$\mathbf{Generalized}_{\mathcal{F}} - \mathbf{open\ sets}$

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Abstract. In this paper we defined and characterized the concept of generalized fuzzy open sets ($\text{generalized}_{\mathcal{F}} -$ open sets) and obtained some significant results in this context with help of various supporting examples.

Key words: Fuzzy open set, fuzzy topological space, $\mathbf{generalized}_{\mathcal{F}} - \mathbf{topological\ space}$

1. Introduction

Azad [1] has introduced the concept of fuzzy semi-open sets in fuzzy topological spaces. Bin Shahana [3-4] has introduced the concept of fuzzy pre-open sets and fuzzy α-open sets in fuzzy topological space. Thakur [13] has introduced the concept of fuzzy semi pre-open sets in fuzzy topological spaces. Beceren [2] introduced and studied the concept of strongly α-continuous functions, strong semi-continuity and fuzzy pre-continuity and investigate various characterizations. Further the author verified that fuzzy strongly α-continuous map is the stronger form of fuzzy α-continuous map. Csaszar [6] introduced the notions of generalized topological spaces. He also introduced the notions of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. Palani Cheety [8] introduced the concept of generalized fuzzy topology and investigates various properties.

In this paper, we have introduced the concept of $\text{generalized}_{\mathcal{F}} -$ open sets in $\text{generalized}_{\mathcal{F}} -$topological space and verify the results with the help of some counter examples.

Some require basic definitions, concepts of $\text{generalized}_{\mathcal{F}} -$ topological space and notations are discussed in Section 2. In section 3, we introduced the concept of $\text{generalized}_{\mathcal{F}} -$Open Sets and established several results. Finally, Section 4 concludes the paper.

2. Preliminaries

Definition 2.1: Let X be a crisp set and let μ be a collection of fuzzy sets on X. Then μ is called $\text{generalized}_{\mathcal{F}} -$ topologyon X if it satisfies following conditions

i) The fuzzy sets 0 and 1 are in μ where $0,1 : X \to I$ are defined as $0(x) = 0$ and $1(x) = 1$ for all $x \in X$

ii) If $\{\lambda_j\}, j \in J$ is any family of fuzzy sets on X where $\lambda_j \in \mu$ then $\cup_{j\in J} \lambda_j \in \mu$

The pair (X, μ) is called $\text{generalized}_{\mathcal{F}} - \text{topological}\ \mathcal{S}\text{pace}$

Definition 2.2: Let (X, μ) be $\text{generalized}_{\mathcal{F}} - \text{topological}\ \mathcal{S}\text{pace}$. The members of the collection μ are called $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ in$\text{generalized}_{\mathcal{F}} - \text{topological}\ \mathcal{S}\text{pace}$. The complement of $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ in X is called $\text{generalized}_{\mathcal{F}} - \mathcal{C}\text{lose}\ \mathcal{S}\text{et}$

Definition 2.3: Let (X, μ) be $\text{generalized}_{\mathcal{F}} - \text{topological}\ \mathcal{S}\text{pace}$. For a fuzzy set A in X the $\mathcal{C}$losure of A is defined as $\text{Cl}_{\mu}(A) = \inf\{K : A \subseteq K, K^C \in \mu\}$. Thus $\text{Cl}_{\mu}(A)$ is the smallest $\mathcal{C}\text{losed}\ \mathcal{S}\text{et}$ in X containing the fuzzy $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ A. From the definition, if follows that $\text{Cl}_{\mu}(A)$ is the intersection of all $\text{generalized}_{\mathcal{F}} - \mathcal{C}\text{losed}\ \mathcal{S}\text{ets}$ in X containing A.

Definition 2.4: Let (X, μ) be $\text{generalized}_{\mathcal{F}} - \text{topological}\ \mathcal{S}\text{pace}$. For a fuzzy $\mathcal{S}$et A in X, the $\mathcal{I}$nterior of A, is defined as $I_{\mu}(A) = \text{Sup}\{Q : Q \subseteq A, Q \in \mu\}$. Thus $I_{\mu}(A)$ is the largest $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ in X contained in the fuzzy $\mathcal{S}$et A. From the definition, if follows that $I_{\mu}(A)$ is the union of all $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ in X contained in A.

