Cable-Connected Satellites System In Elliptical Orbit

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Abstract
The present study aimed to investigate the general nature of satellite system and effect of magnetic field on the system.Two satellites are connected by a light, flexible and inextensible cable in the central gravitational field of the Earth. The equations of motion of the system have been deduced with respect to the centre of mass, which assumed to move along a low altitude equatorial orbit.

Keywords: Cable connected satellites, Magnetic field of earth, free motion.

Introduction

In the numerous analysis of a cable connected satellite system performed and attention has been focused primarily on post deployment behaviours. During deployment and retrieval less attention has been focused on the dynamics and control of the system.One of the main purpose of launching the artificial Earth satellite in orbit is the scientific exploration of the physical problems of the planets. Our country has successfully used her own satellite launcher for communication and weather forecasting purpose through undertaking space research program and launching many artificial Earth satellites. The dynamics of flexible satellites has attracted the attention of many space scientists for the last several years, as it has appeared mainly as a result of the development of gravity gradient stabilized satellites and the cable connected system. In the recent years there has been a tendency towards a very large and flexible space vehicles in response to various mission requirement.

For example, increasing demand of power for operation of the on-board instrumentation, scientific equipment's, communication systems etc. has been reflected in the size of the Solar panels. Cable connected systems in space are mathematical modeling of the real space problems such as space vehicle and astronaut floating in space**.** Two or multi-sectional satellite system connected by a cable manned space capsule attached to its booster by cable and spinning to provide artificial gravity for the astronaut and lastly two satellites at the time of rendezvous in order to transport a man successfully to an orbiting space station. Many space configurations of cable connected system have been proposed and analyzedin order to accomplish passive attitude stabilization in satellites and some of them have been flight tested as well. Recently, a successful attempt was made to launch two-cable connected bodies, a modified Agena vehicle and Gemini capsule in a gravity stabilized earth pointing attitude in consequence of NASA GEMINI programme. The need to account for the disturbance in the analysis of the stability of satellite liberation can be justified by the following Table-I, which shows the value of the relation, (f the Solar radiation pressure and T the atmospheric drag) depending upon the height h of the circular orbit above the Earth's surface.

Table - I

Height (h) of the circular orbit in Kms.	200	300	400	500	600	700	800
$\frac{f}{T}$	0.0002	0.0008	0.018	0.08	0.27	0.8	2.1

For the circular orbit of a satellite the height is above 500 kms. In practice most of the space station are orbiting round the Earth at a height above 700 kms and hence we are interested in studying the effect of Earth's magnetic field on the system.

