A Comparative Study OfDifferent Probability Estimation Methods For ML Estimation Based On Two Parameters Of Lomax Distribution

V.Sastry Ch[1]*

[1]*Department of Statistics, KRU Dr. MRAR College of PG Studies, Nuzvid, A.P, India

***Corresponding Author:** V.Sastry Ch
*Department of Statistics, KRU Dr. MRAR College of PG Studies, Nuzvid, A.P, India

Abstract
In this study, a comparison of performance using different values of αshape parameter and βscale parameter among four different methods Chegodayev, Blom, Gringorten, and VSastry Ch estimator the parameters of Lomax distribution was done. The aim is to find the best methods of estimating the two parameter Lomax distribution. These methods are compared in terms of their fits using the Log-Likehood value and the Mean Square Error (MSE) criteria to select the best method.Results show that the VSastry Ch (VSCH) is the better of themethods.

Keywords: Lomax, Chegodayev, Blom, Gringorten, VSastry Ch, MSE

1. Introduction:
In this study heavy-tailed shaped distribution Lomax distribution considered important in statistical practice. It was introduce in 1954 by Prof. K.S. Lomax. The Pareto Type-II or Lomax distribution is used areas in like Life Medicine Engineering and others and can be motivated in many ways. Also is known as Pareto distribution of the second kind or Person Type VI distribution, It used in the analysis of income data as also business failure data. It describes the lifetime of the decreasing failure rate component as a heavy tailed alternative to the exponential distribution

Section 2,presents the Lomax distribution with discussion of properties and Maximum Likelihood Estimation (MLE). The probability estimators, life data set and the comparison methods are discussioned in section 3. Parameter probability estimation methods applied to a real data set and presented in section 4. Section 5 lists conclusion of the article.

2. The Lomax Distribution:
The Lomax distribution hastwo parameters and denoted by Lomax(α, β) where $\alpha$is a shape parameter and $\beta$is the scaleparameter.The probability density function (p.d.f.)

$$f\left(x;\alpha,\beta\right)=\frac{\alpha}{\beta}\left[1+\frac{x}{\beta}\right]^{-(\alpha+1)};x>0,\alpha,\beta>0 \quad (1)$$

The Lomax distribution, the properties are presented here, Cumulative distribution function (c.d.f):

$$F\left(x;\alpha,\beta\right)=1-\left[1+\frac{x}{\beta}\right]^{-\alpha};x>0,\alpha,\beta>0 \quad (2)$$

The survival function or reliability function:

$$S(x)=\left[1+\frac{x}{\beta}\right]^{-\alpha} \quad (3)$$

The hazard function:

$$H(x)=\frac{\alpha\beta}{1+\alpha x} \quad (4)$$

Mean and variance of Lomax distribution:

$$E\left(X\right)=\frac{\beta}{\alpha-1}\ ;\ \alpha>1 \quad (5)$$

$$Var\left(X\right)=\frac{\beta^2\alpha}{\left(\alpha-1\right)^2\left(\alpha-2\right)}-\frac{\beta^2}{\left(\alpha-1\right)^2}\ ;\ \alpha>2 \quad (6)$$

Inverse Transformation method to generate a sample data for given parameters'

$$x=\beta\left(\left[1-u\right]^{-\frac{1}{\alpha}}-1\right);\quad u:\ U\left[0,1\right] \quad (7)$$

where u~uniform(0,1), and the parameters α, $\beta$are known.

Estimation method:Maximum Likelihood Estimation Method

$$L\left(x;\alpha,\beta\right)=\prod_{i=1}^{n} f\left(x;\alpha,\beta\right)$$

$$=\frac{\alpha^{n}}{\beta^{n}}\prod_{i=1}^{n}\left[1+\frac{x_i}{\beta}\right]^{-(\alpha+1)} \quad (8)$$

$$\ln\left[L\left(x;\alpha,\beta\right)\right]=n\ln\left(\alpha\right)-n\ln\left(\beta\right)-\left(\alpha+1\right)\sum_{i=1}^{n}\ln\left[1+\frac{x_i}{\beta}\right]$$

Estimation for the parameters

$$\hat{\alpha}_{MLE}=\frac{n}{\sum_{i=1}^{n}\ln\left[1+\frac{x_i}{\beta}\right]} \quad (9)$$

$$\hat{\beta}_{MLE}=\frac{1+\alpha}{n\beta}\sum_{i=1}^{n}\left[\frac{x_i}{\beta+x_i}\right]-\frac{1}{\beta} \quad (10)$$

3. Estimation procedures:

The most commonly used approaches within in the Lomax statistics communityfor the estimation of$\alpha$the shape parameter and for$\beta$the scale parameter parameters of the Lomax distribution are described using probability estimators equivalent strength data are ranked in ascending order (i= 1 to n) for the calculation methods.

