Some ξ-Pre-Continuous Maps

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Abstract. In this paper the concept of ξ-pre-continuous and ξ-regular continuous maps in ξ-topological spaces are introduced and all the possible relationships of these maps have been discussed and established. Further we introduce and study ξ-pre-generalized closed sets and ξ-pre-generalized continuity in ξ-topological spaces and investigate various relationship by making the use of some counter examples

Keywords: ξ-regular-continuous maps, totally ξ-pre-continuous maps, strongly ξ-pre continuous maps, totally ξ-regular-continuous maps, strongly ξ-regular-continuous maps, ξ -pre-generalized closed, ξ- generalized-pre closed, ξ -pre-generalized continuous maps, ξ -irresolute, ξ -pre-generalized irresolute.

1 Introduction

In both the pure and applied domains, the importance of general topology is quickly increasing. Information systems are fundamental instruments for generating information understanding in any real-life sector, and topological information collection structures are appropriate mathematical models for both quantitative and qualitative information mathematics. Initially, Mashhour et. al. [21] introduced pre-open sets and pre-continuity in topology. Levine [18] introduced the class of generalized closed (g-closed) sets in topological spaces. The generalized continuity was studied in recent years by Balachandran et.al. Devi et.al and Maki et.al [5,10,20]. Regular open sets have been introduced and investigated by Stone [27]. Miguel Caldas and Cueva introduced and studied the concept of semi-generalized continuous maps in topological spaces [7]. The authors Arya,S. P., Gupta,R Anuradha, Baby Chacko and Singh D [2-3,28] introduced the concept of strongly continuous functions and almost perfectly continuous functions in topological spaces and established the various significant results. Benchalli S.S and Umadevi I Neeli Nour T.M [4, 26] studied the concept of totally semi-continuous functions and semi-totally continuous functions in topological spaces and verify the certain properties of the concept. Bhattacharya,S, [6] introduced and studied the concept of generalized regular closed sets and establish the various characterizations. Nithyanantha and Thangavelu [23] introduced the concept of binary topology between two sets and investigate some of the basic properties, where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. Jamal M. Mustafa [12] studied binary generalized topological spaces and investigate the various relationships of the maps so discussed with some other maps.

1.1 Contribution:

As outline, the concept ξ-pre-continuous and ξ-regular continuous map, ξ-pre-generalized closed sets, ξ-pre-generalized continuous maps and ξ-pre-irresolutes are introduced in ξ-topological spaces and investigate various relationships by making the use of some examples

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1.2 Organization

The rest of the paper structured as follows: Some require basic definitions, concepts of ξ-topological and notations are discussed in Section 2. In section 3, namely ***ξ-Pre-Continuous Maps*** we have introduced several maps and have discussed their relationships also. In section 4, headed by the concept of ***ξ-Regular Continuous Maps*** we introduced several maps and studied their relationships. In section 5, headed by the concept of ***ξ-Pre-Generalized Closed Sets and Maps*** we introduced several closed sets and their maps and verify their relationships. Finally, Section 6 concludes the paper with possible scope of the concept. Throughout the paper $\wp(\Upsilon)$ denotes the power set of Υ.

2. Preliminaries

Some require and important definitions and concepts of *ξ-topological space* and notations have been given in this portion

Definition 2.1: Let Υ_1 and Υ_2 be any two non-void sets. Then ξ-topology (ξ_T) from Υ_1 to Υ_2 is a binary structure $\xi \subseteq \wp(\Upsilon_1) \times \wp(\Upsilon_2)$ satisfying the conditions i.e. $(\emptyset, \emptyset)$, $(\Upsilon_1, \Upsilon_2) \in \xi$ and If $\{(L_\alpha, M_\alpha) ; \alpha \in \Gamma\}$ is a family of elements of ξ, then $(\bigcup_{\alpha\in\Gamma} L_\alpha , \bigcup_{\alpha\in\Gamma} M_\alpha) \in \xi$. If ξ is ξ_T from Υ_1 to Υ_2, then $(\Upsilon_1, \Upsilon_2, \xi)$ is called a ξ-topological space ($\xi_T S$) and the elements of ξ are called the ξ-open subsets of $(\Upsilon_1, \Upsilon_2, \xi)$. The elements of $\Upsilon_1 \times \Upsilon_2$ are called simply ξ-points.

Definition 2.2: Let Υ_1 and Υ_2 be any two non-void set and (L_1, M_1), (L_2, M_2) are the elements of $\wp(\Upsilon_1) \times \wp(\Upsilon_2)$. Then $(L_1, M_1) \subseteq (L_2, M_2)$ only if $L_1 \subseteq L_2$ and $M_1 \subseteq M_2$.

Remark 2.1: Let $\{T_\alpha ; \alpha \in \Lambda\}$ be the family of ξ_T from Υ_1 to Υ_2. Then, $\bigcap_{\alpha\in\Lambda} T_\alpha$ is also ξ_T from Υ_1 to Υ_2. Further $\bigcup_{\alpha\in\Lambda} T_\alpha$ need not be ξ_T.

