Fibonacci Face Prime Labeling On Wheel Related Graphs

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ABSTRACT
In this paper we introduce a new type of labeling, Fibonacci Face Prime Labeling. This work investigates some graphs, such as friendship graph, fan graph, wheel graph and flower graph are Fibonacci Face Prime graph.

KEYWORDS: Wheel graph, Fibonacci Face Prime Labeling.

1. INTRODUCTION AND PRELIMINARIES

This article deals with simple graphs. For notations and terminology we refer to Bondy and Murthy [1]. Fibonacci numbers were introduced by Leonardo of Pisa in his book Liber Abaci 1202. The vertices or edges or both of a graph are assigned by an integer based on some rule that is known as graph labeling, initiated by Rosa in 1967. Koh, Lee and Tan first introduced the labeling using Fibonacci numbers [2]. Roger Entringer introduced the Prime Labeling and discussed it by Tout [3]. The Fibonacci Prime Labeling was introduced by C. Sekar, S. Chandrakala [4]. They were proved cycle related graph, path related graphs are Fibonacci Prime graph. The (1,0,0)-F-Face magic mean labeling was introduced by A. Meena Kumari, S. Arockiaraj [5] and the above works motivates to introduce a new type of graph labeling.

2.MAIN RESULTS

Definition 2.1.
A graph $G(V,E)$ with order n and Let F be a face set of graph with more than two interior faces i.e.$|F| \geq 3$. An injective function $\theta^*: V(G) \to \{F_2, F_3, \dots, F_{n+1}\}$ such that an induced function $\theta': F(G) \to \mathbb{N}$ is defined by $\theta'\big(F(G)\big) = \gcd\,(\theta^*(v_i), \theta^*(v_{i+1})) \forall v_i \in F(G)$, that is the labels of any two vertices in the boundary of each interior face are relatively prime. A graph admits Fibonacci Face Prime labeling then the graph is called as Fibonacci Face Prime graph.

Theorem 2.2. The Friendship graph $Fd_n, n \geq 3$ is a Fibonacci Face Prime graph.
Proof. Let $Fd_n, n \geq 3$ be a Friendship graph with order $2n+1$, size 3n and f_i are interior faces of a Friendship graph with $|F(F_n)| = n$.
Let $V(Fd_n) = \{v_i / 0 \leq i \leq 2n\}$, $E(Fd_n) = \{v_0 v_i / 1 \leq i \leq 2n\} \cup \{v_i v_{i+1} / i = 1,3, \dots, 2n-1\}$ and $F(Fd_n) = \{f_i / 1 \leq i \leq n\}$ be the vertex set, edge set and face set of a Friendship graph $F_n, n \geq 2$.
An injective function $\theta^*: V(Fd_n) \to \{F_2, F_3, \dots, F_{2(n+1)}\}$ is defined by

$$\theta^*(v_i) = F_{i+2}, 0 \leq i \leq n$$

Then the induced function $\theta': F(Fd_n) \to \mathbb{N}$ is defined by

$$\theta'(f_i) = \gcd\big(\theta^*(v_{2i-1}), \theta^*(v_{2i})\big) = \gcd\big(\theta^*(v_0), \theta^*(v_{2i-1})\big) = \gcd\big(\theta^*(v_0), \theta^*(v_{2i})\big) = 1 \text{, where } 1 \leq i \leq n \text{ and } \forall f_i \in F(F_n)$$

Hence the Friendship graph Fd_n , $n \geq 2$ is a Fibonacci Face Prime graph.

Theorem 2.3. The Fan graph $F_n, n \geq 3$ is a Fibonacci Face Prime graph.
Proof. Let $F_n, n \geq 2$ be a fan graph with order n, size 2n-1 and f_i are interior faces of a fan graph with $|F(F_n)| = n$.
Let $V(F_n) = \{v_i / 0 \leq i \leq 2n\}$, $E(F_n) = \{v_0 v_i / 1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\}$ and $F(F_n) = \{f_i / 1 \leq i \leq n\}$ be the vertex set, edge set and face set of a fan graph $F_n, n \geq 2$.
An injective function $\theta^*: V(F_n) \to \{F_2, F_3, \dots, F_{n+1}\}$ is defined by $\theta^*(v_i) = F_{i+2}, 0 \leq i \leq n$
Then the induced function $\theta': F(F_n) \to \mathbb{N}$ is defined by

$$\theta'(f_i) = \gcd\big(\theta^*(v_i), \theta^*(v_{i+1})\big) = \gcd\big(\theta^*(v_0), \theta^*(v_{i+1})\big) = \gcd\big(\theta^*(v_0), \theta^*(v_i)\big) = 1 \text{, where } 1 \leq i \leq n \text{ and } \forall f_i \in F(F_n)$$

Hence the Fan graph F_n , $n \geq 2$ is a Fibonacci Face Prime graph.

