Study Of Anisotropic Cosmological Models In $f\ (R, T)$ Theory Of Gravity

Ghanshyam Sen[1]*, Sudha Agrawal[2]

[1]*Research Scholar AKS University Satna (M.P.), Email: ghanshyamsen342@gmail.com
[2]HOD Department of Mathematics AKS University Satna (M.P.), Email: sudha.agrawal10@gmail.com

***Corresponding Author:** - Ghanshyam Sen
*Research Scholar AKS University Satna (M.P.), Email: ghanshyamsen342@gmail.com

Abstract
The current state of observational data suggests that the cosmos is homogenous and isotropic on a sufficiently vast scale. But this doesn't rule out the potential of some anisotropy occurring in the early stages of the history of the cosmos and then being damped away in subsequent epochs. The homogeneous but anisotropic Bianchi models have gained popularity because of this hypothesis. Second, the issues with the standard CDM model in general relativity have sparked a lot of interest in modified gravity. For this reason, the Bianchi type-I cosmological model has been investigated in f (R, T)-modified gravity in the present research. A model showing a change from early deceleration to late-time acceleration was obtained by assuming a particular form of the deceleration parameter, which was inspired by certain concepts from cosmography. The model speeds up from the outset for a fixed amount of time before slowing down. The model's physical behavior is investigated as well. Both theories' physical and kinematic characteristics for the future of the cosmos are examined at length. For all scenarios, we delved into what makes WEC, DEC, SEC, and energy density tick.

Keywords:- Cosmology model, f (R, T) gravity, cosmological parameters.

INTRODUCTION

However, the most recent finding of the accelerating expansion of the Universe cannot be explained by Einstein's general relativity, which is the foundation for the explanation of most gravitational events known to date. The presence of dark matter and the recent discovery of the universe's accelerated expansion in late time have presented a major theoretical challenge to gravitational theories. The CMBR and supernovae studies put the percentages of conventional baryonic matter, dark matter, and dark energy in our Universe at around 4%, 20%, and 76%, respectively. If the Einstein gravity model of general relativity fails at large scales, then a more generic action may be used to describe the gravitational field, which would help to explain the data. Late acceleration and dark energy have been difficult to explain, but in recent years scientists have turned to tweaks to general relativity to provide an explanation. Gravity with a constant acceleration of f(R) may account for the observed accelerated expansion of the Universe in recent epochs. f(R) theory of gravity is favored above other revisions because of the significance of f(R) models in cosmology. In order to accomplish cosmic acceleration, it has been proposed to substitute a generic function Ricci scalar, f(R), for the Einstein-Hilbert action of general relativity. Numerous applications of f(R), f(G), and f (R, G) gravity have been investigated by Nojiri et al. In a wide variety of studies, scientists have looked at f(R) gravity. Shamir's f(R) gravity model, which he devised, demonstrates how inflation and acceleration may occur at the same time and at the same place in the universe's history.

S.S. Nerkar et.al (2022) In this study, we have explored the f (T) theory of gravity in the setting of a Bianchi type-III universe that is both spatially homogenous and anisotropic. Using the continuity equation and the equation of state parameter to describe the many stages of the cosmos, we have recreated Bianchi type-III cosmological models. We have taken into account several permutations of the matter-dominated, radiation-dominated, and dark-energy phases. One of the models has a constant solution, which may be the cosmological constant, as has been seen. Using standard f (T) models, we have also calculated the equation of state parameter and provided a description of cosmic acceleration.

Bishnu Prasad Brahma et.al (2022) Based on the f (R, T) gravity and a time-dependent displacement field, this study discusses the bulk viscous Bianchi type-V cosmological model with an exponential scale factor in Lyra geometry. We took into account the work of Harko et al. (Phys. Rev. D, 2011, 84, 024020) to establish the model's essence and physical characteristics. The barotropic equation of state for pressure, density, and bulk viscous pressure is proportional to energy density, therefore we have [suggested the linear form f (R, T) = f1(R) + f2(T)]. In addition, we explore the model's kinematic features when bulk viscosity is present. In order to provide an explanation for the recent cosmic acceleration, the evolution of energy conditions and the behavior of that is also explored.

