A Brief Review On Mathematical Modelling In Blood Flow In Stenosed Arteries

Dr. Mokshed Ali[1*]

[1*]Asst. Professor, Department of Mathematics, Barkhetri College, Mukalmua.

***Corresponding Author:** Dr. Mokshed Ali
Gmail: mokshed.ali7@gmail.com.

Abstract:
A mathematical modeluses a mathematical language to describe a system. In Biomechanics, mathematical models are being popularly employed by many researchers round the globe for getting an account of many practical situations. Theoretical models on bloodflow, have also drawn a considerable attention from a good many scientists and researchers. To describe flow of blood inside artery, researchers round the globe used various fluid models. In this paper some frequently used blood flow models have been discussed.

Key words: Mathematical models, stenosis, blood.

Introduction:
Blood is a suspension of millions of cells of different shapes in plasma. Blood has a suspension of cells in an aqueous solution known as plasma. This plasma is made from about 90% water and 7% protein. A milliliter of healthy human blood contains approximately 5 x l0^9 cells of which 95% parts are red cells or erythrocytes. The main function of the red cells is to make oxygen from the lungs available to all the cells of the body. The red cells also remove the carbon dioxide produced by the metabolic processes inthe body. The red blood cells occupy about 45% of the blood in an average man. Of the remaining 5% of the blood, white cells or leukocytes constitute about 1% of the total. These white cells help us to keep our body away from infections.
In 5 liters of blood in the human body, there are approximately 25 x l0^{12} red cells. The average span of life of the redcells is about 120 days and the total number of red cells dying per second is about 2.4xl0^6. A red cell has an average volume of 90p cube meter and surface area of about 140p square meter. It is already reported that blood is a suspension of various tiny particles in a continuous saline solution plasma (Fung,1981: Fung, 1984). It has been pointed out that plasma behaves as a Newtonian fluid (Schliching, 1968) whereas the whole blood, being a suspension of cells and highly viscous in nature, exhibits the property of a non-newtonian fluid (Biswas, 2000: Fung, 1981). Though blood shows a non-Newtonian character at low shear rate but at high shear rate, usually available in large arteries, blood behaves like a Newtonian fluid (Biswas, D.and Chakraborty, U.S.,2010; Merrill, 1965; Taylor, 1959). As blood possesses a finite yield stress as wellas it indicates a power –law behavior, it can be accounted by considering it, in behaving like a Casson fluid as well as power-law fluid. From the aforesaid considerations and to direct our investigation in a proper perspective, in the proceeding text, some of the relevant mathematical models for fluid flow in general and blood flow in particular have been dealt with.

Blood flow through stenosed artery:
The study of blood flow in artery has occupied the attention of the researches for over 150 years. It is the well known that the abnormal growth in the lumen of the arterial wall develop at various locations of the cardiovascular system under diseased condition. Arteriosclerosis or stenosis, as it is called, is one of the most widespread arterial diseases, it has been suggest that the deposit of cholesterol on the arterial wall and proliferation of connective tissue may be responsible for the abnormal growth in the lumen of an artery. The actual causes of stenosis are not well known but its effect on the cardiovascular system can be understood by studying the blood flow in its vicinity.
Atherosclerosis is a degenerative disease in which there is anaccumulation of lipids and other material under the endothelial layers lining the arterial wall, these areas of accumulation arecalled plaques. The greatest predilection for the disease is in proximity to bends, junctions and branches of the large arteries. This suggests a possible role for fluid mechanics in the development and progression of the disease since flow phenomena exhibit unique characteristics in these areas. The incentive for the present research is the fact that 75% of all deaths are due to circulatory disease. Of these the atherosclerosis is the most frequent one. Deposits and blockage is mostly found at bends and bifurcation of human arteries.

Mathematical modeling:
A mathematical model of a system is a symbolic representation involving an abstract mathematical formulation. A mathematical formulation is composed of symbols, and marks no sense outside the mathematics the symbols have

precise mathematical meaning and the manipulation of the symbols is dictated by the rules of logic and mathematics. A mathematical formulation is not the mathematical model by itself. An adequate mathematical model of a system is obtained by relating, on a oneto one basis, the features of an adequate system characterizationwith the variables of an appropriate mathematical formulation or in other words an adequate mathematical model is a mathematical model which is adequate for the purpose in themind of the builder.