Proposition 2.1: Let (X, μ) be $\text{generalized}_{\mathcal{F}} - \text{topological}\ \mathcal{S}\text{pace}$.Then:

i) 0 and 1 are fuzzy $\text{generalized}_{\mathcal{F}} - \mathcal{C}\text{losed}\ \mathcal{S}\text{ets}$ in X.

ii) Arbitrary intersection of $\text{generalized}_{\mathcal{F}} - \mathcal{C}\text{losed}\ \mathcal{S}\text{ets}$ in X is $\text{generalized}_{\mathcal{F}} - \mathcal{C}\text{losed}\ \mathcal{S}\text{et}$ in X.

3. $\text{Generalized}_{\mathcal{F}}$ −Open Sets

Definition 3.1: Let (X, μ) be $\text{generalized}_{\mathcal{F}} - \text{topological}\ \mathcal{S}\text{pace}$. Then a fuzzy set A in X is called $\text{generalized}_{\mathcal{F}} -$

$\alpha - \mathcal{O}$pen $\mathcal{S}$et if $A \subseteq I_\mu(Cl_\mu(I_\mu(A)))$.

Example 3.1: Let $X = \{x_1, x_2\}$ and $A = \{(x_1, 0.3), (x_2, 0.7)\}$, $B = \{(x_1, 0.7), (x_2, 0.4)\}$ and $C = \{(x_1, 0.7), (x_2, 0.7)\}$ are generalized$_{\mathcal{F}} - \mathcal{O}$pen $\mathcal{S}$ets on X. Clearly $\mu = \{0, A, B, C. 1\}$ is generalized$_{\mathcal{F}}$ − topology on X. The set $\lambda = \{(x_1, 0.8), (x_2, 0.7)\}$ is generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et in X.

Proposition 3.1: Every generalized$_{\mathcal{F}} - \mathcal{O}$pen $\mathcal{S}$et is generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et
Proof: Follows from the Definition.

Remark 3.1: In Example 3.1, we see that $\lambda = \{(x_1, 0.8), (x_2, 0.7)\}$ is generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et but not generalized$_{\mathcal{F}} - \mathcal{O}$pen $\mathcal{S}$et in generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace (X, μ).

Proposition 3.2: In generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace arbitrary union of generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$ets is generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et.
Proof: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and let $\{\lambda_J\}_{j \in J}$ be a family of generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$ets in X. Then for each $j \in J$, we have $\lambda_j \subseteq I_\mu(Cl_\mu(I_\mu((\lambda_j))))$.
Substituting $\lambda = \cup_{j \in J} \lambda_j$ we have
$I_\mu(Cl_\mu(I_\mu(\lambda) = I_\mu(Cl_\mu(I_\mu(\cup_{j \in J} \lambda_j))) \supseteq I_\mu(Cl_\mu(I_\mu(\cup_{j \in J} \lambda_j I_\mu(\lambda_j)))) \supseteq I_\mu(\cup_{j \in J} \lambda_j Cl_\mu(I_\mu(\lambda_j)) \quad \supseteq \cup_{j \in J} I_\mu(Cl_\mu(I_\mu(\lambda_j))) \supseteq \cup_{j \in J} \lambda_j = \lambda$.
Thus $\lambda = \cup_{j \in J} \lambda_j$ is generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et in X.