Monsur Rahaman, et.al (2020) The primary focus of this work is on investigating the possibility of compact spherical systems exhibiting anisotropic matter distributions in the context of alternative gravitational theories, particularly f (R,

T) gravity theory. In addition, a significant and doable decision is made about the formulation of f(R, T) gravity. It is assumed that the functional form of f (R, T) is f(R, T)=R+2T, where R and T are the scalar curvature and the trace of the stress-energy tensor, respectively, in order to give the full set of field equations for the anisotropic distribution of matter. To get a complete space-time representation within the astrophysical structure, one might use the embedding class one technique in conjunction with the Eisland condition. In order to characterize the anisotropic distribution of matter inside an astrophysical system, we create an appropriate anisotropic model once we know the space-time geometry. This model uses a new gravitational potential grr, which often produces solutions that are physically justifiable. Several physical tests confirm the model's physical availability, representing the solution's physical features. It's worth noting that we've used the observed mass values for six compact stars to precisely forecast radii for a range of -coupling parameter values. It's clear from this that the solution's anticipated radii are consistent with the measured ones. We have made predictions for the radius of MSP J0740+6620, the most massive neutron star yet detected, over a range of -coupling parameters. As the value decreases from 11 to 1, the anticipated radii decrease in size in a monotonic fashion. Our method yields an M-R curve that can accept compact stars of varying masses, from the very low-mass Her X-1 to the high-mass MSP J0740+6620. Since the conventional implications of general relativity are recovered for =0.0=0.0, the current work concludes that the modified f (R, T) gravity is a suitable theory to explain large astrophysical systems.

S.K. Sahu et.al (2017) Against the background of an anisotropic Bianchi type-III universe, accelerating cosmological models are built in a modified gravity theory termed f (R, T) gravity. In the Einstein-Hilbert action of General Relativity, the Ricci scalar R is replaced by a function f (R, T) of the Ricci scalar R and the trace T of the energy-momentum tensor. Two variations of altering the Einstein-Hilbert action are modeled. By using a fresh technique of integration, we are able to find exact solutions to the field equations. To better understand the dynamical properties of the universe, we have investigated the behavior of the transition from a period of slow expansion to one of rapid growth. Within the framework of this work's formalism, it is discovered that the scale factor is unaffected by the Einstein-Hilbert action's modification. However, the effective dark energy equation of state is profoundly impacted in terms of its dynamics.

MODIFIED F (R, T) GRAVITY

The action of f (R, T) gravity is given by:

$$S=\int\sqrt{-g}\left(\frac{-1}{16\pi G}f(R,T)+L_m\right)d^4x,$$

Where g is the determinant of the metric tensor, gij; f (R, T) is a free function of the Ricci scalar R and the energy-momentum tensor's trace T. T_{ij}—i.e., $\left(T=g^{ij}T_{ij}\right)$; where the Lagrangian density of matter is denoted by Lm. It is important to note that the f (R, T) theory of gravity is an extension of the f(R) theory and a modification of general relativity. The field equations are derived in the same way as in f(R) gravity models: by changing the sum of the field and matter's actions and setting it equal to zero.

Changing the action S in (1) with regard to the metric tensor gij gives us the field equations 1 in f (R, T) gravity, which we express in gravitational units (8G = 1, c = 1):

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+\left(g_{ij}\Box-\nabla_i\nabla_j\right)f_R(R,T)=-T_{ij}-f_T(R,T)T_{ij}-f_T(R,T)\Theta_{ij},$$

Where $f_R(R,T)=\frac{\partial f(R,T)}{\partial R},\ f_T(R,T)=\frac{\partial f(R,T)}{\partial T},\ R_{ij}$, The Ricci tensor is denoted by Rij in this formula, while the energy momentum tensor is denoted by Tij:

$$T_{ij}=\frac{-2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta g^{ij}}.$$

In Equation (2), $\Box=\nabla^i\nabla_i$ is the D'Alembertian operator, where ∇_i represents the covariant derivative and:

$$\Theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{\mu v}\frac{\partial^2L_m}{\partial g^{ij}\partial g^{\mu v}}.$$