Definition 2.3: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be a $\xi_T S$ and $L \subseteq \Upsilon_1, M \subseteq \Upsilon_2$. Then (L, M) is called ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$ if $(\Upsilon_1 \backslash L, \Upsilon_2 \backslash M) \in \xi$.

Proposition 2.1: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then (Υ_1, Υ_2) and $(\emptyset, \emptyset)$ are ξ-closed sets. Similarly if $\{(L_\alpha, M_\alpha): \alpha \in \Gamma\}$ is a family of ξ-closed sets, then $(\bigcap_{\alpha\in\Gamma} L_\alpha , \bigcap_{\alpha\in\Gamma} M_\alpha)$ is ξ-closed.

Definition 2.4: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Let ${(L, M)^{1^*}}_\xi = \bigcap\{L_\alpha: (L_\alpha , M_\alpha)$ is ξ-closed set and $(L, M) \subseteq (L_\alpha , M_\alpha)\}$ and ${(L, M)^{2^*}}_\xi = \bigcap\{M_\alpha: (L_\alpha , M_\alpha)$ is ξ-closed set and $(L, M) \subseteq (L_\alpha , M_\alpha)\}$.Then $({(L, M)^{1^*}}_\xi , {(L, M)^{2^*}}_\xi)$ is ξ-closed set and $(L, M) \subseteq ({(L, M)^{1^*}}_\xi , {(L, M)^{2^*}}_\xi)$. The ordered pair $\left(({(L, M)^{1^*}}_\xi , {(L, M)^{2^*}}_\xi)\right)$ is called *ξ-closure* of (L, M) and is denoted $Cl_\xi(L, M)$ in $\xi_T S$ (X, Y, μ) where $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$.

Proposition 2.2: Let $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Then (L, M) is ξ-open in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = I_\xi(L, M)$ and (L, M) is ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = Cl_\xi(L, M)$.

Proposition 2.3: Let $(L, M) \subseteq (N, P) \subseteq (\Upsilon_1, \Upsilon_2)$ and $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $Cl_\xi(\emptyset, \emptyset) = (\emptyset, \emptyset)$, $Cl_\xi(X, Y) = (X, Y)$, $(L, M) \subseteq Cl_\xi(L, M)$, ${(L, M)^{1^*}}_\xi \subseteq {(N, P)^{1^*}}_\xi$, ${(L, M)^{2^*}}_\xi) \subseteq {(N, P)^{2^*}}_\xi$, $Cl_\xi(L, M) \subseteq Cl_\xi(N, P)$ and $Cl_\xi(Cl_\xi(L, M)) = Cl_\xi(L, M)$

Definition 2.5: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Let ${(L, M)^{1^0}}_\xi = \bigcup\{L_\alpha: (L_\alpha , M_\alpha)$is ξ-open set and $(L, M) \subseteq (L_\alpha , M_\alpha)\}$ and ${(L, M)^{2^0}}_\xi = \bigcup\{M_\alpha: (L_\alpha , M_\alpha)$is ξ-open set and $(L, M) \subseteq (L_\alpha , M_\alpha)\}$.Then $({(L, M)^{1^0}}_\xi , {(L, M)^{2^0}}_\xi)$ is ξ-open set and $({(L, M)^{1^0}}_\xi , {(L, M)^{2^0}}_\xi) \subseteq (L, M)$. The ordered pair $\left(({(L, M)^{1^0}}_\xi , {(L, M)^{2^0}}_\xi)\right)$ is called *ξ-interior* of (L, M) and is denoted $I_\xi(L, M)$ in $\xi_T S$ (X, Y, μ) where $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$.

Proposition 2.4: Let $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Then (L, M) is ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = I_\xi(L, M)$.

Proposition 2.5: Let $(L, M) \subseteq (N, P) \subseteq (\Upsilon_1, \Upsilon_2)$ and $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $I_\xi(\emptyset, \emptyset) = (\emptyset, \emptyset)$, $I_\xi(X, Y) = (X, Y)$, ${(L, M)^{1^0}}_\xi \subseteq {(N, P)^{1^0}}_\xi$, ${(L, M)^{2^0}}_\xi \subseteq {(N, P)^{2^0}}_\xi$, $I_\xi(L, M) \subseteq I_\xi(N, P)$ and $I_\xi(I_\xi(L, M)) = I_\xi(L, M)$

Definition 2.6: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-continuous at $z \in Z$ if for any ξ-open set $(L, M) \in (\Upsilon_1, \Upsilon_2, \xi)$ with $\mathcal{F}(z) \in (L, M)$ then there exists $\mathcal{T}$-open G in $(Z, \mathcal{T})$ such that $z \in G$ and $\mathcal{F}(G) \subseteq (L, M)$. The mapping $\mathcal{F}$ is called ξ-continuous if it is ξ-continuous at each $z \in Z$.

