T}$-open set S such that $S - P \in \mathfrak{T}$ and $P \subseteq S$.

Example 3.1: If $\mathcal{Z} = \{1, 2, 3\}$, $\mathcal{T} = \{\emptyset, \{1, 2\}, \{2, 3\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$. Therefore the sets $\{1\}$ and $\{2\}$ are $\mathcal{T}^*{}_{\mathfrak{T}}$-open sets in $(\mathcal{Z},\mathcal{T},\mathfrak{T})$.

Remark 3.1: Each $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set is $\mathcal{T}^*{}_{\mathfrak{T}}$-open. The converse part is not true which can be seen in Example 3.2.
Example 3.2: The sets $\{1\}$ and $\{2\}$ are $\mathcal{T}^*{}_{\mathfrak{T}}$-open sets in Example 3.1, but not $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) sets in $(\mathcal{Z},\mathcal{T})$.

Remark 3.2: Each $\mathcal{T}^*{}_{\mathfrak{T}}$-open is $\mathcal{T}_{\mathfrak{T}}$-semi-open ($\mathcal{T}_{\mathfrak{T}}$-$\alpha$-open) set. The converse part is not true which can be seen in Example 3.3.

Example 3.3: If $\mathcal{Z} = \{1, 2, 3,4\}$, $\mathcal{T} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}. \{1, 3\}, \{1, 4\}, \{2, 3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{2,3\}\}$. Therefore the set $\{3, 4\}$ is $\mathcal{T}_{\mathfrak{T}}$-semi-open ($\mathcal{T}_{\mathfrak{T}}$- α-open) set but not $\mathcal{T}^*{}_{\mathfrak{T}}$-open in $(\mathcal{Z},\mathcal{T},\mathfrak{T})$.

Remark 3.3: Each $\mathcal{T}^*{}_{\mathfrak{T}}$-open is $\mathcal{T}_{\mathfrak{T}}$-pre-open ($\mathcal{T}_{\mathfrak{T}}$-$\beta$-open) set. The converse part is not true which can be seen in Example 3.4.

Example 3.4: If $\mathcal{Z} = \{1, 2, 3,4\}$, $\mathcal{T} = \{\emptyset, \{1\}, \{3\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{1,2,3\}, \{1,3,4\}, \{1,2,4\}, \{2,3,4\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{1\}\}$. Therefore the set $\{3,4\}$ is $\mathcal{T}_{\mathfrak{T}}$-pre-open ($\mathcal{T}_{\mathfrak{T}}$-$\beta$-open) set but not $\mathcal{T}^*{}_{\mathfrak{T}}$-open in $(\mathcal{Z},\mathcal{T},\mathfrak{T})$.

Definition 3.2: If $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$ is GITS and $P \subseteq \mathcal{Z}$, then P is said to be $\boldsymbol{\mathcal{T}^*}_{\mathfrak{T}}$**-semi-open** set if there is any $\mathcal{T}$-open set S such that $S - \mathfrak{T}_{\mathcal{T}}(P) \in \mathfrak{T}$ and $P \subseteq S$.

Example 3.5: If $\mathcal{Z} = \{1, 2, 3\}$, $\mathcal{T} = \{\emptyset, \{1, 2\}, \{2, 3\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{2, 3\}\}$. Therefore the sets $\{2\}$ and $\{3\}$ are $\mathcal{T}^*{}_{\mathfrak{T}}$-semi-open sets in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Remark 3.4: Each $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set is $\mathcal{T}^*{}_{\mathfrak{T}}$-semi-open. The converse part is not true which can be seen in Example 3.6.

Example 3.6: The sets $\{2\}$ and $\{3\}$ are $\mathcal{T}^*{}_{\mathfrak{T}}$-semi-open in Example 3.5, but not $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) sets in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Remark 3.5: Each $\mathcal{T}^*{}_{\mathfrak{T}}$-semi-open is $\mathcal{T}_{\mathfrak{T}}$-semi-open ($\mathcal{T}_{\mathfrak{T}}$-$\alpha$-open) set. The converse is not true which can be seen in Example 3.7.

Example 3.7: If $\mathcal{Z} = \{1, 2, 3,4\}$, $\mathcal{T} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}. \{1, 3\}, \{1, 4\}, \{2, 3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{2,3\}\}$. Therefore the set $\{3,4\}$ is $\mathcal{T}_{\mathfrak{T}}$-semi-open ($\mathcal{T}_{\mathfrak{T}}$-$\alpha$-open) set but not $\mathcal{T}^*{}_{\mathfrak{T}}$-semi-open in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Definition 3.3: If $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$ is GITS and $P \subseteq \mathcal{Z}$, then P is said to be $\boldsymbol{\mathcal{T}^*}_{\mathfrak{T}}$**-$\boldsymbol{\alpha}$-open** if there is any $\mathcal{T}$-open set S such that $S - \mathcal{C}_{\mathcal{T}}(\mathfrak{T}_{\mathcal{T}}(P)) \in \mathfrak{T}$ and $P \subseteq S$.