An essential link between the Ricci scalar R and the trace T of the energy-momentum tensor is obtained during contraction through Equation (3):

$$f_R(R,T)R+3\Box f_R(R,T)-2f(R,T)=-T-f_T(R,T)(T+\Theta),$$

Where $\Theta = \Theta_i^i$. Lagrangian density Lm of matter is assumed to rely simply on the metric tensor component rather than its derivatives, simplifying Equation (3) to the form:

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}}.$$

The matter energy-momentum tensor, with ideal fluid dispersion, has the form:

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij},$$

Where is the fluid's energy density and p is its pressure. In this case, the four-velocity vector u i that $u^i u_i = -1$ and $u^i \nabla_j u_i = 0$. given that Lm = p, we may rewrite Equation as:

$$\Theta_{ij} = -p g_{ij} - 2T_{ij}.$$

As a result, the version of Equation (4) in the field is:

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - \left(\nabla_i\nabla_j - \Box g_{ij}\right)f_R(R,T) = -T_{ij} + f_T(R,T)\left(T_{ij} + p g_{ij}\right).$$

There are a few different versions of the function f (R, T) that Harko and co. have studied:

$$f(R,T) = \begin{cases} R + 2f_1(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases}$$

For the sake of this investigation, we will focus on the first version of f (R, T), namely, f (R, = R + 2 f1(T), and we will set f1(T) = T, where is a constant chosen at random. Taking into account the energy-momentum tensor (7), we may simplify Equation (6) to:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -(1+2\lambda)T_{ij} + \lambda(T + 2p)g_{ij}.$$

To include the cosmological constant into Einstein's field equations, we get:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} + \Lambda g_{ij}.$$

We may determine that = (T) = (T + 2p) by comparing Equations (7) and (8) and setting the parameter to a modest value. Therefore, the field equations in the f (R, T) theory of gravity, where the cosmological parameter (T) is a variable, may be written as:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -(1+2\lambda)T_{ij} + \Lambda g_{ij}.$$

The energy-momentum tensor's trace T in the case of a perfect fluid may be expressed as T = 3p. To express the cosmic parameter mathematically, we have:

$$\Lambda = \lambda(\rho - p).$$

It is evident from Equation that the general energy conservation rule does not hold in the f (R, T) theory, which follows from Equation. Shabani and Zaiae have pointed out that, from a thermodynamic point of view, the non-conservation of energy implies an irreversible matter production process. Particle physics at its most basic level is thought to provide a plausible explanation for this procedure. The generation of these particles is proportional to the transfer of energy from the gravitational field to the newly formed matter particles. The effects of energy conservation were studied by the same authors in a different publication. It was determined that late-time stable accelerating solutions are not ubiquitous in f (R, T) gravity if energy conservation holds. A vast class of solutions with late-time acceleration and stability may be found, however, when energy is not conserved.

Let's start by checking to see whether energy conservation holds in this situation. As a result of the Bianchi identities, the divergence on the left-hand side of Equation (9 is zero. As a corollary, this necessitates that the RHS likewise exhibit no divergence. This demonstrates that the conventional energy conservation rule does not apply in any circumstance save one. This is when $\frac{d\rho}{dt} = \frac{d\bar{p}}{dt}$ There is a lack of energy conservation otherwise, as is the case in this study.

COSMOLOGICAL SOLUTION

The connection between the two observable parameters q and H

$$q = -1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{H}\right)$$

This equation, on integration, gives the scale factor a(t) as

$$a(t) = \mathrm{e}^{\delta} \exp \int \frac{\mathrm{d}t}{\int (1+q)\mathrm{d}t + r},$$

Where r and may be any constants chosen at random. Abdussattar and Prajapati suggested the following selection of q for a(t)'s final determination as

$$q = -\frac{\alpha}{t^2} + (\beta - 1),$$

Where > 0 is a time-squared parameter and > 1 is a constant with no dimensions. Various models emerge depending on the values of and. The scale factor is obtained by integrating eq. with q supplied by eq. and = 0, r = 0.

$$a(t) = \left(t^2 + \frac{\alpha}{\beta}\right)^{1/2\beta}$$

The non-singular bouncing model with a(t) given by has been extensively examined by Abdussattar and Prajapati, who have also provided a graphical representation of the change of various cosmological parameters for fixed values of those parameters.