Proposition 2.6: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-continuous if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

3. ξ-Pre-Continuous Maps (ξPCM)

In this section, the concept of *ξ-pre-continuous maps, totally ξ-pre-continuous maps* and *strongly ξ-pre-continuous maps* in $\xi_T S$ have been introduced and established the relationships between these maps and some other maps by making the use of some counter examples.

Definition 3.1: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $(L, M) \subseteq (\Upsilon_1, \Upsilon_2, \xi)$ is said to *ξ-pre-open set* (ξPOS) if $(L, M) \subseteq I_\xi(Cl_\xi(L, M))$. The complement of *ξ-pre-open set* is *ξ-pre-closed set* denoted as (ξPCS).
Definition 3.2: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called *ξ-pre-continuous map* (ξPCM) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-pre-open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Definition 3.3: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is said to be

i) Totally ξ-*continuous map* $(T\xi CM)$ if $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$ for every ξ-open set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.
ii) Totally ξ-*pre-continuous map* $(T\xi PCM)$ if $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}$-pre-clopen in $(\mathrm{Z},\mathcal{T})$ for every ξ-open set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.
iii) Strongly ξ-*continuous map* $(S\xi CM)$ if $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$ for every ξ-set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.
iv) Strongly ξ-*pre-continuous ma*$(S\xi PCM)p$ if $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}$-pre-clopen in $(\mathrm{Z},\mathcal{T})$ for every ξ-set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

Proposition 3.1:
i) Every strongly ξ-*continuous map* in ξ_TS is totally ξ-*continuous map*
ii) Every strongly ξ-*pre-continuous map* in ξ_TS is totally ξ-*pre-continuous map*
iii) Every totally ξ-*pre-continuous map* in ξ_TS is totally ξ-*continuous map*
Proof: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is ξ_TS and $(\mathrm{Z},\mathcal{T})$ be $\mathrm{G_T}$ and the map $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ is strongly ξ-*continuous map*. Therefore $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$. Thus for every ξ-open set (R,S), $\mathcal{F}^{-1}(\mathrm{R},\mathrm{S})$ is $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$. Hence $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ is totally ξ-*continuous map*. The proof of (ii) and (iii) are quite analogous.

Remark 3.1: The converse of Proposition 3.1 need not be true shown in Example 3.1, Example 3.2 and Example 3.3.

Example 3.1: Let $\mathrm{Z}=\{1,2,3\}$, $\Upsilon_1=\{\mathrm{m}_1,\mathrm{m}_2\}$ and $\Upsilon_2=\{\mathrm{l}_1,\mathrm{l}_2\}$. Then $\mathcal{T}=\{\emptyset,\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\mathrm{Z}\}$ and $\xi=\{(\emptyset,\emptyset),(\{\mathrm{m}_1\},\{\mathrm{l}_2\}),(\{\mathrm{m}_2\},\{\Upsilon_2\}),(\Upsilon_1,\Upsilon_2)\}$. Clearly $\mathcal{T}$ is $\mathrm{G_T}$ on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ by $\mathcal{F}(1)=(\mathrm{m}_2,\emptyset)=\mathcal{F}(3)$ and $\mathcal{F}(2)=(\emptyset,\mathrm{l}_2)$. Therefore $\mathcal{F}^{-1}(\emptyset,\ \emptyset)=\emptyset$, $\mathcal{F}^{-1}(\{\mathrm{m}_1\},\{\mathrm{l}_1\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\mathrm{m}_2\},\{\Upsilon_2\})=\{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2)=\mathrm{Z}$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$. Hence $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ is totally ξ-*continuous map* but not strongly ξ-*continuous map* because $\mathcal{F}^{-1}(\{\mathrm{m}_2\},\{\emptyset\})=\{1,3\}$ and $\mathcal{F}^{-1}(\{\emptyset\},\{\mathrm{l}_2\})=\{2\}$, where $\{1,3\}$ and $\{2\}$ are not $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$.

Example 3.2: In Example 3.1 the $\mathcal{T}$-pre-clopen in $(\mathrm{Z},\mathcal{T})$ are $\emptyset,\{1\},\{3\},\{1,2\},\{2,3\}$ and Z. Now $\mathcal{F}^{-1}(\emptyset,\ \emptyset)=\emptyset$, $\mathcal{F}^{-1}(\{\mathrm{m}_1\},\{\mathrm{l}_2\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\mathrm{m}_2\},\{\Upsilon_2\})=\{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2)=\mathrm{Z}$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-pre-clopen in $(\mathrm{Z},\mathcal{T})$. Hence $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ is totally ξ-*pre-continuous map* but not strongly ξ-*pre-continuous map* because $\mathcal{F}^{-1}(\{\mathrm{m}_2\},\{\emptyset\})=\{1,3\}$ and $\mathcal{F}^{-1}(\{\emptyset\},\{\mathrm{l}_2\})=\{2\}$, where $\{2\}$ and $\{1,3\}$ are not $\mathcal{T}$-pre-clopen in $(\mathrm{Z},\mathcal{T})$.