Theorem 2.4. The Wheel graph $W_n, n \geq 3$ is a Fibonacci Face Prime graph.
Proof. Let $W_n, n \geq 3$ be a Wheel graph with order n+1 and f_i , where $1 \leq i \leq n$ are interior faces of a wheel graph with $|F(W_n)| = n$.
Let $(W_n) = \{w_i / 0 \leq i \leq n\}$, $E(W_n) = \{w_0 w_i / 1 \leq i \leq n\} \cup \{w_i w_{i+1} / 1 \leq i \leq n-1\} \cup \{w_n w_1\}$ and $F(W_n) = \{f_i / 1 \leq i \leq n\}$.

An injective function $\theta^*: V(W_n) \to \{F_2, F_3, \dots, F_{n+2}\}$ is defined by
Case 1. If $n \equiv 0(mod3)$ and $n \equiv 2(mod3)$

$$\theta^*(w_i) = F_{i+2}, \text{where } 0 \le i \le n$$

Then the induced function $\theta': F(G) \to \mathbb{N}$ is defined by
$\theta'(f_i) = \gcd(\theta^*(w_i), \theta^*(w_{i+1})) = \gcd(F_{i+2}, F_{i+3}) = 1$, where $1 \le i \le n-1$
$\theta'(f_i) = \gcd(\theta^*(w_0), \theta^*(w_i)) = \gcd(F_2, F_{i+2}) = 1$, where $1 \le i \le n$
$\theta'(f_n) = \gcd(\theta^*(w_0), \theta^*(w_n)) = \gcd(F_2, F_{n+2}) = 1$
$\theta'(f_n) = \gcd(\theta^*(w_1), \theta^*(w_n)) = \gcd(F_3, F_{n+2}) = 1$

Case 2. If $n \equiv 1(mod3)$

$$\theta^*(w_i) = F_{i+2}, \text{where } 0 \le i \le n-2;\ \theta^*(w_{n-1}) = F_{n+2};\ \theta^*(w_n) = F_{n+1}$$

Then the induced function $\theta': F(G) \to \mathbb{N}$ is defined by
$\theta'(f_i) = \gcd(\theta^*(w_i), \theta^*(w_{i+1})) = \gcd(F_{i+2}, F_{i+3}) = 1$, where $1 \le i \le n-3$
$\theta'(f_i) = \gcd(\theta^*(w_0), \theta^*(w_i)) = \gcd(F_2, F_{i+2}) = 1$, where $1 \le i \le n-2$
$\theta'(f_{n-2}) = \gcd(\theta^*(w_{n-1}), \theta^*(w_{n-2})) = \gcd(F_{n+2}, F_n) = 1$
$\theta'(f_{n-1}) = \gcd(\theta^*(w_0), \theta^*(w_{n-1})) = \gcd(F_2, F_{n+2}) = 1$
$\theta'(f_{n-1}) = \gcd(\theta^*(w_{n-1}), \theta^*(w_n)) = \gcd(F_{n+2}, F_{n+1}) = 1$
$\theta'(f_n) = \gcd(\theta^*(w_0), \theta^*(w_1)) = \gcd(F_2, F_3) = 1$
$\theta'(f_n) = \gcd(\theta^*(w_0), \theta^*(w_n)) = \gcd(F_2, F_{n+1}) = 1$
$\theta'(f_n) = \gcd(\theta^*(w_1), \theta^*(w_n)) = \gcd(F_3, F_{n+1}) = 1$
Therefore, $\theta'(F(W_n)) = \gcd\ (\theta^*(v_i), \theta^*(v_{i+1}))\forall v_i \in F(W_n)$.
Hence the wheel graph W_n , $n \ge 3$ is a Fibonacci Face Prime graph.