Equations with a1, a2, a3,, and p as unknowns are notoriously hard to answer exactly. Assuming a3 = Vb, where b is a constant, allows us to find a unique solution to the system. Then, we get the precise equation for the scale factors from eqs. and:

$$a_1(t) = \left(t^2 + \frac{\alpha}{\beta}\right)^{(3+3mb-3b)/2\beta(m+2)}$$

$$a_2(t) = \left(t^2 + \frac{\alpha}{\beta}\right)^{(3+3m-3b-6mb)/2\beta(m+2)}$$

$$a_3(t) = \left(t^2 + \frac{\alpha}{\beta}\gamma\right)^{3b/2\beta}$$

PHYSICAL PROPERTIES OF THE MODELS

Hubble's parameter and the deceleration parameter determine the expansion rate of the cosmos as a function of time. It is possible to derive more precise kinematical descriptions of cosmic expansions by considering an expanded set of parameters with higher order time derivatives of the scale factor. It is discovered that the volume of space is

$$V = AB^2 = (\mathrm{e}^{\beta t} - 1)^{\frac{3}{\beta}}$$

The preceding equation shows that at time t = 0, the spatial volume is zero in both models. It proves that the big bang scenario is the starting point for the development of our cosmos. The average scale factor also vanishes once (21) is applied.

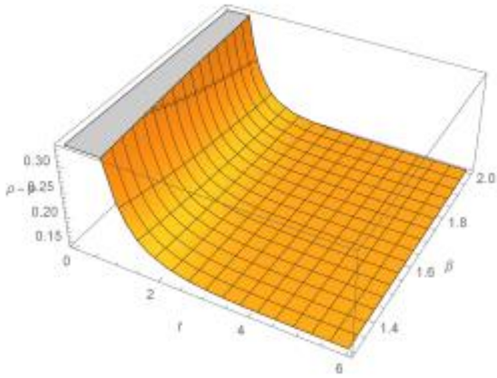

Fig. 1. Behaviour of WEC versus t and β with μ = 5

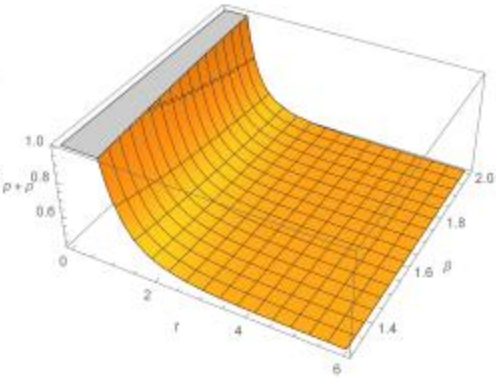

Fig. 2. Behaviour of DEC versus t and β with μ = 5.

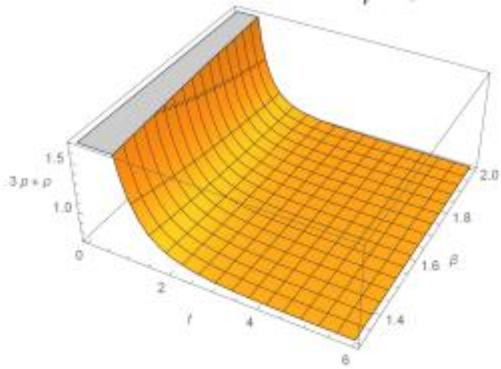

Fig. 3. Behaviour of SEC versus t and β with μ = 5.

at the initial epoch. Hence, both models have a point type singularity. The spatial volume increases with time. The Hubble's parameter H, expansion scalar θ and shear scalar σ 2 become

$$H = \frac{1}{3}(H_1 + 2H_2) = e^{\beta t}(e^{\beta t} - 1)^{-1}$$

$$\theta = 3H = 3e^{\beta t}(e^{\beta t} - 1)^{-1}$$

$$\sigma^2 = \frac{1}{2}\left(H_1^2 + 2H_2^2 - \frac{\theta^2}{3}\right) = \frac{3}{4}e^{2\beta t}(e^{\beta t} - 1)^{-2}$$