Example 3.8: If $\mathcal{Z} = \{1, 2, 3\}$, $\mathcal{T} = \{\emptyset, \{1,3\}, \{2, 3\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{2, 3\}\}$. Therefore the set $\{3\}$ is $\mathcal{T}^*{}_{\mathfrak{T}}$-$\alpha$-open set in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Remark 3.6: Each $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set is $\mathcal{T}^*{}_{\mathfrak{T}}$-$\alpha$-open. The converse part is not true which can be seen in Example 3.9.

Example 3.9: The set $\{3\}$ is $\mathcal{T}^*{}_{\mathfrak{T}}$-$\alpha$-open in Example 3.8, but not $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Remark 3.7: Each $\mathcal{T}^*{}_{\mathfrak{T}}$-$\alpha$-open set is $\mathcal{T}_{\mathfrak{T}}$-semi-open ($\mathcal{T}_{\mathfrak{T}}$-$\alpha$-open) set. The converse part is not true which can be seen in Example 3.10.

Example 3.10: If $\mathcal{Z} = \{1, 2, 3,4\}$, $\mathcal{T} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}. \{1, 3\}, \{1, 4\}, \{2, 3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{2,3\}\}$. Therefore the set $\{3,4\}$ is $\mathcal{T}_{\mathfrak{T}}$-semi-open ($\mathcal{T}_{\mathfrak{T}}$-$\alpha$-open) set but not $\mathcal{T}^*{}_{\mathfrak{T}}$-$\alpha$-open in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Remark 3.8: Each $\mathcal{T}^*{}_{\mathfrak{T}}$-semi-open set is $\mathcal{T}^*{}_{\mathfrak{T}}$-$\alpha$-open. The converse part is not true which can be seen in Example 3.11.

Example 3.11: If $\mathcal{Z} = \{1, 2, 3,4\}$, $\mathcal{T} = \{\emptyset, \{1\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{1\}\}$. Therefore the set $\{2,3,4\}$ is $\mathcal{T}^*{}_{\mathfrak{T}}$-$\alpha$-open but not $\mathcal{T}^*{}_{\mathfrak{T}}$-semi-open set in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Definition 3.4: If $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$ is GITS and $P \subseteq \mathcal{Z}$, then P is said to be $\boldsymbol{\mathcal{T}^*}_{\mathfrak{T}}$**-pre-open** if there is any $\mathcal{T}$-open set S such that $S - \mathcal{C}_{\mathcal{T}}(P) \in \mathfrak{T}$ and $P \subseteq S$.

Example 3.12: If $\mathcal{Z} = \{1, 2, 3\}$,$\mathcal{T} = \{\emptyset, \{1, 2\}, \{2, 3\}, \mathcal{Z}\}and\mathfrak{T} = \{\emptyset, \{2\}, \{1,2\}\}$. Therefore the set $\{1\}$ is $\mathcal{T}^*{}_{\mathfrak{T}}$-pre-open set in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Remark 3.9: Each $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set is $\mathcal{T}^*{}_{\mathfrak{T}}$-pre-open. The converse part is not true which can be seen in 3.13.

Example 3.13: The set $\{1\}$ is $\mathcal{T}^*{}_{\mathfrak{T}}$-pre-open set in Example 3.12, but not $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Remark 3.10: Each $\mathcal{T}^*{}_{\mathfrak{T}}$-pre-open set is $\mathcal{T}_{\mathfrak{T}}$-semi-open ($\mathcal{T}_{\mathfrak{T}}$-$\alpha$-open) set. The converse part is not true which can be seen in Example 3.14.

Example 3.14: If $\mathcal{Z} = \{1, 2, 3,4\}$, $\mathcal{T} = \{\emptyset, \{1, 3\}, \{1, 4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{2,3\}\}$. Therefore $\{2, 4\}$ is $\mathcal{T}_{\mathfrak{T}}$-semi-open ($\mathcal{T}_{\mathfrak{T}}$-$\alpha$-open) set but not $\mathcal{T}^*{}_{\mathfrak{T}}$-pre-open set in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Definition 3.5: If $(\mathcal{Z},\mathcal{T},\mathfrak{I})$ is GITS and $P \subseteq \mathcal{Z}$, then P is said to be $\boldsymbol{\mathcal{T}^*{}_{\mathfrak{I}}}$**-**$\boldsymbol{\beta}$**-open** if there is any $\mathcal{T}$-open set S such that $S - \mathfrak{I}_{\mathcal{T}}(\mathcal{C}_{\mathcal{T}}(P)) \in \mathfrak{I}$ and $P \subseteq S$.
Example 3.15: The set $\{1\}$ in Example 4.12 is $\mathcal{T}^*{}_{\mathfrak{I}}$-$\beta$-open set in $(\mathcal{Z},\mathcal{T},\mathfrak{I})$.