Example 3.3: Let $\mathrm{Z}=\{1,2,3\}$, $\Upsilon_1=\{\mathrm{m}_1,\mathrm{m}_2\}$ and $\Upsilon_2=\{\mathrm{l}_1,\mathrm{l}_2\}$. Then $\mathcal{T}=\{\emptyset,\{1\},\{1,2\},\{2,3\},\mathrm{Z}\}$ and $\xi=\{(\emptyset,\emptyset),(\{\mathrm{m}_1\},\{\mathrm{l}_2\}),(\{\mathrm{m}_2\},\{\Upsilon_2\}),(\Upsilon_1,\Upsilon_2)\}$. Clearly $\mathcal{T}$ is $\mathrm{G_T}$ on Z and ξ is ξ_T from Υ_1 to Υ_2. The$\mathcal{T}$-pre-clopen in $(\mathrm{Z},\mathcal{T})$ are $\emptyset,\{1\},\{2\},\{1,3\},\{2,3\}$ and Z. Now define $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ by $\mathcal{F}(1)=(\mathrm{m}_1,\mathrm{l}_2)=\mathcal{F}(3)$ and $\mathcal{F}(2)=(\emptyset,\mathrm{l}_2)$. Therefore $\mathcal{F}^{-1}(\emptyset,\ \emptyset)=\emptyset$, $\mathcal{F}^{-1}(\{\mathrm{m}_1\},\{\mathrm{l}_2\})=\{1,3\}$, $\mathcal{F}^{-1}(\{\mathrm{m}_2\},\{\Upsilon_2\})=\{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2)=\mathrm{Z}$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-pre-clopen in $(\mathrm{Z},\mathcal{T})$. Hence $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ is totally ξ-*pre-continuous map* but not ξ-*continuous map* because $\{1,3\}$ is $\mathcal{T}$-pre-clopen but not $\mathcal{T}$-open in $(\mathrm{Z},\mathcal{T})$.

Proposition 3.2:
i) Every ξ-*continuous map* in ξ_TS is totally ξ-*pre-continuous map*
ii) Every totally ξ-*continuous map* in ξ_TS is totally ξ-*pre-continuous map*
iii) Every strongly ξ-*continuous map* in ξ_TS is strongly ξ-*pre-continuous map*
Proof: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is ξ_TS and $(\mathrm{Z},\mathcal{T})$ be $\mathrm{G_T}$ and the map $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ is ξ-*continuous map*. Therefore $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}$-open in $(\mathrm{Z},\mathcal{T})$ for every ξ-open set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$. Since every $\mathcal{T}$-open is $\mathcal{T}$-pre-open in $(\mathrm{Z},\mathcal{T})$. Therefore, $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ is totally ξ-*pre-continuous map*. The proof of (ii) and (iii) are quite analogous.

Remark 3.2: The converse of Proposition 3.2 need not be true shown in Example 3.5, Example 3.6 and Example 3.7.

Example 3.5: Let $\mathrm{Z}=\{1,2,3\}$, $\Upsilon_1=\{\mathrm{m}_1,\mathrm{m}_2\}$ and $\Upsilon_2=\{\mathrm{l}_1,\mathrm{l}_2\}$. Then $\mathcal{T}=\{\emptyset,\{1,2\},\{2,3\},\mathrm{Z}\}$ and $\xi=\{(\emptyset,\emptyset),(\{\mathrm{m}_1\},\{\mathrm{l}_2\}),(\{\mathrm{m}_2\},\{\Upsilon_2\}),(\Upsilon_1,\Upsilon_2)\}$. Clearly $\mathcal{T}$ is $\mathrm{G_T}$ on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ by $\mathcal{F}(1)=(\mathrm{m}_1,\mathrm{l}_2)=\mathcal{F}(3)$ and $\mathcal{F}(2)=(\mathrm{m}_2,\mathrm{l}_1)$. Therefore $\mathcal{F}^{-1}(\emptyset,\ \emptyset)=\emptyset$, $\mathcal{F}^{-1}(\{\mathrm{m}_1\},\ \{\mathrm{l}_2\})=\{1,3\}$, $\mathcal{F}^{-1}(\{\mathrm{m}_2\},\{\Upsilon_2\})=\{2\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2)=\mathrm{Z}$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-pre-open in $(\mathrm{Z},\mathcal{T})$. Hence $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ is ξ-*pre-continuous map* but not ξ-*continuous map* because $\{1,3\}$ and $\{2\}$ are not $\mathcal{T}$-open in $(\mathrm{Z},\mathcal{T})$.

