Study of Mechanical Behavior of a Plate with Environment-Based Boundaries

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Abstract:
Finding a general mathematical solution for a rectangular plate within specific environment-based boundaries is the study's main goal. In this case, the temperature is maintained at zero on the edge x = 0, while the homogeneous boundary condition of the third kind is maintained at zero on the edges y = -b, b. the expression of the stress, displacement, and temperature functions as determined analytically.

Key-words: Rectangular Plate, Temperature, Stresses

INTRODUCTION
According to Adams and Bert (1999); Tanigawa, Komatsubara (1997) and Vihak (1998), the heating temperature and thermal stresses are examined in transient thermoelastic direct issues. A thin rectangular plate's inverse thermoelastic issue is examined in Khobragade and Wankhede (2002).
In order to find the temperature, displacement, and stress functions of a thin rectangular plate occupying the space D, the inverse unsteady-state thermoelastic problem is attempted to be solved here D: $0 \le x \le a$, $-b \le y \le b$ with homogeneous boundary conditions of the third kind is maintained at zero on the edges y = -b, b, and on the edge x = 0.

STATEMENT OF THE PROBLEM
Consider a thin rectangular plate occupying the space D : $0 \le x \le a$, $-b \le y \le b$. The displacement components u_x and u_y in the x and y- direction represented in the integral form as in [2] are $u_x = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial x^2} \right) + \alpha T \right] dx$

(1)

$$u_y = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2} \right) + \alpha T \right] dy \qquad (2)$$

ν and α are the Poisson’s ratio and the linear coefficient of thermal expansion of the material of the plate respectively and U(x,y, t) is the Airy’s stress function which satisfy the following relation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 U = -\alpha E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) T \qquad (3)$$

where E is the Young’s modulus of elasticity and T is the temperature of the plate satisfying the differential equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{k}\frac{\partial T}{\partial t} \qquad (4)$$

subject to the initial condition

$$T(x, y, 0) = 0 \qquad (5)$$

the boundary conditions

$$T(0, y, t) = 0 \qquad (6)$$

$$\left[T(x,y,t)+k_1\frac{\partial T(x,y,t)}{\partial y}\right]_{y=b}=0 \tag{7}$$

$$\left[T(x,y,t)+k_2\frac{\partial T(x,y,t)}{\partial y}\right]_{y=-b}=0 \tag{8}$$

where k is the thermal diffusivity of the material of the plate and k_1 and k_2 are the radiation constants on the two edges y = b, -b of the rectangular plate respectively.

The stress components in terms of U are given by

$$\sigma_{xx}=\frac{\partial^2 U}{\partial y^2} \tag{9}$$

$$\sigma_{yy}=\frac{\partial^2 U}{\partial x^2} \tag{10}$$

$$\sigma_{xy}=-\frac{\partial^2 U}{\partial x\partial y} \tag{11}$$

The equations (1) to (11) constitute the mathematical formulation of the problem under consideration.

SOLUTION OF THE PROBLEM

Applying finite Marchi- Fasulo integral transform to the equations (4), (5),(6) and using (7),(8) one obtains

$$\frac{d^2\overline{T}}{dx^2}-a_n{}^2\overline{T}=\frac{1}{k}\frac{d\overline{T}}{dt} \tag{12}$$

where the eigen values a_n are the solutions of the equation

$$\begin{aligned}&[\alpha_1 a\cos(ab)+\beta_1\sin(ab)][\beta_2\cos(ab)+\alpha_2 a\sin(ab)]\\&=[\alpha_2 a\cos(ab)-\beta_2\sin(ab)][\beta_1\cos(ab)-\alpha_1 a\sin(ab)]\end{aligned} \tag{13}$$

$$\overline{T}(x,n,0)=0 \tag{14}$$

$$\overline{T}(0,n,t)=0 \tag{15}$$

$$\overline{T}(\xi,n,t)=\overline{f}(n,t) \tag{16}$$

where $\overline{T}$ denotes the Marchi- Fasulo integral transform of T and n is a Marchi- Fasulo integral transform parameter, α_1, α_2, β_1 and β_2 are constants.