Now we are giving the process by which a mathematical model building occurred in the following flow charts viz. simplified mathematical modeling arid the detailed mathematical modeling?

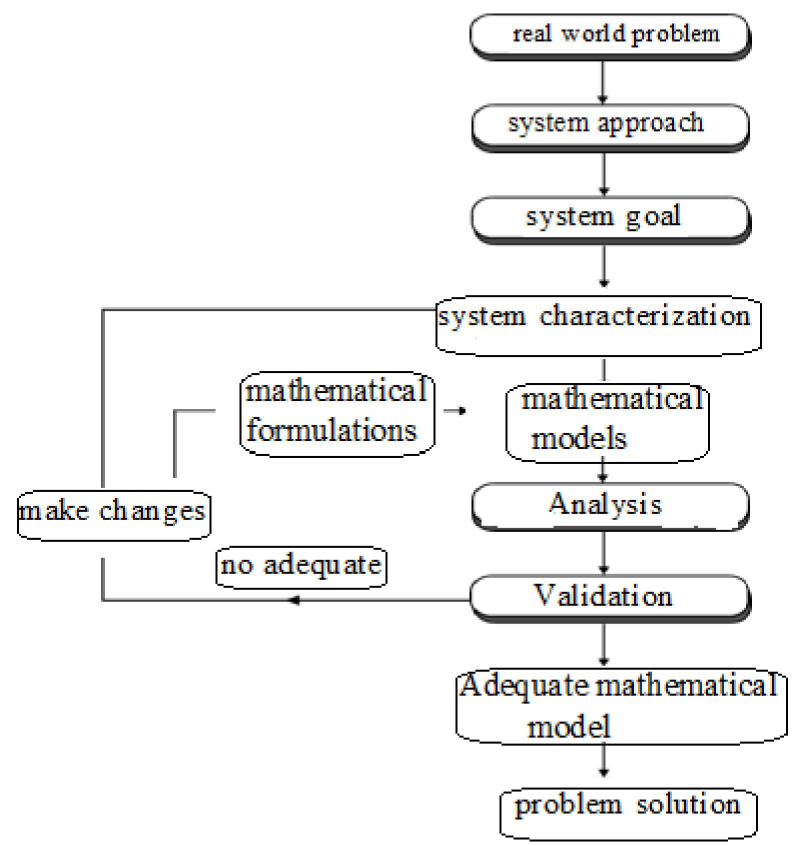


Fig. 1 Mathematical modeling process (simplified)

Newtonian Fluid Models:

It is already said that at high shear rate and large diameter arteries, blood behaves likes a Newtonian fluid where in shear stress versus rate of strain relation is linear (Fung, 1981; Taylor, 1959). Young (1968) has examined the effects of stenosis on flow behaviour of blood. It has been noticed that resistance to flow and wall shear stress increase with the increase of stenosis size. Forrester and Young (1970) extended the work of Young (1968), including the effects of flow separation in an artery with mild stenosis. Results of experimental work on models of arterial stenosis, have been presented by Young and Tsai (1973). Sanyal and Maiti (1998) have addressed the pulsatile blood flow through a stenosed artery. They have discussed numerical solutions of axial velocity profile and pressure gradient graphically. Jayaraman and Tewari (1995) have studied blood flow in a catheterized curved artery. They assumed blood as an incompressible Newtonian fluid and usual zero-slip boundary condition is used to analyze the flow. Sarkar and Jayaraman (1997) have putforward a model of blood flow through a catheterized stenosed artery by considering blood as a Newtonian fluid.

The importance of slip velocity at flow boundaries has reported by many authors both theoretically and experimentally (Vand, 1948; Bloch, 1962; Brunn, 1975; Nubar, 1967; Bugliarello and Hayden, 1962; Bennet, 1967). Chaturani and Biswas (1984) have examined the effect of slip velocity on blood flow through a stenosed artery. Pulsatile flow of blood in a stenosed artery by using an axial velocity slip at the stenosed wall and by considering blood as Newtonian fluid, has been dealt by Biswas and Chakraborty (2010).