Proposition 3.3: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace. Then a fuzzy set λ_1 is generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et in X iff for each fuzzy point $x_r \in \lambda_1$ there exists generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et λ_2 in X such that $x_r \in \lambda_2$ and $\lambda_2 \subseteq \lambda_1$.
Proof: Let λ_1 be generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et in X. If x_r is fuzzy $\mathcal{P}$oint in X and $x_r \in \lambda_1$, then clearly generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et λ_1 itself satisfies the desired condition. Conversely suppose λ_1 is a fuzzy set in X having the property that for each fuzzy $\mathcal{P}$oint $x_r \in \lambda_1$, $x \in X$ there exists generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et say λ_2 in X such that $x_r \in \lambda_2$ and $\lambda_2 \subseteq \lambda_1$. Then we can see that λ_1 will be the union of all such generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$ets. Hence from Proposition 4.2, it follows that λ_1 is generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et in X.

Definition 3.2: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace. Then a fuzzy set λ in X is called generalized$_{\mathcal{F}} - \alpha - \mathcal{C}$losed $\mathcal{S}$et if λ^c is generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et in X.

Proposition 3.4: In generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace arbitrary intersection of generalized$_{\mathcal{F}} - \alpha - \mathcal{C}$losed $\mathcal{S}$ets is generalized$_{\mathcal{F}} - \alpha - \mathcal{C}$losed $\mathcal{S}$et.
Proof: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and let $\{\lambda_j\}_{j \in J}$ be a family of generalized$_{\mathcal{F}} - \alpha - \mathcal{C}$losed $\mathcal{S}$ets in X. Then $\{\lambda_j{}^c\}_{j \in J}$ is a family of generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$ets in X. Therefore $\cup_{j \in J} \lambda_j{}^c = (\cap_{j \in J} \lambda_j)^c$ is generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et in X. Thus $\cap_{j \in J} \lambda_j$ is generalized$_{\mathcal{F}} - \alpha - \mathcal{C}$losed $\mathcal{S}$et in X.

Definition 3.3: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and λ be a fuzzy set in X. Then α –$\mathcal{I}$nterior of λ is denoted by $\alpha - I_\mu(\lambda)$ and is defined to be the union of all generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$ets in X contained in λ.

Remark: 3.1: Since $\alpha - I_\mu(\lambda)$ is generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen $\mathcal{S}$et in X, $\alpha - I_\mu(\lambda)$ is the largest generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen set in X contained in λ, i.e. any generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen set in X contained in λ will also be contained in $\alpha - I_\mu(\lambda)$.

Proposition 3.5: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and λ be a fuzzy set in X. Then λ is generalized$_{\mathcal{F}} - \alpha - \mathcal{O}$pen iff $\alpha - I_\mu(\lambda) = \lambda$.
Proof: Follows from the definition.

Proposition 3.6: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and let A and B be fuzzy sets in X. Then
i) $\alpha - I_\mu(0) = 0$, $\alpha - I_\mu(1) = 1$,
ii) $A \subseteq B \Longrightarrow \alpha - I_\mu(A) \subseteq \alpha - I_\mu(B)$,
iii) $\alpha - I_\mu(A) \cup \alpha - I_\mu(B) \subseteq \alpha - I_\mu(A \cup B)$,
iv) $\alpha - I_\mu(A \cap B) \subseteq \alpha - I_\mu(A) \cap \alpha - I_\mu(B)$,
v) $\alpha - I_\mu(\alpha - I_\mu(A)) = \alpha - I_\mu(A)$.
Proof: Follows from the definition.

Definition 3.4: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and λ be a fuzzy set in X. Then α –clouser of λ is denoted by $\alpha - Cl_\mu(\lambda)$ and is defined to be the intersection of all generalized$_{\mathcal{F}}$ − α − closed $\mathcal{S}$ets in X containing λ .

Remark 3.2: Since $\alpha - Cl_\mu(\lambda)$ is generalized$_{\mathcal{F}}$ − α − closed $\mathcal{S}$et in X, $\alpha - Cl_\mu(\lambda)$ is the smallest generalized$_{\mathcal{F}}$ − $\mathcal{C}$losed $\mathcal{S}$et in X containing λ, i.e. any generalized$_{\mathcal{F}}$ − α − closedset in X containing λ will also be contained in $\alpha - Cl_\mu(\lambda)$.