Remark 3.11: Each $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set is $\mathcal{T}^*{}_{\mathfrak{I}}$-$\beta$-open. The converse part is not true which can be seen in Example 3.12.

Remark 3.12: Each $\mathcal{T}^*{}_{\mathfrak{I}}$-$\beta$-open set is $\mathcal{T}_{\mathfrak{I}}$-semi-open ($\mathcal{T}_{\mathfrak{I}}$-$\alpha$-open) set. The converse part is not true which can be seen in Example 3.16.

Example 3.16: The set $\{2,4\}$ in Example 3.14 is $\mathcal{T}_{\mathfrak{I}}$-semi-open ($\mathcal{T}_{\mathfrak{I}}$-$\alpha$-open) set but not $\mathcal{T}^*{}_{\mathfrak{I}}$-$\beta$-open set in $(\mathcal{Z},\mathcal{T},\mathfrak{I})$.

Remark 3.13: In general $\mathcal{T}^*{}_{\mathfrak{I}}$-open sets and $\mathcal{T}^*{}_{\mathfrak{I}}$-semi-open sets are independent which can be seen in Example 3.17 and Example 3.18.

Example 3.17: If $\mathcal{Z} = \{1,2,3,4\}$, $\mathcal{T} = \{\emptyset,\{1\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\}, \{1,3,4\},\mathcal{Z}\}$ and $\mathfrak{I} = \{\emptyset,\{1\}\}$. Therefore the set $\{2,3,4\}$ is $\mathcal{T}^*{}_{\mathfrak{I}}$-open set but not $\mathcal{T}^*{}_{\mathfrak{I}}$-semi-open in $(\mathcal{Z},\mathcal{T},\mathfrak{I})$.

Example 3.18: If $\mathcal{Z} = \{1,2,3\}$, $\mathcal{T} = \{\emptyset,\{1,3\},\{2,3\},\mathcal{Z}\}$ and $\mathfrak{I} = \{\emptyset,\{2,3\}\}$. Therefore the sets $\{2\}$ and $\{3\}$ are $\mathcal{T}^*{}_{\mathfrak{I}}$-semi-open sets but not $\mathcal{T}^*{}_{\mathfrak{I}}$-open sets in $(\mathcal{Z},\mathcal{T},\mathfrak{I})$.

Remark 3.14: In general $\mathcal{T}^*{}_{\mathfrak{I}}$-open sets and $\mathcal{T}^*{}_{\mathfrak{I}}$-$\alpha$-open sets are independent which can be seen in Example 3.19 and Example 3.20.

Example 3.19: If $\mathcal{Z} = \{1,2,3\}$, $\mathcal{T} = \{\emptyset,\{1,2\},\{1,3\},\mathcal{Z}\}$ and $\mathfrak{I} = \{\emptyset,\mathcal{Z}\}$. Therefore the set $\{2,3\}$ is $\mathcal{T}^*{}_{\mathfrak{I}}$-$\alpha$-open set but not $\mathcal{T}^*{}_{\mathfrak{I}}$-open in $(\mathcal{Z},\mathcal{T},\mathfrak{I})$.

Example 3.20: If $\mathcal{Z} = \{1,2,3\}$, $\mathcal{T} = \{\emptyset,\{1,3\},\{2,3\},\mathcal{Z}\}$ and $\mathfrak{I} = \{\emptyset,\{2\}\}$. Therefore the set $\{3\}$ are $\mathcal{T}^*{}_{\mathfrak{I}}$-open set but not $\mathcal{T}^*{}_{\mathfrak{I}}$-$\alpha$-open in $(\mathcal{Z},\mathcal{T},\mathfrak{I})$.

Remark 3.15: In general $\mathcal{T}^*{}_{\mathfrak{I}}$-semi-open sets and $\mathcal{T}^*{}_{\mathfrak{I}}$-pre-open sets are independent which can be seen in Example 3.21 and Example 3.22.