Example 3.6: In Example 3.5, the $\mathcal{T}$-pre-clopen in $(\mathrm{Z},\mathcal{T})$ are $\emptyset$, $\{2\},\{1,3\}$ and Z. Now $\mathcal{F}^{-1}(\emptyset,\ \emptyset)=\emptyset$, $\mathcal{F}^{-1}(\{\mathrm{m}_1\},\{\mathrm{l}_2\})=\{1,3\}$, $\mathcal{F}^{-1}(\{\mathrm{m}_2\},\{\Upsilon_2\})=\{2\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2)=\mathrm{Z}$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-pre-clopen in $(\mathrm{Z},\mathcal{T})$. Hence $\mathcal{F}:(\mathrm{Z},\mathcal{T})\to\Upsilon_1\times\Upsilon_2$ is totally ξ-*pre-continuous map* but not totally ξ-*continuous map* because $\{1,3\}$ and $\{2\}$ are not $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$.

Example 3.7: In Example 3.5, the $\mathcal{T}$-clopen sets in $(\mathrm{Z},\mathcal{T})$ are $\emptyset$, $\{2\},\{1,3\}$ and Z.. Now $\mathcal{F}^{-1}(\emptyset,\ \emptyset)=\emptyset$, $\mathcal{F}^{-1}(\{\mathrm{m}_1\},\{\mathrm{l}_1\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\mathrm{m}_1\},\ \{\Upsilon_2\})=\{1,3\}$, $\mathcal{F}^{-1}(\{\mathrm{m}_2\},\ \{\Upsilon_2\})=\{2\}$, $\mathcal{F}^{-1}(\{\emptyset\},\{\mathrm{l}_1\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\},\{\mathrm{l}_2\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\},\ \{\Upsilon_2\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\mathrm{m}_1\},\ \{\emptyset\})=\{\emptyset\}$, $\mathcal{F}^{-1}(\{\mathrm{m}_1\},\{\ \mathrm{l}_2\})=\{1,3\}$, $\mathcal{F}^{-1}(\{\mathrm{m}_1\},\{\Upsilon_2\})=\{1,3\}$, $\mathcal{F}^{-1}(\{\mathrm{m}_2\},\emptyset)=$

$\{\emptyset\}$, $\mathcal{F}^{-1}(\{m_2\},\{l_1\}) = \{2\}$, $\mathcal{F}^{-1}(\{m_2\},\{l_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\}, \{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{l_1\}) = \{2\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{l_2\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1, \Upsilon_2) = Z$. This shows that the inverse image of every ξ-set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-pre-clopen in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is strongly *ξ-pre-continuous map* but not strongly *ξ-continuous map* because $\{1,3\}$ and $\{2\}$ are not $\mathcal{T}$-open in $(Z, \mathcal{T})$.

Relationships of Various ξ -continuous maps that we discussed in this section:

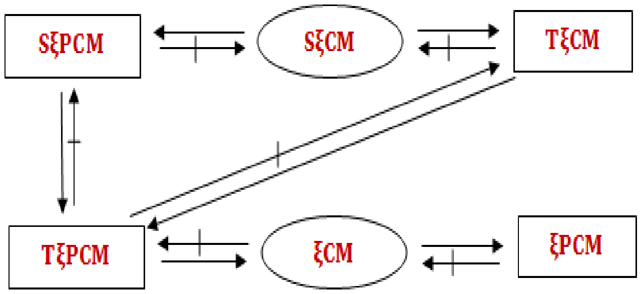

Figure-1

4. ξ-Regular Continuous Maps (ξRCM)

In this section, we have introduced and studied the concepts of *ξ-regular-continuous maps*, *totally ξ-regular-continuous maps and strongly ξ-regular-continuous maps.* Further, the relationships of these maps with some other maps have been established by making the use of some counter examples.

Definition 4.1: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $(L, M) \subseteq (\Upsilon_1, \Upsilon_2, \xi)$ is said to *ξ-regular-open set* (ξROS) if $(L, M) = I_\xi(Cl_\xi(L, M))$. The complement of *$\xi$-regular-open set* is *$\xi$-regular-closed set* denoted as (ξRCS).

Definition 4.2: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is said to be

i) *ξ-regular-continuous map* (ξRCM) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-regular-open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

ii) Totally *ξ-regular-continuous map* $(T\xi RCM)$ if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-regular-clopen in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

iii) Strongly *ξ-regular-continuous map* $(S\xi RCM)$ if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-regular-clopen in $(Z, \mathcal{T})$ for every ξ-set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Proposition 4.1:

i) Every strongly *ξ-regular-continuous map* in $\xi_T S$ is strongly *ξ-continuous map*

ii) Every totally *ξ-regular-continuous map* in $\xi_T S$ is totally *ξ-continuous map*

iii) Every *ξ-regular-continuous map* in $\xi_T S$ is *ξ-continuous map*

Proof: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T and the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is strongly *ξ-regular-continuous map*. Therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-regular-clopen in $(Z, \mathcal{T})$. Since every $\mathcal{T}$-regular-clopen is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$. Thus $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is strongly *ξ-continuous map*. The proof of (ii) and (iii) are quite analogous.