From the above equations, we can observe that the Hubble factor, scalar expansion and At t = 0, the shear scalar diverges, and as t decreases, it becomes finite. The isotropic condition is mentioned below. $\frac{\sigma^2}{\theta^2}$ illustrates that the model does not evolve toward isotropy by remaining constant (from early to late time),
f (R, T) anisotropic cosmological models with a tunable acceleration

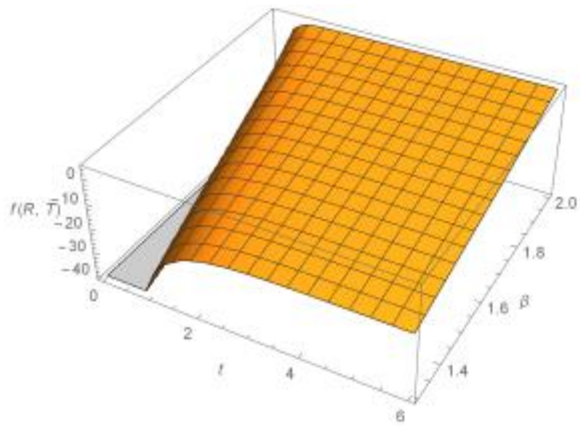

Fig. 4. Behaviour of f(R, T) versus t and β with μ = 5.

Universe. The anisotropy parameter

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = 6\left(\frac{\sigma}{\theta}\right)^2 = \frac{1}{2}$$

For our models, the anisotropic parameter is kept fixed. The following equation shows that the cosmos in our simulations is expanding and speeding up after the big bang.

Jerk parameter:

One of the key quantities in characterizing the mechanics of the cosmos is the jerk parameter. The cosmic jerk parameter j characterizes the models that are close to CDM. The value of jerk for the flat CDM model is j = 1. The jerk parameter is defined as the dimensional-free third derivative of the scale factor a with respect to the cosmological time t.

$$j = \frac{a^2}{\dot{a}^3}\frac{d^3 a}{dt^3}$$

As a deceleration parameter version of the aforementioned statement, we have $j = q + 2q^2 - \frac{\dot{q}}{H}$

Thus, the jerk parameter for our models is

$$j = 1 - 3\beta e^{-\beta t} + 2\beta^2 e^{-2\beta t} + \beta^2 e^{-2\beta t}(e^{\beta t} - 1)$$

Clearly, our result does not coincide with the value shown in Fig.5. $j = 2.16^{+0.81}_{-0.75}$, obtained from combination of three kinematical data sets: the gold sample data of type I a supernovae, X-ray galaxy cluster distance measurements, and SNLS project SNIa data. In Fig. 5, the jerk parameter is shown as a function of. As far as anybody can tell, the jerk parameter is always going to be positive and equal to

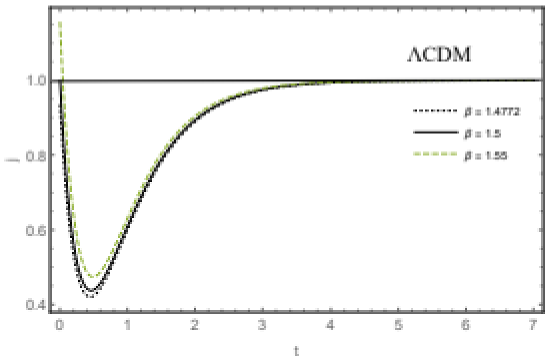

Fig. 5 Behaviour of Jerk parameter versus t with different β.

For the examined values of, the CDM model is valid at t 5.5. For the numbers shown in table, it is noteworthy to notice that our model is similar to the CDM model I.

r − s parameter:

The state-finder pair {r, s} is defined as

$$r = \frac{\dddot{a}}{aH^3}, \qquad s = \frac{r-1}{3(q-\frac{1}{2})}$$

The state-finder pair is a geometrical diagnostic parameter that is built directly from a space-time metric, and it is more model-independent than physical variables that rely on the characteristics of physical fields to describe DE. The r, s state-finder pair for the flat CDM model is 1, 0. Our model's state-finder parameter has the following values:

$$r = 1 - 3\beta e^{-\beta t} + 2\beta^2 e^{-2\beta t} + \beta^2 e^{-2\beta t}(e^{\beta t} - 1)$$

$$s = \frac{1}{6\beta - 9e^{\beta t}}\left[2\beta^2 e^{-\beta t}(e^{\beta t} - 1) + 4\beta^2 e^{-\beta t} - 6\beta\right]$$