Proposition 3.7: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and λ be a fuzzy set in X. Then, λ is generalized$_{\mathcal{F}}$ − α − closed iff $\alpha - Cl_\mu(\lambda) = \lambda$

Proof: Follows from the Definition

Proposition 3.8: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and let A and B be fuzzy sets in X. Then

i) $\alpha - Cl_\mu(0) = 0$.

ii) $\alpha - Cl_\mu(1) = 1$.

iii) If $A \subseteq B$ then $\alpha - Cl_\mu(A) \subseteq \alpha - Cl_\mu(B)$.

iv) $\alpha - Cl_\mu(A) \cup \alpha - Cl_\mu(B) \subseteq \alpha - Cl_\mu(A \cup B)$.

v) $\alpha - Cl_\mu\left(\alpha - Cl_\mu(A)\right) = \alpha - Cl_\mu(A)$

Proof: Follows from the Definition

Definition 3.5: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace. Then a fuzzy set λ in X is called generalized$_{\mathcal{F}}$ − $\mathcal{S}$emi $\mathcal{O}$pen $\mathcal{S}$et if $\lambda \subseteq Cl_\mu(I_\mu(\lambda))$.

Example 3.2: Let $X = \{x_1, x_2\}$. Suppose$A = \{(x_1, 0.3), (x_2, 0.7)\}$, $B = \{(x_1, 0.7), (x_2, 0.4)\}$ and $C = \{(x_1, 0.7), (x_2, 0.7)\}$ are fuzzy $\mathcal{S}$ets on X. Clearly $\mu = \{0, A, B, C. 1\}$ is generalized$_{\mathcal{F}}$ − topology on X. The set $\lambda = \{(x_1, 0.8), (x_2, 0.7)\}$ is generalized$_{\mathcal{F}}$ − $\mathcal{S}$emi $\mathcal{O}$pen $\mathcal{S}$et in X.

Proposition 3.9: Every generalized$_{\mathcal{F}}$ − $\mathcal{O}$pen $\mathcal{S}$e isgeneralized$_{\mathcal{F}}$ − $\mathcal{S}$emi $\mathcal{O}$pen $\mathcal{S}$et

Proof: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and λ be generalized$_{\mathcal{F}}$ − $\mathcal{O}$pen $\mathcal{S}$et in X. Then $I_\mu(\lambda) = \lambda$. Since $\lambda \subseteq Cl_\mu(\lambda)$.we have $\lambda = I_\mu(\lambda) \subseteq Cl_\mu\left(I_\mu(\lambda)\right)$, i.e, $\lambda \subseteq Cl_\mu\left(I_\mu(\lambda)\right)$. Hence λ is generalized$_{\mathcal{F}}$ − $\mathcal{S}$emi $\mathcal{O}$pen $\mathcal{S}$et in X.

Remark 3.3: In Example 3.2, we see that $\lambda = \{(x_1, 0.8), (x_2, 0.7)\}$ is generalized$_{\mathcal{F}}$ − semi − $\mathcal{O}$pen $\mathcal{S}$et but not generalized$_{\mathcal{F}}$ − $\mathcal{O}$pen $\mathcal{S}$et in generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace (X, μ).

Definition 3.6: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace . Then a fuzzy set λ in X is called generalized$_{\mathcal{F}}$ − semi − $\mathcal{C}$losed $\mathcal{S}$et if λ^c is generalized$_{\mathcal{F}}$ − semi − $\mathcal{O}$pen $\mathcal{S}$et in X.

Proposition 3.10: In generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace arbitrary intersection of generalized$_{\mathcal{F}}$ − semi − $\mathcal{C}$losed $\mathcal{S}$ets is generalized$_{\mathcal{F}}$ − semi − $\mathcal{C}$losed $\mathcal{S}$et.