The annular flow of blood in a catheterized tapered artery, is investigated by Biswas and Chakraborty (2009). Biswas and Laskar (2012) have studied a Newtonian fluid model of blood flow, in a catheterized multistenosed artery with an axial velocity slip condition at the stenotic wall. Recently, Mathematical modeling of blood (Newtonian fluid) flow

throughinclined non-uniform stenosed artery, has been proposed by Biswas and Paul (2013).
Non-Newtonian Fluid Models:
A good deal of studies have revealed that in the vicinity of a stenosis, the shear rate of blood is low and so, non-Newtonian nature of blood in that section is pretty important (Misra *et al.*, 2008). In view of above, many researchers who have considered blood as a non-Newtonian fluid, while investigating its flow behaviour inside a stenosed artery, are incorporated here.

A. Casson Model
Due to the presence of substances like protein, fibrinogen and globulin in aqueous base plasma, human red blood cells form rouleaux (Fung, 1981;Boyd, 1963). As shear rate tends to zero, there forms a big aggregate which then behaves like a solid-viscoelastic or visco-plastic. If the rouleaux behave like a plastic then there exists a yield stress in Casson‟s fluid (Fung, 1981). Charm and Kurland (1965) have reported the utility of Casson‟s equation which can be used to analyze blood flow over a wide range of hematocrit and shear rates. The flow of Casson fluid in tapered tubes is dealt by Oka (1973). Chaturani and Samy (1982) have taken a stenosed two-layered mode with peripheral layer of Newtonian fluid and the core region of Casson fluid. Dash *et al*. (1995) have studied the changed flow pattern and estimated the increase of flow resistance in a narrow catheterized artery, by using the Casson fluid model of blood. Casson fluid models of blood flow for different flow situations and geometries have investigated by many authors (Chaturani and Palanisamy, 1990; Biswas and Bhattacherjee, 2003; Nagarani and Sarojamma, 2008). Sankar (2009) has proposed a two-layered pulsatile flow of blood inside a catheterized artery with core region as Casson fluid and plasma in a constricted rigid artery, by considering blood as a Casson fluid, has been studied by Verma *et al*. (2011). Recently, a mathematical model for blood flow in a catheterized stenosed artery has been examined by Sarojamma *et al*. (2012).

B. Bingham Plastic Models
A fluid that does not flow below a certain yield stress and above that stress, the increase in shear stress is proportional to the shear rate, is called the Bingham plastic fluid (Fung, 1984). This fluid is in the class of visco-inelastic non-Newtonian fluid (Kapur *et al*., 1982). Bingham plastic models are examined by several authors (MohanRao,1965; Kapur and Dikshit, 1965). Steady flow of Bingham plastic fluid through a circular tube subject to a pressure gradient and the usual zero-slip condition (Schlichting, 1968) at boundary, is investigated in the models (Fung, 1981). Biswas and Bhattacherjee (2003) have modeled the steady annular flow of Bingham fluid in a catheterized stenosed artery. Biswas and Chakraborty (2010) have studied a two-layered Bingham plastic fluid in a stenosed artery with a velocity slip condition at the boundary wall.

Conclusion:
Apart from above mentioned models, many other models viz. polar fluid model, Bingham plastic model, couple stress fluid model etc have been dealt with by many researchers over the years. Since the nature of blood is very complicated, it is not easy to choose an appropriate fluid model for it. However, many sincere attempts are being made in recent times to describe blood flow in arteries more accurately.