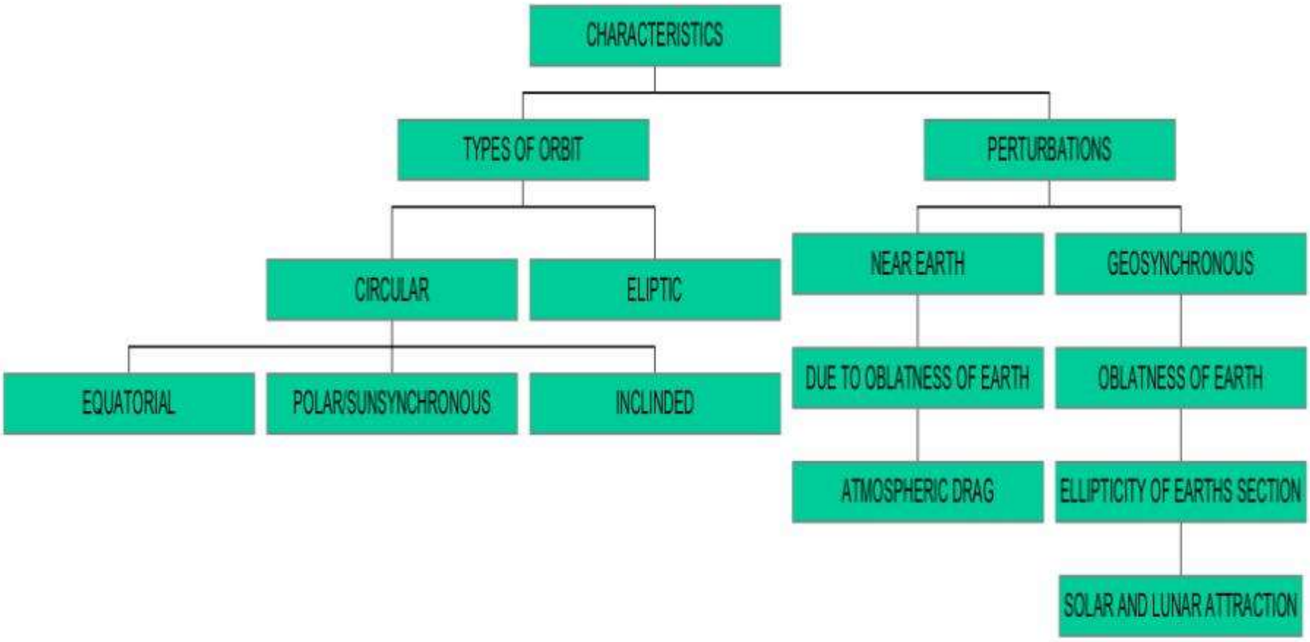
- The particles may move with loose cable may be called as free motion (i.e. when the equality sign doesn't occur in the equation of the constraint).In this case the motion of one of the satellite will be determined in conformity with the two body problem.
- The particles move with tight cable may be derived as 'constrained motion'. In this case, as soon as the cable becomes taut, the free motion of the satellite ceases and the system starts moving like a single body in its ensuing motion. This type of motion is called non-evolutional motion.

- The problem becomes complex due to the fact that the cable, which is assumed to be flexible may become loose after some time and the motion will take place alternately with loose and tight cable. Thus the particles may move in phase with alternately free and constrained motion. This type of motion is called evolutional motion.
- The perturbing force due to Earth's magnetic field results from the interaction between spacecraft's residue magnetic field and the geomagnetic field. The perturbing force is arising out due to magnetic moments eddy currents and hysterisisof these the spacecraft magnetic moment is usually the dominant source of disturbing effect.
- matical idealization of the above mentioned real space system has been achieved on the on the following natural and logical assumptions amounting to be abstractive of the problem.

The satellites have been considered as material particles moving in the gravitational field of the Earth. The Cable connecting satellites is assumed to be a light, flexible and inextensible.

Central gravitational force is considered as the main force governing the motion of the system..

Earth resource satellites where accurate mapping and swath on a day to day basis is desirable, near recursive sin synchronous orbits are taken. Near polar orbits are designed for meteorological studies. The exact planning of orbit, however, depends upon visibility, life time, thermal and power systems, Launching facilities, tracking, elementary and ground systems other than the mission objectives.



Equation of motion

The satellites are considered to be charged material particles and the motion of the system is studied relative to their centre of mass, under the assumption that the later moves along

Keplerian equatorial orbit. While investigating the relative motion of the system of two cable connected satellites, it is assumed that the particles are subjected to impacts of absolutely nonelastic in nature, when the cable is tightened up. Beletsky (1969) Beletesky and Novikova (1969) studied the motion of a system of two satellites connected by a light, flexible and inextensible string in the central gravitational field of force, relative to the centre of mass, which is assumed to move along a Keplerian elliptical orbit under the assumption that the two satellites are moving in the plane of motion of the centre of mass. The same problem in its general form was further investigated by Singh (1971, 1973).