Proof: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and let $\{\lambda_j\}_{j\in J}$ be a family of generalized$_{\mathcal{F}}$ − semi − $\mathcal{C}$losed $\mathcal{S}$ets in X. Then $\{\lambda_j{}^c\}_{j\in J}$ is a family of generalized$_{\mathcal{F}}$ − semi − $\mathcal{O}$pen $\mathcal{S}$ets in X. Therefore $\cup_{j\in J} \lambda_j{}^c = (\cap_{j\in J} \lambda_j)^c$ isgeneralized$_{\mathcal{F}}$ − semi − $\mathcal{O}$pen $\mathcal{S}$et in X. Hence $\cap_{j\in J} \lambda_j$ is generalized$_{\mathcal{F}}$ − semi − $\mathcal{C}$losed $\mathcal{S}$et in X.

Definition 3.7: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace . Then a fuzzy $\mathcal{S}$et λ in X is called generalized$_{\mathcal{F}}$ − $\mathcal{P}$re − $\mathcal{O}$pen $\mathcal{S}$et if $\lambda \subseteq i_\mu(cl_\mu(\lambda))$.

Example 3.3: Let $X = \{x_1, x_2\}$. Suppose $A = \{(x_1, 0.3), (x_2, 0.7)\}$, $B = \{(x_1, 0.7), (x_2, 0.4)\}$ and $C = \{(x_1, 0.7), (x_2, 0.7)\}$ are generalized$_{\mathcal{F}}$ − $\mathcal{O}$pen $\mathcal{S}$ets on X. Clearly $\mu = \{0, A, B, C. 1\}$ isgeneralized$_{\mathcal{F}}$ − $\mathcal{T}$opology on X. The set $\lambda = \{(x_1, 0.8), (x_2, 0.7)\}$ is generalized$_{\mathcal{F}}$ − $\mathcal{P}$re − $\mathcal{O}$pen $\mathcal{S}$et in X.

Proposition 3.11: Every generalized$_{\mathcal{F}}$ − $\mathcal{O}$pen $\mathcal{S}$et in generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace is generalized$_{\mathcal{F}}$ − $\mathcal{P}$re − $\mathcal{O}$pen $\mathcal{S}$et.

Proof: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and λ be generalized$_{\mathcal{F}}$ − $\mathcal{O}$pen $\mathcal{S}$et in X. Then $l_\mu(\lambda) = \lambda$. Since $\lambda \subseteq CI_\mu(\lambda)$.We have $\lambda = l_\mu(\lambda) \subseteq I_\mu\left(Cl_\mu(\lambda)\right)$. i.e , $\lambda \subseteq I_\mu\left(Cl_\mu(\lambda)\right)$. Hence λ is generalized$_{\mathcal{F}}$ − $\mathcal{P}$re − $\mathcal{O}$pen $\mathcal{S}$et in X.

Remark 3.4: The converse of proposition 3.11 is not true as illustrated in Example 3.6

Example 3.4: In Example 3.3, $\lambda = \{(x_1, 0.8), (x_2, 0.7)\}$ is generalized$_{\mathcal{F}}$ − $\mathcal{P}$re − $\mathcal{O}$pen $\mathcal{S}$et but not Generalized$_{\mathcal{F}}$ −

$\mathcal{O}$pen $\mathcal{S}$et in X.

Definition 3.8: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace. Then a fuzzy set λ in X is called generalized$_{\mathcal{F}}$ – Semi − Pre − Open Set if $\lambda \subseteq Cl_{\mu}(I_{\mu}(Cl_{\mu}(\lambda)))$.

Definition 3.9: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and λ be a fuzzy set in X. Then sp − $\mathcal{I}$nterior of λ is denoted by $sp - I_{\mu}(\lambda)$ and is defined to be the union of all generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{O}$pen $\mathcal{S}$et in X contained in λ..

Remark 3.5: Since $sp - I_{\mu}(\lambda)$ is generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{O}$pen $\mathcal{S}$et in X. Hence $sp - I_{\mu}(\lambda)$ is the largest generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{O}$pen set in X contained in i.e. any generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{O}$pen sets in X contained in λ will also be contained in $sp - I_{\mu}(\lambda)$

Proposition 3.12: Every generalized$_{\mathcal{F}}$ − $\mathcal{O}$pen $\mathcal{S}$et is generalized$_{\mathcal{F}}$ – Semi − Pre − Open Set.