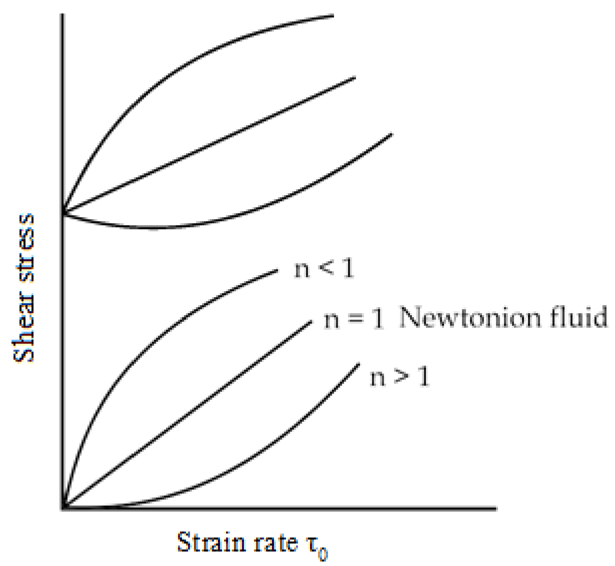
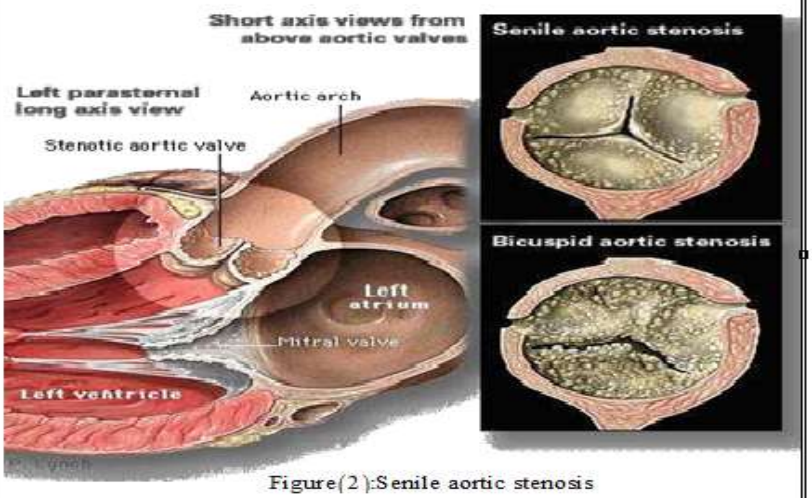