Applying Laplace transform to the equations (12), (15),(16) and using (14) one obtains

$$\frac{d^2\overline{T}^*}{dx^2}-q^2\overline{T}^*=0 \tag{17}$$

where $q^2=a_n{}^2+\frac{s}{k}$ (18)

$$\overline{T}^*(0,n,s)=0 \tag{19}$$

$$\overline{T}^*(\xi,n,s)=\overline{f}^*(n,s) \tag{20}$$

where $\overline{T}^*$ denotes the Laplace transform of $\overline{T}$ and s is a Laplace transform parameter.

The equation (17) is a second order differential equation whose solution is in the form

$$\overline{T}^*(x,n,s)=Ae^{qx}+Be^{-qx} \tag{21}$$

where A, B are arbitrary constants can be obtained by (19) and (20).

Applying inverse formula of finite Marchi-Fasulo integral transform Laplace transform one obtain the expressions for the temperature T(x, y, t) and the unknown function g(y, t) as

$$T(x,y,t)=\frac{2k\pi}{\xi^2}\sum_{n=1}^{\infty}\frac{P_n(y)}{\lambda_n}\sum_{m=1}^{\infty}(-1)^{m+1}m\sin\left(\frac{m\pi}{\xi}\right)x\ \times\int_0^t\overline{f}(n,t')e^{-k\left(a_n^2+\frac{m^2\pi^2}{\xi^2}\right)(t-t')}dt' \tag{22}$$

$$\overline{f}(n,t)=\int_{-b}^{b}f(y,t)P_n(y)\,dy \quad , \quad \lambda_n=\int_{-b}^{b}P_n^{\,2}(y)\,dy$$

$$P_n(y)=Q_n\cos(a_n y)-W_n\sin(a_n y)$$

$$Q_n=a_n(\alpha_1+\alpha_2)\cos(a_n b)+(\beta_1-\beta_2)\sin(a_n b)$$

$$W_n=(\beta_1+\beta_2)\cos(a_n b)+(\alpha_2-\alpha_1)a_n\sin(a_n b)$$

The equations (22) is the desired solutions of the given problem with $\beta_1 = \beta_2 = 1$ and $\alpha_1 = k_1$, $\alpha_2 = k_2$.

Substituting the value of T(x,y, t) from (22) in (3) one obtains the expression for Airy's stress function U(x,y,t) as

$$U(x,y,t)=-\alpha E\left(\frac{2k\pi}{\xi^2}\right)\sum_{n=1}^{\infty}\frac{P_n(y)}{\lambda_n}\sum_{m=1}^{\infty}(-1)^{m+1}m\sin\left(\frac{m\pi}{\xi}\right)x\ \times\int_0^t\overline{f}(n,t')e^{-k\left(a_n^2+\frac{m^2\pi^2}{\xi^2}\right)(t-t')}dt' \tag{23}$$

DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting the value of U(x,y,t) from (23) in (1) and (2) one obtains the thermoelastic displacement functions u_x and u_y as

$$\begin{aligned}u_x=&\left(\frac{2\alpha k}{\xi}\right)\sum_{n=1}^{\infty}\frac{P_n''(y)-P_n(y)}{\lambda_n}\sum_{m=1}^{\infty}(-1)^{m+1}\left[\cos\left(\frac{m\pi}{\xi}\right)a-1\right]\times\int_0^t\overline{f}(n,t')e^{-k\left(a_n^2+\frac{m^2\pi^2}{\xi^2}\right)(t-t')}dt'\\&+\left(\frac{2\alpha k\nu\pi^2}{\xi^3}\right)\sum_{n=1}^{\infty}\frac{P_n(y)}{\lambda_n}\sum_{m=1}^{\infty}(-1)^{m+1}m^2\left[\cos\left(\frac{m\pi}{\xi}\right)a-1\right]\times\int_0^t\overline{f}(n,t')e^{-k\left(a_n^2+\frac{m^2\pi^2}{\xi^2}\right)(t-t')}dt'\end{aligned} \tag{24}$$