Proof: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and λ be generalized$_{\mathcal{F}}$ − $\mathcal{O}$pen $\mathcal{S}$et in X. Since $\lambda \subseteq Cl_{\mu}(\lambda)$, we have $\lambda = I_{\mu}(\lambda) \subseteq I_{\mu}(Cl_{\mu}(\lambda))$, i.e., $\lambda \subseteq I_{\mu}(Cl_{\mu}(\lambda))$. This implies $\lambda \subseteq Cl_{\mu}(I_{\mu}(Cl_{\mu}(\lambda)))$. Hence λ is generalized$_{\mathcal{F}}$ − semi − $\mathcal{P}$re − $\mathcal{O}$pen $\mathcal{S}$et in X.

Remark 3.6: The converse of Proposition 3.12 is not necessarily true. In Example 3.3 we have $\lambda = \{(x_1, 0.8), (x_2, 0.7)\}$ is generalized$_{\mathcal{F}}$ – Semi − Pre − Open Set but not generalized$_{\mathcal{F}}$ − $\mathcal{O}$pen $\mathcal{S}$et in X.

Proposition 3.13: Every generalized$_{\mathcal{F}}$ − $\mathcal{S}$emi $\mathcal{O}$pen $\mathcal{S}$et is generalized$_{\mathcal{F}}$ – Semi − Pre − Open Set

Proof: Let (X, μ) be X be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and let λ be generalized$_{\mathcal{F}}$ – Semi − Open Set in X. Then $\lambda \subseteq Cl_{\mu}(I_{\mu}((\lambda))$. Since $\lambda \subseteq Cl_{\mu}(\lambda)$, we have $\lambda \subseteq Cl_{\mu}(I_{\mu}\left(Cl_{\mu}(\lambda)\right))$. This implies λ is generalized$_{\mathcal{F}}$ – Semi − Pre − Open Set in X. This proves the result.

Proposition 3.14: Every generalized$_{\mathcal{F}}$ − $\mathcal{P}$re − $\mathcal{O}$pen $\mathcal{S}$et is generalized$_{\mathcal{F}}$ – Semi − Pre − Open Set.

Proof: Follows from the Definition.

Remark 3.7: The converse of Proposition 3.14 is not necessarily true, which is illustrated in the have following Example.

Example 3.7: Let $X = \{x_1, x_2\}$ Suppose $A = \{(x_1, 0.5), (x_2, 0.3)\}$, $B = \{(x_1, 0.3), (x_2, 0.4)$, $C = \{(x_1, 0.5), (x_2, 0.4)\}$ are fuzzy sets on X. Then we see that collection $\mu = \{0, A, B, C, 1\}$ is generalized$_{\mathcal{F}}$ − topology $\mathcal{S}$pac on X. The set $D = \{(x_1, 0.4), (x_2, 0.5)$ is generalized$_{\mathcal{F}}$ – Semi − Pre − Open Set but not generalized$_{\mathcal{F}}$ − $\mathcal{P}$re − $\mathcal{O}$pen $\mathcal{S}$et in X.

Proposition 3.15: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace. Then a fuzzy $\mathcal{S}$et λ_1 is generalized$_{\mathcal{F}}$ – Semi − Pre − Open Set in X iff for each fuzzy point $x_r \in \lambda_1$ there exists generalized$_{\mathcal{F}}$ – Semi − Pre − Open Sets λ_2 in X such that $x_r \in \lambda_2$ and $\lambda_2 \subseteq \lambda_1$.

Proof: Similar to Proposition 3.3.

Definition 3.10: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace. Then a fuzzy set λ in X is called generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{C}$losed $\mathcal{S}$et if λ^c is generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{O}$pen $\mathcal{S}$et in X.