Theorem 2.5. The Flower graph $Fl_n, n \ge 3$ is a Fibonacci Face Prime graph.
Proof. Let $Fl_n, n \ge 3$ be a flower graph with order $2n+1$ and f_i are interior faces of a flower graph with $|F(Fl_n)| = 2n$.
Let $V(Fl_n) = \{w_i / 0 \le i \le 2n\}$, $E(Fl_n) = \{w_0 w_i /\ i\ = 1, 3, \dots, 2n-1\} \cup \{w_i w_{i+1} / 1 \le i \le 2n-1\} \cup \{w_i w_{i+2}\ /\ i\ = 1, 3, \dots, 2n-2\} \cup \{w_{2n-1} w_1\}$ and $F(Fl_n) = \{f_i / 1 \le i \le n\}$.
An injective function $\theta^*: V(Fl_n) \to \{F_2, F_3, \dots, F_{2(n+1)}\}$ is defined by
Case 1. If $n \equiv 0(mod3)$ and $n \equiv 2(mod3)$
$\theta^*(w_i) = F_{i+2}$, where $0 \le i \le 2n$
Then the induced function $\theta': F(Fl_n) \to \mathbb{N}$ is defined by
$\theta'(f_{2i-1}) = \gcd(\theta^*(w_0), \theta^*(w_{2i-1})) = \gcd(\theta^*(w_0), \theta^*(w_{2i})) = \gcd(\theta^*(w_{2i-1}), \theta^*(w_{2i}))$
$= \gcd(F_2, F_{2i+1}) = \gcd(F_2, F_{2i+2}) = \gcd(F_{2i+1}, F_{2i+2}) = 1$, where $1 \le i \le n$
$\theta'(f_{2i}) = \gcd(\theta^*(w_0), \theta^*(w_{2i-1})) = \gcd(\theta^*(w_0), \theta^*(w_{2i+1})) = \gcd(\theta^*(w_{2i-1}), \theta^*(w_{2i+1}))$
$= \gcd(F_2, F_{2i+1}) = \gcd(F_2, F_{2i+3}) = \gcd(F_{2i+1}, F_{2i+3}) = 1$, where $1 \le i \le n-1$
$\theta'(f_{2n}) = \gcd(\theta^*(w_0), \theta^*(w_{2n-1})) = \gcd(\theta^*(w_0), \theta^*(w_{2n+1})) = \gcd(\theta^*(w_{2n-1}), \theta^*(w_{2n+1}))$

$$= \gcd(F_2, F_{2n+1}) = \gcd(F_2, F_{2n+3}) = \gcd(F_{2n+1}, F_{2n+3}) = 1$$

Case 2. If $n \equiv 1(mod3)$
$\theta^*(w_i) = F_{i+2}$, where $1 \le i \le 2n-2$
$\theta^*(w_{2n-1}) = F_{2n+2}$; $\theta^*(w_{2n}) = F_{2n+1}$
Then the induced function $\theta': F(G_n) \to \mathbb{N}$ is defined by
$\theta'(f_{2i-1}) = \gcd(\theta^*(w_0), \theta^*(w_{2i-1})) = \gcd(\theta^*(w_0), \theta^*(w_{2i})) = \gcd(\theta^*(w_{2i-1}), \theta^*(w_{2i}))$
$= \gcd(F_2, F_{2i+1}) = \gcd(F_2, F_{2i+2}) = \gcd(F_{2i+1}, F_{2i+2}) = 1$, where $1 \le i \le n-1$
$\theta'(f_{2i}) = \gcd(\theta^*(w_0), \theta^*(w_{2i-1})) = \gcd(\theta^*(w_0), \theta^*(w_{2i+1})) = \gcd(\theta^*(w_{2i-1}), \theta^*(w_{2i+1}))$
$= \gcd(F_2, F_{2i+1}) = \gcd(F_2, F_{2i+3}) = \gcd(F_{2i+1}, F_{2i+3}) = 1$, where $1 \le i \le n-1$
$\theta'(f_{2n-1}) = \gcd(\theta^*(w_0), \theta^*(w_{2n-1})) = \gcd(\theta^*(w_0), \theta^*(w_{2n})) = \gcd(\theta^*(w_{2n-1}), \theta^*(w_{2n}))$