Let the two satellites of the system be of masses m_1 and m_2 respectively, with their radii vectors $\vec{r}_1$ and $\vec{r}_2$ with respect to the earth's centre E, as shown in Fig. 3.1 and l be the length of the cable connecting them. The vector equations of motion of the system obtained with the aid of Lagrange's equations of motion of the first kind can be written as:

$$m_1\ddot{\vec{r}}_1 + m_1\mu\frac{\vec{r}_1}{r_1^{\,3}} + \lambda\left(\vec{r}_1 - \vec{r}_2\right) + 3\mu\frac{m_1k_2}{r_1^{\,5}}\vec{r}_1 = Q_1\left(\vec{r}_1 \Lambda \vec{H}\right)$$

$$m_2\ddot{\vec{r}}_2 + m_2\mu\frac{\vec{r}_2}{r_2^{\,3}} + \lambda\left(\vec{r}_2 - \vec{r}_1\right) + 3\mu\frac{m_2k_2}{r_2^{\,5}}\vec{r}_2 = Q_2\left(\vec{r}_2 \Lambda \vec{H}\right) \qquad \ldots (3.1)$$

Where the dot denotes differentiation with respect to t, □ denotes the Lagrange's multiplier and □denotes the product of the gravitational constant and the mass of the earth where *c* is the centre of mass of the system, H the intensity of the Earth's magnetic field for equatorial satellites.

$$\text{where Q} = \left[\frac{\text{charge } q_i \text{ of the ith particle}}{\text{velocity of light}}\right] \quad i = 1,\, 2;$$

Here, the Earth's potential for equatorial satellites is given as follows:

$$V = \frac{\mu}{\vec{R}} + \overline{\in}\,\frac{R_e^{\,2}\mu}{3R^3}$$

where *R* is the radius vector of the satellites from the attracting centre:

$$\overline{\in} = \alpha_R - \frac{m}{2} \quad ;$$

where $m = \dfrac{\omega^2 R_e}{g_e}$, where R_e is the equatorial radius of the Earth and g_e is the acceleration due to gravity at the equator of the Earth, where □$_R$ is the Oblateness of the Earth.

$$= \frac{R_e - R_p}{R_e} \;;$$

where R_p is the polar radius of the Earth:

$$K_2 = \frac{\overline{\in}\,R_e^{\,2}}{3}.$$

Let v be the true anomaly of the centre of mass of the system, □$_1$ *and* □$_2$are assumed to be the radius vectors of m_1 and m_2 respectively with respect to the centre of mass c. The constraint condition is given by:

$$\left|\vec{r}_1 - \vec{r}_2\right|^2 \le l^2 \;; \qquad \ldots (3.2)$$

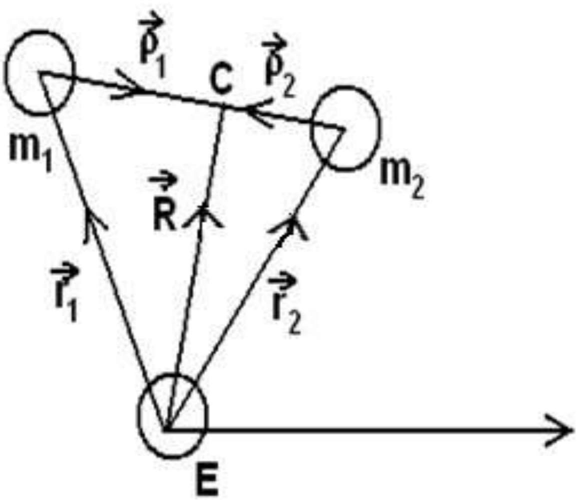


Fig. 3.1 The system under the combined influence of Earth magnetic field and Oblateness of the Earth

For free motion the strict inequality sign holds in (3.2), and hence □=0in equation (3.1). For the equality sign in (3.2), the motion of the system is under the active constraint and □ which is always positive must be determined in course of solving the problem.

Conclusion

It is assumed that the centre of mass of the system moves along Keplerian elliptical orbit in particular case this may be considered a circular one. As the general solution of the system of equation is beyond reach. Hence analyse the behaviour of the system in the neighbourhood of some particular solutions.

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