Proposition 3.16: In generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace arbitrary intersection of generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{C}$losed $\mathcal{S}$ets is generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{C}$losed $\mathcal{S}$et.

Proof: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and let $\{\lambda_j\}_{j \in J}$ be a family of generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{C}$losed $\mathcal{S}$ets in X. Then $\{\lambda_j{}^c\}_{j \in J}$ is a family of generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{O}$pen $\mathcal{S}$ets in X. Therefore $\cup_{j \in J} \lambda_j{}^c = (\cap_{j \in J} \lambda_j)^c$ is generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{O}$pen $\mathcal{S}$et in X. Hence $\cap_{j \in J} \lambda_j$ is generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{C}$losed $\mathcal{S}$et in X.

Proposition 3.18: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace and λ be a fuzzy set in X. Then λ is generalized$_{\mathcal{F}}$ − semi − pre − $\mathcal{O}$pen iff $sp - I_{\mu}(\lambda) = \lambda$.

Proof: Follows from the Definition.

Proposition 3.19: Let (X, μ) be generalized$_{\mathcal{F}}$ − topological $\mathcal{S}$pace Let A and B be fuzzy sets in X. Then

i) $sp - I_{\mu}(0) = 0$, $sp - I_{\mu}(1) = 1$,

ii) $A \subseteq B \Longrightarrow sp - I_{\mu}(A) \subseteq sp - I_{\mu}(B)$,

iii) $sp - I_{\mu}(A) \cup sp - I_{\mu}(B) \subseteq sp - I_{\mu}(A \cup B)$

iv) $sp - I_{\mu}(A \cap B) \subseteq sp - I_{\mu}(A) \cap sp - I_{\mu}(B)$,.

v) $sp - I_{\mu}\,(sp - I_{\mu}(A)) = sp - I_{\mu}(A)$.

Definition 3.11: Let (X, μ) be $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$ and λ be a fuzzy set in X. Then sp –clouser of λ is denoted by $sp - Cl_{\mu}(\lambda)$ and is defined to be the intersection of all $generalized_{\mathcal{F}} - semi - pre - closed\ \mathcal{S}ets$ in X containing λ.

Remark 3.8: Since $sp - Cl_{\mu}(\lambda)$ is $generalized_{\mathcal{F}} - semi - pre - closed\ \mathcal{S}et$ in X. Hence $sp - Cl_{\mu}(\lambda)$ is the smallest $generalized_{\mathcal{F}} - semi - pre - closed$ set in X containing in i.e. any $generalized_{\mathcal{F}} - semi - pre - closed$ set in X containing in λ will also be containing in $sp - Cl_{\mu}(\lambda)$

Proposition 3.20: Let (X, μ) be $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$ and λ be a fuzzy $\mathcal{S}et$ in X. Then λ is $generalized_{\mathcal{F}} - semi - pre - closed$ iff $sp - Cl_{\mu}(\lambda) = \lambda$.

Proposition 3.21: Let (X, μ) be $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$ and λ be $generalized_{\mathcal{F}} - closed\ \mathcal{S}et$ in X. Let A and B be fuzzy set in $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$ X. Then

i) $sp - Cl_{\mu}(0) = 0$, $sp - Cl_{\mu}(1) = 1$

ii) $A \subseteq B \Longrightarrow sp - Cl_{\mu}(A) \subseteq sp - Cl_{\mu}(B)$.

iii) $sp - Cl_{\mu}(A) \cup sp - Cl_{\mu}(B) \subseteq sp - Cl_{\mu}(A \cup B)$

iv) $sp - Cl_{\mu}(A \cap B) \subseteq sp - Cl_{\mu}(A) \cap sp - Cl_{\mu}(B)$.

v) $sp - Cl_{\mu}\,(sp - Cl_{\mu}(A)) = sp - Cl_{\mu}(A)$.

Proof: Follows from the Definition.

4. Conclusion

In this Paper we have studied a new concept of generalized fuzzy open sets in generalized fuzzy topological space in which many important results have been obtained. Further we have established the relationships with the help of some counter examples.

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