Remark 4.1: The converse of (iii) in Proposition 4.1 is not true seen in Example 4.1.

Example 4.1: Let $Z = \{1, 2, 3, 4\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{2\}, \{2,3\}, \{3,4\}, \{2,3,4\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\},\{l_1\}), (\{m_1\},\{\Upsilon_2\}), (\{m_2\},\{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(2) = (m_1, l_1) = \mathcal{F}(3)$ and $\mathcal{F}(1) = \mathcal{F}(4) = (m_1, l_2)$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{l_1\}) = \{2,3\}$, $\mathcal{F}^{-1}(\{m_1\},\{\Upsilon_2\}) = \{2,3\}$, $\mathcal{F}^{-1}(\{m_2\},\{\Upsilon_2\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1, \Upsilon_2) = Z$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-open in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is *ξ-continuous map* but not *ξ-regular-continuous map* because $\{2,3\}$ is not $\mathcal{T}$-regular-open in $(Z, \mathcal{T})$.

Proposition 4.2:

i) Every strongly *ξ-regular-continuous map* in $\xi_T S$ is totally *ξ-regular-continuous map*

ii) Every totally *ξ-regular-continuous map* in $\xi_T S$ is *ξ-regular-continuous map*

Proof: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T and the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is strongly *ξ-regular-continuous map*. Therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-regular-clopen in $(Z, \mathcal{T})$ for every ξ-set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$. Thus for every ξ-open set (R, S), $\mathcal{F}^{-1}(R, S)$ is $\mathcal{T}$-regular-clopen in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is totally *ξ-regular-continuous map*. The proof of (ii) and (iii) are quite analogous.

Relationships of Various ξ -continuous maps that we discussed in this section:

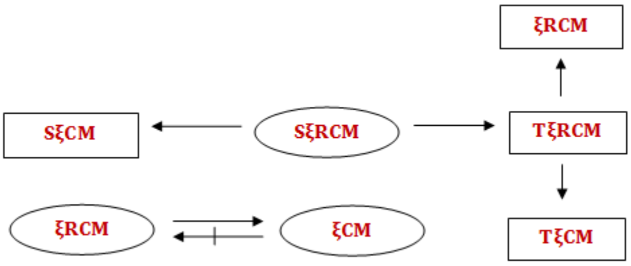

Figure-2

5. ξ-Pre-Generalized Closed Sets and Maps

In this section, we have introduced and studied the concepts of *ξ-pre-generalized closed set*, *ξ-generalized pre-closed set*, *ξ-pre-generalized maps and ξ-pre-irresolutes.* Further, the relationships of these maps with some other maps have been established by making the use of some counter examples.

Definition 5.1: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $(L, M) \subseteq (\Upsilon_1, \Upsilon_2, \xi)$ is said to be

i) *ξ-semi-open set* (ξSOS) if $(L, M) \subseteq Cl_\xi(I_\xi(L, M))$

ii) *ξ-pre-open set* (ξPOS) if $(L, M) \subseteq I_\xi(Cl_\xi(L, M))$.

iii) *ξ-α-open set* ($\xi\alpha OS$) if $(L, M) \subseteq I_\xi(Cl_\xi\left(I_\xi(L, M)\right))$.

Definition 5.2: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2, \xi)$, then $pCl_\xi(L, M) = (L, M) \cup Cl_\xi\left(I_\xi(L, M)\right)$

Definition 5.3: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2, \xi)$, then

i) (L, M) is*ξ-pre-generalized closed set* ($\xi PGCS$) if $pCl_\xi(L, M) \subseteq (U, V)$ whenver $(L, M) \subseteq (U, V)$ and (U, V) is ξ-pre-open set in $(\Upsilon_1, \Upsilon_2, \xi)$

ii) (L, M) is*ξ-generalized pre-closed set* ($\xi GPCS$) if $pCl_\xi(L, M) \subseteq (U, V)$ whenver $(L, M) \subseteq (U, V)$ and (U, V) is ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$

iii) (L, M) is *ξ^*-closed set*($\xi^* CS$) if $Cl_\xi(L, M) \subseteq (U, V)$ whenver $(L, M) \subseteq (U, V)$ and (U, V) is ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$

Proposition 5.1: Every *ξ-generalized pre-closed set* in $\xi_T S$ *ξ-pre-generalized closed*

Proof: Follows from definition

Remark 5.1: The Converse of Proposition 5.1 is not true in general shown in Example 5.1.