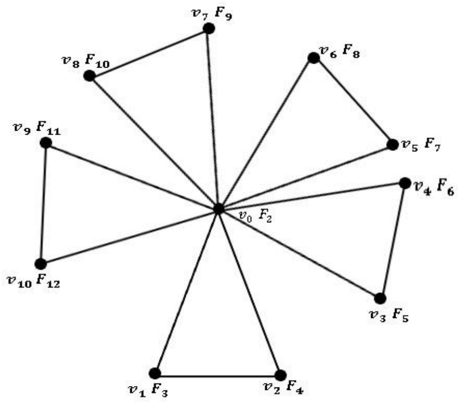
$$= \gcd(F_2, F_{2n+2}) = \gcd(F_2, F_{2n+1}) = \gcd(F_{2n+1}, F_{2n+2}) = 1$$

$\theta'(f_{2n}) = \gcd(\theta^*(w_0), \theta^*(w_{2n-1})) = \gcd(\theta^*(w_0), \theta^*(w_1)) = \gcd(\theta^*(w_{2n-1}), \theta^*(w_{2n+1}))$

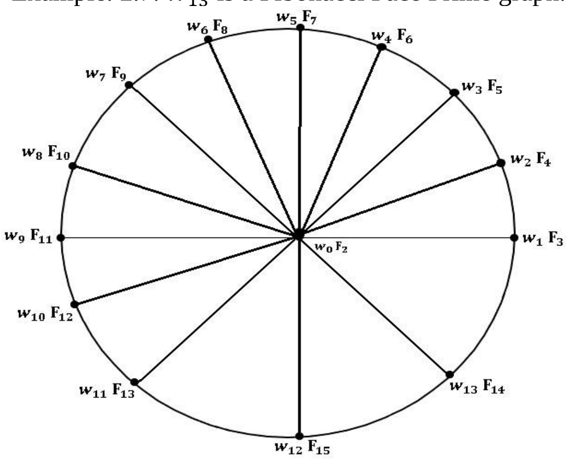
$$= \gcd(F_2, F_{2n+2}) = \gcd(F_2, F_3) = \gcd(F_{2n+2}, F_3) = 1$$

Therefore, $\theta'(F(Fl_n)) = \gcd(\theta^*(v_i), \theta^*(v_{i+1})) = 1 \forall v_i \in F(Fl_n)$.
Hence the flower graph Fl_n , $n \ge 3$ is a Fibonacci Face Prime graph.

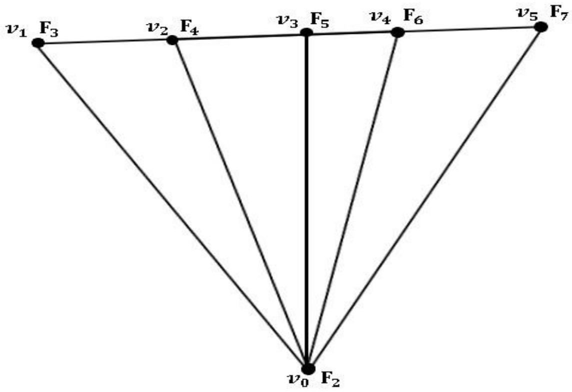
Example. 2.6. Fd_5 is a Fibonacci Face Prime graph.



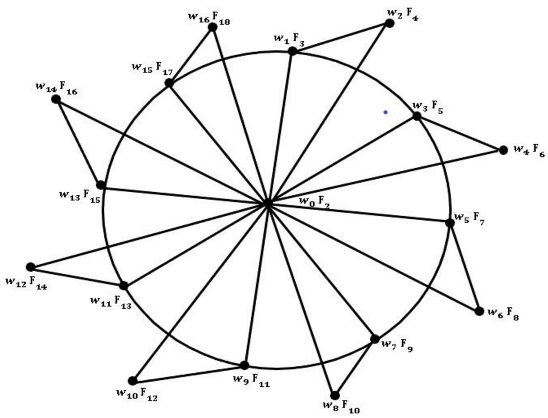
Example. 2.7. W_{13} is a Fibonacci Face Prime graph.



Example. 2.8. F_5 is a Fibonacci Face Prime graph.



Example. 2.9. Fl_8 is a Fibonacci Face Prime graph.



3. CONCLUSION

In this paper we investigate the fan graph, friendship graph, wheel graph, flower graph are Fibonacci Face Prime graph.

3. REFERENCE

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