Themost commonly usedprobability estimation (Lomax statistics) methods and new method VSastry Ch estimatorare shows in thefollowing Table.

Table -1: Probability Estimation Methods

Method	$F\left(x_i\right)$
Chegodayev estimator	$F_{\text{Cheg}}=\frac{i-0.3}{n+0.4}, i=1,2,3,\ldots,n$
Blom (1958) estimator	$F_{\text{Blom}}=\frac{i-0.375}{n+0.25}, i=1,2,3,\ldots,n$
Gringorten (1963) estimator	$F_{\text{Grin}}=\frac{i-0.44}{n+0.12}, i=1,2,3,\ldots,n$
VSastry Ch estimator	$F_{\text{VSCH}}=\frac{i-0.49}{n+0.21}, i=1,2,3,\ldots,n$

Statistical criteria

MSE is a measure of the accuracy of the estimator of 2 parameter Lomax distribution. MSE can be calculated as below

$$MSE=\frac{\sum_{i=1}^{n}\left(\hat{F}\left(x_i\right)-F\left(x_i\right)\right)^2}{n}$$

Where

$\hat{F}\left(x_i\right)$= value of the cumulative distribution function (CDF) of the Lomax distribution evaluated atweekly stock price $i$in a week by using different estimations,

$F\left(x_i\right)$= observed cumulative probability of tree $i$in a weekly stock price.

n =number of trees in a plot
In this study, testing and evaluation computations were computed using theR4.1.2 software version a language different statistical package

4. Parameter probability estimation method Results:

Real Data set

The data used for this study is the weekly stock prices (N = 100 weeks) collected from Cornerstone Insurance Company PLC, a public liability company listed in the Nigerian Stock Exchange from Felix NN and Chukwudi AU (2014)

Table - 2 : Weekly stock prices (read row-wise)

1.03	1.06	0.99	1.03	0.99	0.95	0.96	0.98	0.93	1.05
0.92	0.99	0.97	0.96	0.91	0.94	0.97	0.99	1.15	1.27
1.46	1.83	2.31	2.49	2.73	2.70	2.52	2.49	2.76	3.00
3.18	3.88	3.84	3.79	3.76	3.75	3.89	4.04	4.70	4.34
4.55	4.20	4.19	4.12	4.13	3.77	3.25	3.14	3.12	2.82
3.24	3.44	3.50	3.64	3.72	3.68	3.41	3.24	3.26	3.42
3.38	4.02	4.21	4.23	4.04	4.11	4.28	4.84	4.46	4.87
5.00	5.91	7.36	7.34	7.23	7.19	6.79	6.03	5.97	5.69
6.42	6.23	5.86	5.46	4.71	4.32	4.79	4.62	4.54	4.22
4.28	4.08	3.95	4.16	3.50	3.65	3.22	3.50	3.97	2.96

The shape and scale parameters of the Lomax distribution a numerical simulation study same carried by a in order to investigate the behavior carried out. A simulation study has been conducted to explore the performances of the different methods discussed in this article. The main objective of the study is to compare the performance of the four probability estimators methods of estimate the shape (α) = 0.5 (0.5) 3 and scale (β) = 0.5 (0.5) 3 parameters of two parameter Lomax distribution. Using the values of Log- Likelihood and MSE the estimates are comparing.

Implementation:
To explore the performances of the different methods discussed in this article. The main objective of the study is to compare the performance of 4 probability estimation methods areChegodayev estimator, Blom (1958) estimator, Gringorten (1963) estimator, and VSastry Ch estimator of estimate the shape (α) = 0.5 (0.5) 3 and scale (β) = 0.5 (0.5) 3 parameters of two parameter Lomax distribution
Out of 4 methods VSastry Ch estimator values seen to be the best when the MSE value is minimum value. They are presented hereunder in the Table-3.