Example 5.1: Let $\Upsilon_1 = \{m_1, m_2, m_3\}$ and $\Upsilon_2 = \{l_1, l_2, l_3\}$. Then $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1, l_2\}), (\{m_2, m_3\}, \{l_3\}), (\{l_1, l_3\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly ξ is ξ_T from Υ_1 to Υ_2. Now consider, $(\{m_1, m_2\}, \{l_1, l_2\}) \subseteq (\{m_1, m_2\}, \{\Upsilon_2\})$. Therefore $pCl_\xi(\{m_1, m_2\}, \{l_1, l_2\}) = (\{m_1, m_2\}, \{l_1, l_2\}) \subseteq (\{m_1, m_2\}, \{\Upsilon_2\})$, where $(\{m_1, m_2\}, \{\Upsilon_2\})$ is ξ -pre-open. Therefore $(\{m_1, m_2\}, \{l_1, l_2\})$ is ξ-pre-generalized closed but not ξ-generalized pre-closed because $(\{m_1, m_2\}, \{\Upsilon_2\})$ is ξ-pre-open but not ξ-open.

Proposition 5.2: Every *$\mathcal{T}$-pre-closed set* in $\xi_T S$ is *$\mathcal{T}$-pre-generalized closed*

Proof: Obvious

Remark 5.2: The converse of Proposition 5.2 is not true in general shown in Example 5.2.

Example 5.2: Let $Z = \{1, 2, 3,4\}$. Then $\mathcal{T} = \{\{\emptyset, \{1\}, \{1,2\}, \{1,2,3\}, \{3,4\}, \{1,3,4\} Z\}$ is G_T on Z. Consider the set $\{1,3\} \subseteq \{1,2,3\}$. Therefore $p - \mathcal{T}_g(\{1,3\}) = \{1,2,3\} \subseteq \{1,2,3\}$, where $\{1,2,3\}$ is $\mathcal{T}$-pre-open. Therefore the set $\{1,3\}$ is $\mathcal{T}$-pre-generalized closed but not $\mathcal{T}$-pre-closed.

Remark 5.3: In general ξ^*-closed set and ξ-pre-generalized closed set in $\xi_T S$ are independent shown in Example 5.3 and Example 5.4.

Example 5.3: Let $\Upsilon_1 = \{m_1, m_2, m_3\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\xi = \{(\emptyset, \emptyset), (\{\emptyset\}, \{l_2\}) , (\{\Upsilon_1\}, \{l_1\}), (\{m_1, m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)$ is ξ_T from Υ_1 to Υ_2. Clearly the sets $(\emptyset, \emptyset)$, $(\{\Upsilon_1\}, \{l_1\}), (\{\emptyset\}, \{l_2\})$ and (Υ_1, Υ_2) are ξ-closed sets in $(\Upsilon_1, \Upsilon_2, \xi)$. Let $(\{m_2\}, \{\Upsilon_2\}) \in \wp(\Upsilon_1) \times \wp(\Upsilon_2)$. Then $Cl_\xi(\{m_2\}, \{\Upsilon_2\}) = (\Upsilon_1, \Upsilon_2) \subseteq (\Upsilon_1, \Upsilon_2)$ where $(\{m_2\}, \{\Upsilon_2\}) \subseteq (\Upsilon_1, \Upsilon_2)$ and (Υ_1, Υ_2) is ξ-open. Therefore the set $(\{m_2\}, \{\Upsilon_2\})$ is ξ^*-closed set but not ξ-pre-generalized closed set because $(\{m_2\}, \{\Upsilon_2\}) \subseteq (\{m_1, m_2\}, \{\Upsilon_2\})$ and $pCl_\xi(\{m_2\}, \{\Upsilon_2\}) = (\Upsilon_1, \Upsilon_2) \nsubseteq (\{m_1, m_2\}, \{\Upsilon_2\})$, where $(\{m_1, m_2\}, \{\Upsilon_2\})$ is ξ-pre-open.

Example 5.4: Let $\Upsilon_1 = \{m_1, m_2, m_3\}$ and $\Upsilon_2 = \{l_1, l_2, l_3\}$. Then $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1, l_2\}), (\{m_2, m_3\}, \{l_3\}), (\{m_1, m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$ is ξ_T from Υ_1 to Υ_2. Consider the set $(\{m_1, m_3\}, \{l_1, l_2\}) \subseteq (\{m_1, m_3\}, \{\Upsilon_2\})$. Therefore $pCl_\xi(\{m_1, m_3\}, \{l_1, l_2\}) = (\{m_1, m_3\}, \{l_1, l_2\}) \subseteq (\{m_1, m_3\}, \{\Upsilon_2\})$, where $(\{m_1, m_3\}, \{\Upsilon_2\})$ is ξ -pre-open. Therefore $(\{m_1, m_3\}, \{l_1, l_2\})$ is ξ-pre-generalized closed set open but not ξ^*-closed set because $Cl_\xi(\{m_1, m_3\}, \{l_1, l_2\}) = (\Upsilon_1, \Upsilon_2) \nsubseteq (\{m_1, m_3\}, \{\Upsilon_2\})$ where $(\{m_1, m_3\}, \{l_1, l_2\}) \subseteq (\{m_1, m_3\}, \{\Upsilon_2\})$ and $(\{m_1, m_3\}, \{\Upsilon_2\})$ is ξ-open.