From the expressions of r and s parameters, we found that {r, s} = {1, 0} only when $t = \frac{1}{\beta}\ln\left(\frac{\beta}{3-\beta}\right)$.The variation of β and t for {r, s} = {1, 0} is presented in table-I. For the set of values of (β, t) our models represent ΛCDM models, which are presented in table-I.

CONCLUSION

In this research, we explored a Bianchi type-I space-time in the context of f (R, T) gravity, which is both spatially homogenous and anisotropic. Some precise solutions of an anisotropic and homogeneous Bianchi type-I space-time have been investigated for the case when f (R, T) = R + 2 f1(T), where f1(T) = T. The topic of whether limits exist on the coupling parameter might be posed. It is fascinating to notice that solar system tests, which are based on the vacuum field equations (in which the energy momentum tensor is zero and hence T = 0 for the trace), impose no constraints on the value of. For a range of observational data and a change from deceleration to acceleration, Nagpal et al. have shown that the value = 65 permits structure creation. The following stronger constraint is the result of Bhattacharjee and Sahoo's investigation of big bang nucleosynthesis bounds in f (R, T) gravity. From the helium and deuterium abundances, we get 0.42 0.07. The Lithium issue is still there, just as it was with the classic model. The developed model might provide insight into f (R, T) cosmology. Two different cosmological models have been provided, one for each value of f(R, T).

REFERENCE

1. S.S. Nerkar et.al "Dark Energy Bianchi Type-III Cosmological Models in f(T) Theory of Gravity" Bulgarian Journal of Physics vol. 49 (2022) 209–218
2. Bishnu Prasad Brahma et.al "Bulk Viscous Bianchi Type-V Cosmological Model in f (R, T) Theory of Gravity" Volume 9 - 2022 | https://doi.org/10.3389/fspas.2022.831431
3. Rahaman, M., Singh, K.N., Errehymy, A. et al. Anisotropic Karmarkar stars in f (R, T)-gravity. Eur. Phys. J. C 80, 272 (2020). https://doi.org/10.1140/epjc/s10052-020-7842-9
4. S.K. Sahu et.al "Cosmic transit and anisotropic models in f(R,T) gravity" Chinese Journal of Physics Volume 55, Issue 3, June 2017, Pages 862-869
5. D. D. Pawar , V. J. Dagwal. (2014) Int. J. Theor. Phys, DOI 10.1007/s10773-014-2396-y.
6. S.Bhattacharaya,T.M. Karade, (1993) Astrophys. Space Sci. 202, 69 .
7. D.R.K.Reddy, M.V.S. Rao, (2006), ASS, 305, 183.
8. D.R.K Reddy,R.L.Naidu,K.S.Adhav (2007) Astrophys. Space Sci.307-365.
9. V. U. M. Rao and K. V. S. Sireesha,(2012) The African Review of Physics, vol. 7. Article ID 0042.
10. P.K. Sahoo, B. Mishra, (2013) International Journal of Pure and Applied Mathematics , 82 , 1 , 87-94.
11. Nawsad Ali (2013) J. Astrophys. Astr. 34, 259-271.
12. M. Visser, Cosmography: Cosmology without the Einstein equations, Gen. Relativ. Grav., 37 (2005) 1541-1548.
13. D. Rapetti, S. W. Allen, M. A. Amin, R. D. Blandford, A kinematical approach to dark energy studies, Mon. Not. R. Astron. Soc., 375 (2007) 1510-1520, arXiv:astroph/0605683.
14. A. G. Riess et al., Type Ia Supernova Discoveries at z > 1 From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution, Astrophys. J., 607 (2004) 665-687.
15. P. Astier et al., The Supernova Legacy Survey: measurement of ΩM, ΩΛ and ω from the first year data set, Astron. Astrophys., 447 (2006) 31-48.