Results and Discussion

Table – 3 Best values Estimated of Lomax distribution including Log Likelihood and MSE values

Sl.No	α	β	$\hat{\alpha}$	$\hat{\beta}$	LL Value	MSE
1	0.5	0.5	0.0082	0.5055	417.6965	116.6417
2	1	0.5	0.0124	0.5196	379.1309	482.8522
3	1.5	0.5	0.0103	0.5480	402.4232	1122.3823
4	2	0.5	0.0186	0.5654	346.7814	2092.2682
5	2.5	0.5	0.0138	0.5855	379.9252	3395.8534
6	3	0.5	0.0148	0.6085	376.16	5109.0649
7	0.5	1	0.0092	1.0019	472.3671	29.9172
8	1	1	0.0158	1.0106	418.4908	121.6340
9	1.5	1	0.0223	1.0318	386.4962	279.0394
10	2	1	0.0299	1.0457	358.3639	506.9475
11	2.5	1	0.0109	1.0592	460.5738	794.4370
12	3	1	0.0087	1.0728	484.3459	1166.8125
13	0.5	1.5	0.0073	1.5009	534.2508	13.2962
14	1	1.5	0.0362	1.506	374.8029	56.2341
15	1.5	1.5	0.0427	1.5145	359.0031	126.4464
16	2	1.5	0.0405	1.5342	365.3975	224.4619
17	2.5	1.5	0.024	1.5455	418.36	348.8701
18	3	1.5	0.027	1.5611	407.7953	509.2308
19	0.5	2	0.0095	2.0008	536.8749	7.5717
20	1	2	0.009	2.0052	542.525	30.0253
21	1.5	2	0.0521	2.0131	367.0111	71.7451
22	2	2	0.031	2.0276	419.4249	124.2860

23	2.5	2	0.0457	2.0368	381.1315	197.3485
24	3	2	0.0232	2.0473	449.4214	281.6825
25	0.5	2.5	0.0339	2.5004	431.2715	5.3308
26	1	2.5	0.0641	2.5026	367.8084	21.3507
27	1.5	2.5	0.0131	2.5072	526.7802	43.5982
28	2	2.5	0.0121	2.5161	535.3325	77.7861
29	2.5	2.5	0.01	2.5301	554.6143	122.1355
30	3	2.5	0.0094	2.5395	561.4153	177.1539
31	0.5	3	0.0638	3.0003	386.0942	4.1328
32	1	3	0.0394	3.0022	434.3042	14.1553
33	1.5	3	0.0105	3.006	567.147	30.1310
34	2	3	0.0524	3.0136	406.2901	55.9978
35	2.5	3	0.0665	3.0288	382.8722	88.1790
36	3	3	0.0681	3.0324	380.7315	126.9589

The following observations can be made on how the parameters α , β and MSE of the estimators vary with respect from Table-3.

For all methods of estimations, it is clear the MSEs decreased as parameters are increased. The shape (α) and scale (β) values are same in almost all MSE value but some values are nearly.The scale (β) = 0.5 (0.5) 3 parameter increased the MSEs value decreased. The shape (α) = 0.5 (0.5) 3 values very large difference and scale (β) = 0.5

(0.5) 3 are very close values.Shape (α) parameter value increasing MSE value to increased and LL value zigzag value (flatuvation). The scale (β) parameter value increasing MSE value decreasing and LL valve also zigzag (flatuvation). The shape (α) value large and scale (β) parameter values increasing MSE value decreased. The shape (α) value increasing and scale (β) parameter values large value MSE value increased.

5. Conclusion:

Many distributions have been developed and applied to describe the various phenomena. The MLE method is employed for estimating the model parameters. We hope that the probability estimation models will attract wider application in areas such as engineering, survival, banking and lifetime data.

Overall, VSastry Ch was found to be the most effective method for all parameter combination i.e, shape (α) = 0.5 (0.5) 3 and scale (β) = 0.5 (0.5) 3, combinations investigated in the study because the it provided the best MSE. Remaining methods are Gringortenestimator, Blom estimator, Chegodayev estimator methods are order to MLE estimation for all parameter combinations ascending order values MSE values.

Therefore, this confirms that the conclusions of studies that a “2 parameter Lomax distribution is are accurate in describing Cornerstone Insurance Company PLCdata than a Lomax distribution and is always more conservative in the tail of the distribution” is valid for the strength data used in the present studystatistical analysis.

Reference:

1. Kao HJ., A summary of some new tehniques on failure analysis, Proceedings of the sixth National Symposium on Reliability and Quality Control in elektronics, p. 190-201, 1960.
2. Lomax, K. S. (1954). Business failures. Another example of the analysis of failure data. JASA, 49, 847-852.
3. K.S. Lomax, Business failures. Another example of the analysis of failure data, J. Amer. Statist. Assoc. 49 (1954), pp. 847–852.
4. Lingappaiah, G. S. (1987), On the Pareto distribution of the second kind (Lomax distribution) , Revista de Mathemática e Estatística, Vol. 4/5, pp. 63–68.
5. Nayak,T.K., 1987. Multivariate lomax distribution—properties and usefulness in reliability theory. J.Appl. Probab. 24, 170–177.
6. Balakrishnan, N., Ahsanullah, M. (1994). Relations for single and product moments of record values from Lomax distribution. Sankhya B 56:140–146.
7. Childs, A., Balakrishnan, N., Moshref, M. (2001). Order statistics from non-identical right truncated Lomax random variables with applications. Statist. Pap. 42(2):187–206.
8. Petropoulos,C. and Kourouklis,S.(2004), Improved estimation of extreme quantiles in the multivariate Lomax (Pareto II) distribution, Metrika, 60,15-24.
9. S. Nadarajah, Sums, products, and ratios for the bivariate Lomax distribution, Comput. Stat. Data Anal. 49 (2005), pp. 109–129.