Definition 5.4: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is said to be
i) *ξ-pre-generalized continuous map ($\xi PGCM$)* $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$ -pre-generalized closed in $(Z, \mathcal{T})$ for every ξ-closed set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.
ii) *ξ-pre-irresolute (ξPI)* $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$ -pre-closed in $(Z, \mathcal{T})$ for every ξ-pre-closed set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.
iii) *ξ-pre-generalized irresolute (ξPGI)* $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$ -pre-generalized closed in $(Z, \mathcal{T})$ for every ξ-pre-generalized closed set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Proposition 5.3: Every *ξ-pre-continuous map* in $\xi_T S$ is *ξ-pre-generalized continuous*

Proof: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T and the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is *ξ-pre-continuous map*. Therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-pre-closed in $(Z, \mathcal{T})$ for every ξ-closed set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$. Since every $\mathcal{T}$-pre-closed is $\mathcal{T}$-pre-generalized closed in $(Z, \mathcal{T})$. Thus $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-pre-generalized closed in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is *ξ-pre-generalized continuous map*.

Remark 5.4: The Converse of Proposition 5.3 is not true in general shown in Example 5.5.

Example 5.5: Let $Z = \{1, 2, 3,4\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{1\}, \{1,2\}, \{1,2,3\}, \{3,4\}, \{1,3,4\}\ Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_1) = \mathcal{F}(3)$ and $\mathcal{F}(2) = (m_2, \emptyset)$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{l_1\}) = \{1,3\}$, $\mathcal{F}^{-1}(\{m_2\}, \{\Upsilon_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_1\}, \{\emptyset\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1, \Upsilon_2) = Z$. This shows that the inverse image of every ξ-closed set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-pre-generalized closed in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is *ξ-pre-generalized continuous map* but not *ξ-pre-continuous map* because $\{1,3\}$ is not $\mathcal{T}$-pre-closed in $(Z, \mathcal{T})$.

Proposition 5.4: Every *ξ-pre-continuous map* in $\xi_T S$ is *ξ-pre-generalized irresolute.*

Proof: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$ be G_T and the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is *ξ-pre-continuous map*. Therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-pre-closed in $(Z, \mathcal{T})$ for every ξ-closed set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$. Since every $\mathcal{T}$-pre-closed is $\mathcal{T}$-pre-generalized closed in $(Z, \mathcal{T})$ and like wise every ξ-pre-closed set is ξ-pre-generalized closed set in $(\Upsilon_1, \Upsilon_2, \xi)$. Thus $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-pre-generalized closed in $(Z, \mathcal{T})$ for every ξ-pre-generalized closed set in $(\Upsilon_1, \Upsilon_2, \xi)$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is *ξ-pre-generalized irresolute*.

Remark 5.5: The Converse of Proposition 5.4 is not true in general which can be easily seen from Example 5.5.

Proposition 5.5: Every *ξ-pre-irresolute* in $\xi_T S$ is *ξ-pre-generalized irresolute.*
Proof: Follows from definition, while the converse need not be true in general shown in Example 5.6.

Example 5.6: In Example 5.5, $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is *ξ-pre-generalized irresolute* but not *ξ-pre-irresolute*.

Relationships of Various ξ -continuous maps that we discussed in this section:

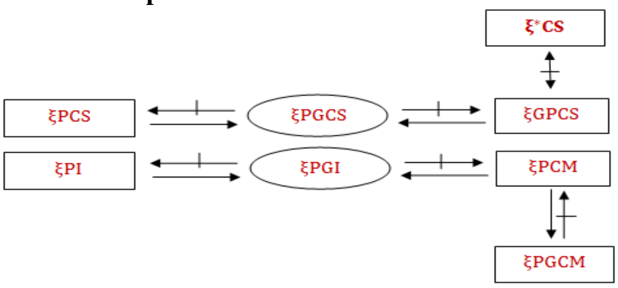

Figure-3

6. Conclusion

In this paper, a very useful concept of *ξ-pre-continuous maps*, *totally ξ-pre-continuous maps* and *strongly ξ-pre-continuous maps* in ξ-topological spaces have been introduced and established the relationships between these maps and some other maps. Further the concepts of *ξ-regular-continuous maps*, *totally ξ-regular-continuous maps and strongly ξ-regular-continuous maps* have been introduced along with some concepts of *ξ-pre-generalized closed set*, *ξ-generalized* *pre-closed set, ξ-pre-generalized maps and ξ-pre-irresolutes with* the relationships of these particular types of sets and maps in ξ-topological spaces. All the relationships have been verified by making the use of some examples.

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