Example 3.21: If $\mathcal{Z} = \{1,2,3,4\}$, $\mathcal{T} = \{\emptyset,\{1\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\}, \{1,3,4\},\mathcal{Z}\}$ and $\mathfrak{I} = \{\emptyset,\{1,2,4\}\}$. Therefore the set $\{2,4\}$ is $\mathcal{T}^*{}_{\mathfrak{I}}$-semi-open set but not $\mathcal{T}^*{}_{\mathfrak{I}}$-pre-open in $(\mathcal{Z},\mathcal{T},\mathfrak{I})$.

Example 3.22: If $\mathcal{Z} = \{1,2,3,4\}$, $\mathcal{T} = \{\emptyset,\{1\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\}, \{1,3,4\},\mathcal{Z}\}$ and $\mathfrak{I} = \{\emptyset,\{3\}\}$. Therefore the set $\{2,4\}$ is $\mathcal{T}^*{}_{\mathfrak{I}}$-semi-open set but not $\mathcal{T}^*{}_{\mathfrak{I}}$-pre-open in $(\mathcal{Z},\mathcal{T},\mathfrak{I})$.

Remark 3.16: Each $\mathcal{T}^*{}_{\mathfrak{I}}$-open set is $\mathcal{T}_{\mathfrak{I}}$-pre-open ($\mathcal{T}_{\mathfrak{I}}$-$\beta$-open) set. The converse part is not true in general which can be seen in Example 3.23.

Example 3.23: If $\mathcal{Z} = \{1,2,3,4\}$, $\mathcal{T} = \{\emptyset,\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\mathcal{Z}\}$ and $\mathfrak{I} = \{\emptyset,\{2\}\}$. Therefore the set $\{1\}$ is $\mathcal{T}_{\mathfrak{I}}$-pre-open ($\mathcal{T}_{\mathfrak{I}}$-$\beta$-open) set but not $\mathcal{T}^*{}_{\mathfrak{I}}$-open in $(\mathcal{Z},\mathcal{T},\mathfrak{I})$.

Remark 3.17: The following implications as shown in figure-1 and figure-2 are the direct consequences of the definitions of various $\mathcal{T}^*{}_{\mathfrak{I}}$-open sets that we discussed in this section

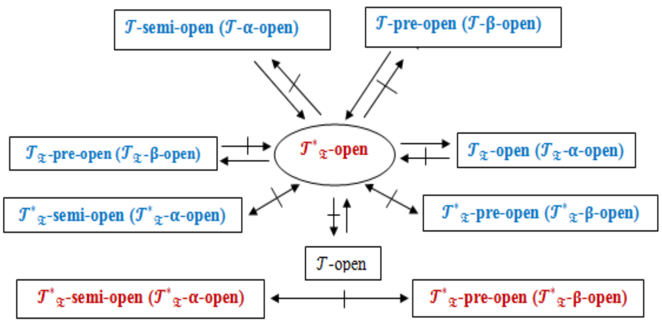

Figure-1: Relationships of $\mathcal{T}^*{}_{\mathfrak{I}}$-open sets

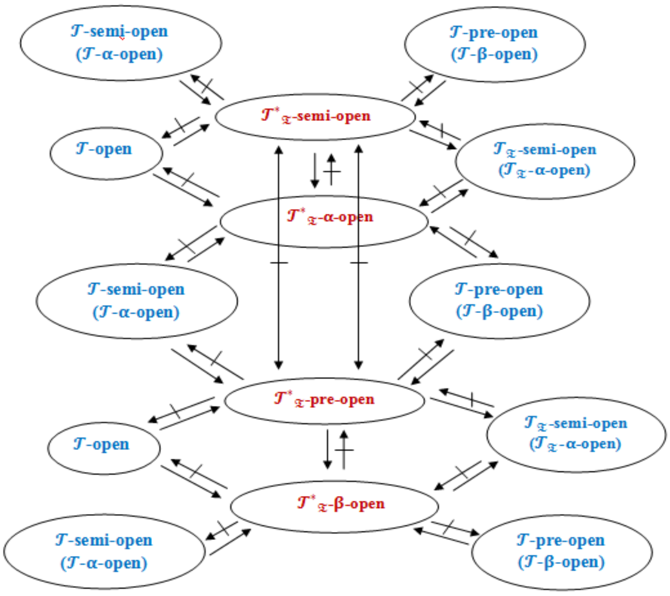

Figure-2: Relationships of $\mathcal{T}^*_{\mathfrak{X}}$-open sets

4. Conclusion

We aimed to introduce the concept of $\mathcal{T}^*_{\mathfrak{X}}$-open sets in this paper. Then we established the relationship between above discuses and the results have been shown by several counter examples. Further the study has provided the certain classification of other special class of open set which are more useful in studying the behaviour of some other open and closed sets in generalized topological spaces and generalized ideal topological spaces.

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