10. El-Bassiouny, A. H., Abdo, N. F. and Shahen, H. S. (2015). Exponential lomax distribution, International Journal of Computer Applications, 121(13), 24–29.
11. M. E. Ghitany, F. A. Al-Awadhi, L. A. Alkhalfan, Marshall-Olkin extended Lomax distribution and its application to censored data, Communications in Statistics-Theory and Methods, 36 (2007), 1855–1866.
12. Gupta RC, Ghitany ME, Al-Mutairi DK (2010) Estimation of reliability from Marshall–Olkin extended Lomax distributions. J Stat Comput Simul 80:937–947
13. Punathumparambath, B. (2011). Estimation of P(X > Y) for the double Lomax distribution. ProbStat Forum, 4, 1-11.
14. Cramer, E. and Schmiedt, A.B. (2011), Progressively type-II censored competing risks data from Lomax distributions , Computational Statistics and Data Analysis, Vol. 55, pp. 1285–1303.
15. E. Cramer and A. B. Schmiedt, Progressively type-II censored competing risks data from Lomax distributions, Computational Statistics and Data Analysis, vol. 55, no. 3, pp. 1285–1303, 2011.
16. D. E. Giles, H. Feng, R. T. Godwin, (2013), On the bias of the maximum likelihood estimator for the two-parameter lomax distribution, Communications in Statistics-Theory and Methods 42 (11) 1934–1950.
17. M. M. El-Din, H. M. Okasha, and B. Al-Zahrani (2013), Empirical Bayes Estimators of Reliability Performances Using Progressive Type-II Censoring From Lomax Model, Journal of Advanced Research in Applied Mathematics, vol. 5(1), 74–83.
18. Lemonte, A. J. and Cordeiro, G. M. (2013). An extended Lomax distribution. Statistics, 47, 800–816.
19. Cordeiro, G.M., Ortega, E.M. and Popovi ´c, B. z. (2013). The gamma Lomax distribution. Journal of Statistical Computation and Simulation, 85(2), 305–319.
20. Al-Zahrania B, Sagorb H (2014) The Poisson-Lomax distribution. Rev Colomb de Estad 37(1):223–243
21. Felix NN and Chukwudi AU (2014), A comparison of methods for the estimation of weibull distribution parameters, MetodoloskiZvezki, Vol.11, No.1, pg 65-78
22. Tahir MH, Cordeiro GM, Mansoor M, Zubair M (2015) The Weibull Lomax distribution: properties and applications. Hacettepe J Math Stat 44(2):461–480
23. Cordeiro, G.M., Ortega, E.M.M. and Popovi´c, B.V. (2015). The gamma-Lomax distribution. Journal of Statistical Computation and Simulation, 85, 305–319.
24. Afify AZ, Nofal ZM, Yousof HM, Gebaly YM, Butt NS (2015) The transmuted Weibull Lomax distribution: properties and application. Pak J Stat Oper Res 11:135–152
25. Kilany, N. M. (2016). Weighted Lomax distribution. Springer Plus, 5(1), article no. 1862.
26. M. Tahir, M. A. Hussain, G. M. Cordeiro, G. Hamedani, M. Mansoor, M. Zubair, The gumbel-lomax distribution: properties and applications, Journal of Statistical Theory and Applications 15 (1) (2016) 61–79.
27. Devi, B., Kumar, P., and Kour, K., 2017. Entropy of Lomax Probability Distribution and its Order Statistics, International Journal of Statistics and System **12**: 175-181.
28. Hassan, A. S. and Al-Ghamdi, A. S. (2009). Optimum step stress accelerated life testing for Lomax distribution. Journal of Applied Sciences Research, 5, 2153-2164.
29. Holland, O., Golaup, A. and Aghvami, A. H. (2006). Traffic characteristics of aggregated module downloads for mobile terminal reconfiguration. IEE Proceedings – Communications, 135, 683690.
30. Labban J.A. (2005). Modified and Simplified Method for Finite Difference (Divided). Al-Qadisiya Journal for Science, 10(2), 244-251.