Fig.3 *Stress – strainraterelation for non – Newtonian Fluid*



Figure(2):Senile aortic stenosis

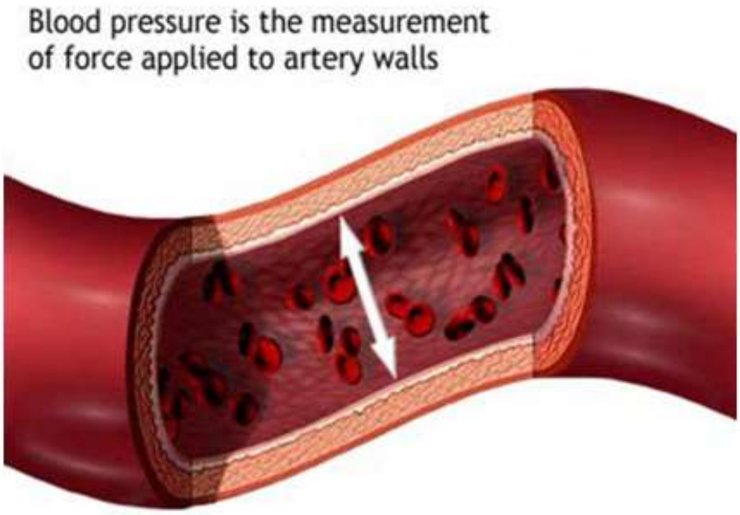


Fig.4 Blood pressure

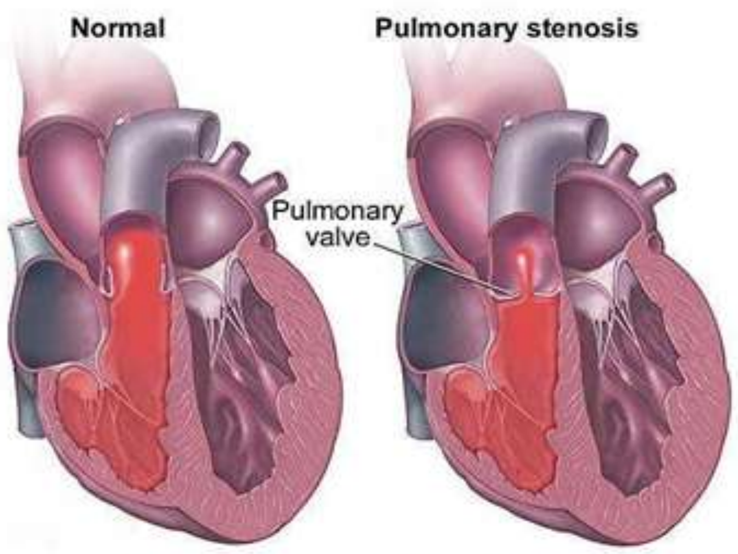


Fig.5: Pulmonary stenosis in heart

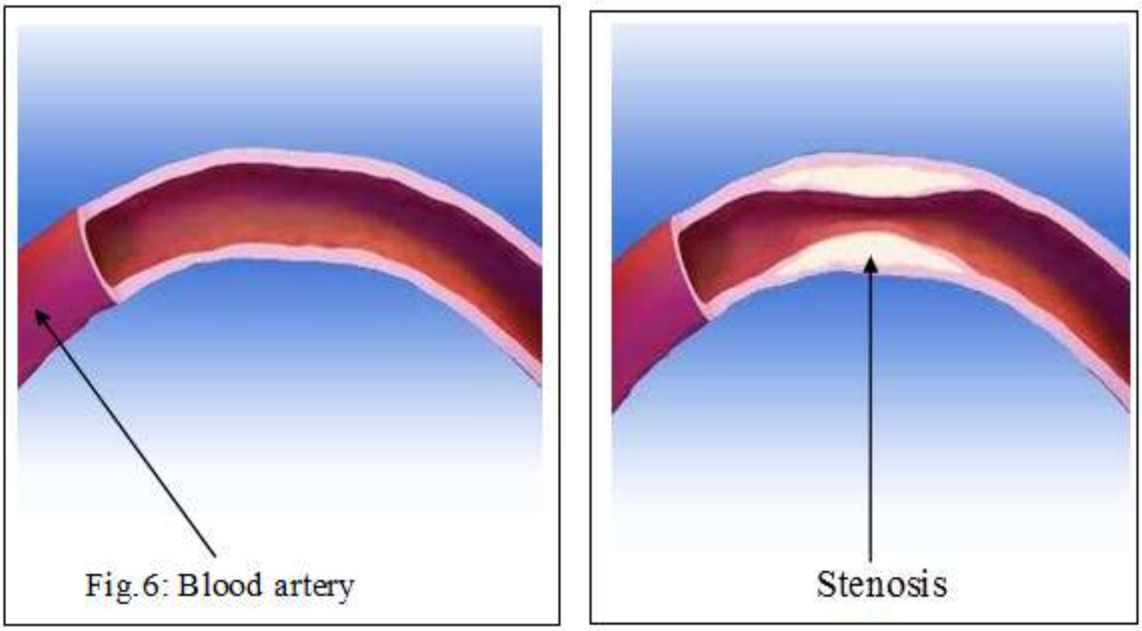


Fig.6: Blood artery

References:

1. Biswas, D(2000): Blood flow Models: A comparative Study. Mittal Publications, New Delhi, India
2. Fung, Y.C. (1981) Biomechanics: Mechanical properties of Living Tissues, Springer-Verlag, New York Inc.
3. Mac Donald, D.A. (1979). On steady flow through modeled vascular stenosis. J. Biomech. 12:13-20
4. Biswas, D.and Chakraborty, U.S.(2010).A Brief Review on Blood Flow Modeling in Arteries, Assam University Journal of Science &Technology,Silchar,India.Vol.6,pp.10-15
5. Chaturani, P. and Biswas, D (1983), Effects of slip in Flow Through stenosed tube, Physiological Fluid Dynamics: Proc. Of 1^{st} international Conf. on Physiological Fluid Dynamics ,September 5-7,pp. 75-80,IIT-Madras
6. Bugliarello, G.; Hayden, J.W. (1962) High speed micro cinematographic studies of blood flow in vitro, sciences, 138: 981-983.
7. Boyd, W. (1963) Text Book of pathology: Structure and Functions in Diseases. Lea and Febigar, Philadelphia
8. Brunn, P. (1975) The velocity slip of polar fluids. Rheol.Acta 14,1039-1054.
9. Biswas, D.; Chakraborty, U.S. (2009). Steady Flow of Blood through a Catheterized Tapered Artery with stenosis. A Theoretical Model .Assam University Journal of Science and Technology. 4(2): 7-16
10. Biswas, D. and Paul, M. (2013) Study of Blood flow inside an Inclined non-uniform stenosed artery, International Journal of Mathematical Archive-4(5), 1-10
11. Chaturani, P. and D. Biswas, (1983).A theoretical study of blood flow through stenosed arteries with velocity slip at the wall. Proc. First Internat. Symposium on Physiological fluid Dynamics,IIT Madras, India, pp: 23-26
12. Schlichting, H., (1968) Boundary Layer Theory, Mc Graw-Hill Book Company, New York
13. Biswas, D and Laskar,R.B (2012): A mathematical model for blood flow inside a catheterized Multistenosed artery, Assam university Journal of Science and technology, Vol.10, pp.53-64
14. Chaturani, P. and Biswas, D (1984), Effects of slip in Flow Through stenosed tube, Physiological Fluid Dynamics: Proc. Of 1^{st} international Conf. on Physiological Fluid Dynamics ,September 5-7,pp. 75-80,IIT-Madras
15. Nubar, Y. (1967): Effects of slip on the Rheology of a composite Fluid: Application to Blood Flow, Rheology, Vol.4,pp.133-150
16. Jayaraman and Tewari, (1995): Flow in Catheterized curved Arery. Medical and Biological Engineering and Computing (IFBME). 33: 1-6
17. . Nagarani, P. ; Sarojamma, G. (2008). Effect of Body Accleration on Pulsatile Flow of Casson Fluid through a mild stenosed Artery. Korea Australia Rheology Journal. 20(4): 189-196
18. Biswas, D; Bhattacherjee, A (2003). " Blood flow in Anular Region of a Catheterized Stenosed Artery". Proc. 30^{th} NC-FMFP, 104-110.
19. Sankar, D.S. (2009) "A two-fluid model for pulsatile flow in Catheterized blood vessels", International Journal of non-linear Mechanics.
20. Fung,Y.C. (1984): Biodynamics Circulation, Springer-Verlag, New York INC.
21. Merrill, E.W. (1965). "Rheology of human Blood and Some Speculations on its Role in vascular Homeostasis Biomechanical Mechanisms in vascular Homeostasis and Intravascular Thrombosis".P.N. sawyer (ed.). Appleton century Crofts. New York, 127-137.
22. Taylor, M.G. (1959). The Influence of Anomalous viscosity of Blood upon its Oscillatory flow. Physics in Medicine and Biology.3:273-290
23. Young, D.F. (1968). Effect of a Time-dependent stenosis on Flow Through a tube. J. Engng. Ind. Trans. AsME.90:248-254.

24. Young, D.F.; Tsai, F.Y. (1973): Flow Characteristics in Models of Arterial Stenosis-1: Steady Flow.J. Biomech.6(4): 395-410.
25. Chaturani, P.; Biswas, D. (1984). A Comparative Study of Poiseuille Flow of a Polar Fluid under Boundary conditions with Applications to Blood Flow. Rheol. Acta. 23(4): 435-446.
26. Chaturani, P.; Samy, S.P. (1982). ‘A Two-layered Model for Blood flow through stenosed Arteries’. Prc. 11^{th} NC on Fluid Mech. And Fluid Power, I.C.F. 16.9
27. Forrester, J.H.; Young, D.F. (1970). Flow Through a Converging- Diverging Tube and its Application in occlusive vascular Disease-1. J. Biomech. 3: 297-306.
28. Oka, S. (1973). Pressure Development in a non-Newtonian flow through a tapered tube. Biorheology.10: 207-212.
29. Mohan Rao, D.K. (1965). Helical Flow of a Bingham Plastic. J.Indian Inst. Sci.47: 97-105.
30. Chaturani, P and Palanisamy, V. (2009). Casson Fluid Model for Pulsatile Flow of Blood Under Periodic Body Acceleration, Bio-rheology, Vol.27, pp. 619-630 (1990).
31. Vand, V. (1984). Viscosity of Solutions and Suspensions, J.Phys. Colloid Chem., 52, 277-321.
32. Bloch, E.H. (1962). A Quantitative study of hemodynamics in the Living Macro-vascular system, Am. J. Anat.110,125-153.
33. Available online: http://biology.about.com/library/organs/heart/blcapillary.htm (last accessed on 26-04-2009)