$$\begin{aligned}u_y=&\left(\frac{2\alpha k\pi^3}{\xi^4}\right)\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left(\int_{-b}^{b}P_n(y)dy\right)\sum_{m=1}^{\infty}(-1)^{m+1}m^3\sin\left(\frac{m\pi}{\xi}\right)x\ \times\int_0^t\overline{f}(n,t')e^{-k\left(a_n^2+\frac{m^2\pi^2}{\xi^2}\right)(t-t')}dt'\\&+\left(\frac{2\alpha k\nu\pi}{\xi^2}\right)\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left(\int_{-b}^{b}P_n''(y)dy\right)\sum_{m=1}^{\infty}(-1)^{m+1}m\sin\left(\frac{m\pi}{\xi}\right)x\ \times\int_0^t\overline{f}(n,t')e^{-k\left(a_n^2+\frac{m^2\pi^2}{\xi^2}\right)(t-t')}dt'\\&+\left(\frac{2\alpha k\pi}{\xi^2}\right)\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left(\int_{-b}^{b}P_n(y)dy\right)\sum_{m=1}^{\infty}(-1)^{m+1}m\sin\left(\frac{m\pi}{\xi}\right)x\ \times\int_0^t\overline{f}(n,t')e^{-k\left(a_n^2+\frac{m^2\pi^2}{\xi^2}\right)(t-t')}dt'\end{aligned} \tag{25}$$

DETERMINATION OF STRESS FUNCTIONS

Using (23), in (9) , (10) and (11) the stress functions are obtained as

$$\sigma_{xx}=-\alpha E\left(\frac{2k\pi}{\xi^2}\right)\sum_{n=1}^{\infty}\frac{P_n''(y)}{\lambda_n}\sum_{m=1}^{\infty}(-1)^{m+1}m\sin\left(\frac{m\pi}{\xi}\right)x\ \times\int_0^t\overline{f}(n,t')e^{-k\left(a_n^2+\frac{m^2\pi^2}{\xi^2}\right)(t-t')}dt' \tag{26}$$

$$\sigma_{yy}=\alpha E\left(\frac{2k\pi^3}{\xi^4}\right)\sum_{n=1}^{\infty}\frac{P_n(y)}{\lambda_n}\sum_{m=1}^{\infty}(-1)^{m+1}m^3\sin\left(\frac{m\pi}{\xi}\right)x\ \times\int_0^t\overline{f}(n,t')e^{-k\left(a_n^2+\frac{m^2\pi^2}{\xi^2}\right)(t-t')}dt' \tag{27}$$

$$\sigma_{xy} = -\alpha E\left(\frac{2k\pi^2}{\xi^3}\right)\sum_{n=1}^{\infty}\frac{P_n'(y)}{\lambda_n}\sum_{m=1}^{\infty}(-1)^{m+1}m^2\cos\left(\frac{m\pi}{\xi}\right)x \times \int_0^t \overline{f}(n,t')e^{-k\left(a_n{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t-t')}dt' \qquad (28)$$

SPECIAL CASE

Set $f(y,t) = (1-e^{-t})(y-b)^2(y+b)^2\xi$ (29)

Applying finite Marchi-Fasulo integral transform equation (29) one obtains

$$\overline{f}(n,t) = \int_{-b}^{b}(1-e^{-t})(y-b)^2(y+b)^2\xi P_n(y)dy$$

$$= 4(k_1+k_2)\xi(1-e^{-t})\left[\frac{(a_n b)\cos^2(a_n b) - \cos(a_n b)\sin(a_n b)}{a_n{}^2}\right] \qquad (30)$$

Substituting the value of $\overline{f}(n,t)$ from (30) in the equations (22) to (30) we find the response of the temperature, displacement and stresses as a special case.

CONCLUSION

The author of this paper covered in detail the transient thermoelastic impact of a thin rectangular plate on the edge x = a, where the temperature of the rectangular plate is maintained at zero on the edge x = 0 and the homogeneous boundary condition of the third kind is maintained at zero on the edges y = -b, b. The analytical findings are obtained using the Laplace transform and the finite Marchi-Fasulo integral transform algorithms. The acquired temperature, displacement, and thermal stresses can be used in engineering applications to develop practical machines or structures. It is possible to derive any specific example of interest by giving the parameters and functions appropriate values.

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