Determination Of Optimal Inventory Level Using Suitable Inventory Models

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Abstract:
To find the ideal inventory level, appropriate models for a variety of real-world systems are built in inventory control. This work analyzes a novel kind of inventory model that makes use of the so-called Setting the Clock Back of Zero (SCBZ) characteristic. There are two machines M_1 and M_2 in series and the output of M_1 is the input of M_2. Hence a reserve inventory between M_1 and M_2 is maintained. The method of obtaining the optimal size of reserve inventory $\hat{s}$, assuming cost of excess inventory, cost of shortage and when the rate of consumption of M_2 is a constant, has already been attempted. In this paper, it is assumed that the repair time of M_1 is a random variable and the distribution of the same undergoes a parametric change after a truncation point X_0, which is taken to be a random variable. The optimal size of the reserve inventory is obtained under the above said assumption. Numerical illustration also provided.

Keywords: Reserve inventory, Truncation point, SCBZ property.

INTRODUCTION

A system which has two machines M_1 and M_2 are in series is considered. The output of M_1 is the input of M_2. The breakdown of M_1 results the idle time of M_2, since there is no input to the Machine M_2, whenever the Machine M_1 breakdown, it leads to the showdown of M_2 and this state continues till the Machine M_1 gets repaired. The idle time of M_2 is very costly and hence, to avoid the idle time of M_2, a reserve inventory is maintained in between M_1 and M_2.When there is huge inventory is kept as a reserve then it takes more carrying cost and when there is less inventory is kept, then it recurs idle time cost of M_2.Since the duration of repair time of $M_1$is high then the reserve inventory will be exhausted by M_2 .In order to balance these costs the optimal inventory must be maintained. The repair time of M_1 is a random variable and after the repair of M_1 is over, it supplies to the reserve inventory .During the repair time of M_1, the Machine M_2 gets the input from reserve.

Hanssman [1] discusses the very basic model of figuring out the best reserve inventory between two machines operating in series. The model for three machines has been described by Ramachandran et al. [2]. According to S. Sachithanantham et al. [3], the repair time of Machine M1 is a random variable with an exponential probability distribution, which satisfies the SCBZ Property. This model is used to determine the ideal reserve inventory between the two machines. Talwaker and Raja Rao discusses the SCBZ property [4]. S. Sachithanantham et al [5] discussed the optimal reserved inventory model between two Machines under the assumption that the probability distribution of repair time of M_1 is exponential, which satisfies the SCBZ property and the truncation point X_0 is taken assumed to be random variable. In this model the author assumed that the probability function of the truncation point was exponential.

The similar model was examined by Ramerthilagam et al. [6] under the supposition that the truncation point is a random variable and that its probability function has a uniform distribution. Among three machines, Venkatesan et al. [7] have talked about the ideal reserved inventory. L. Hentry et al.'s model [8] for figuring out the best reserve inventory between two machines is predicated on the idea that Machine M1's repair time truncation point follows a target generalized distribution. In their discussion of the same model, S. Sachithanantham et al. [9] assumed that the truncation point on the repair time is a random variable that follows a mixed exponential distribution. They also assumed that the probability function of Machine M1's repair time follows an exponential distribution that satisfies SCBZ Properties.

DETERMINE OPTIMAL INVENTORY LEVELS

The process of ascertaining the ideal stock levels to save expenses and increase a company's profitability is known as inventory optimization. It entails striking a balance between the expenses of stock-outs and lost sales as a result of inadequate inventory. Finding the ideal balance between having too little inventory, which can result in stock-outs and lost sales, and too much inventory, which requires handling and storage expenses and ties up cash, is the aim of inventory optimization.

Inventory optimization is the process of making data-driven decisions about inventory levels by utilizing a variety of methodologies, including statistical analysis, inventory management systems, and demand forecasts.

When deciding on the ideal inventory levels, businesses also take lead time, safety stock, reorder point, and carrying expenses into account. Businesses can enhance customer service, boost profitability, cut expenses, and better manage their working capital by optimizing their inventory.

Importance of Inventory Optimization:
Because inventory management may help firms stay competitive by lowering costs, satisfying consumer expectations, and increasing profits, it is crucial in today's business environment. Businesses may remain competitive and stay ahead of the curve by putting smart inventory optimization tactics into practice in an increasingly complicated and competitive environment. Inventory optimization has become increasingly important in today’s business environment for several reasons:

1. **Optimize Inventory to Meet Customer Demands:** By accurately forecasting customer demand and optimizing inventory levels, businesses can ensure that they have the right products available when customers need them.
2. **Reduce Costs:** Inventory optimization helps minimize costs associated with inventory management. By avoiding excessive inventory, businesses can reduce carrying costs such as storage, insurance, and obsolescence.
3. **Increase Profitability:** Effective inventory optimization directly impacts a company’s profitability. By aligning inventory levels with customer demand, businesses can reduce inventory carrying costs and minimize the risk of obsolescence.
4. **Be Competitive:** Inventory optimization plays a crucial role in maintaining competitiveness in the market. By having the right products available when customers want them, businesses can respond quickly to market trends and customer demands. This improves customer satisfaction and helps businesses gain a competitive edge over rivals.

By effectively managing inventory levels, businesses can achieve a balance between customer satisfaction and cost optimization, driving long-term success and sustainable growth.

Objective of Inventory Optimization:
Finding the ideal balance between having too little inventory, which can result in stock-outs and lost sales, and too much inventory, which requires handling and storage expenses and ties up money, is the major goal of inventory optimization. The ultimate aim is to reduce the total cost of inventory, which takes into account the expenses associated with stock outs and lost sales in addition to the cost of maintaining inventory. In order to achieve this goal, the following objectives are typically pursued through inventory optimization:

Minimize Stock-Outs:
The goal of inventory optimization is to reduce the quantity of stock-outs, which happen when a company cannot supply the demand of its customers for a specific good. Businesses may guarantee that they have the correct products in stock at the right time to prevent stock-outs and boost customer satisfaction by optimizing inventory levels.

Reduce Holding Costs:
The expenses incurred in processing and storing inventory, including storage, are referred to as holding costs. By lowering the amount of inventory required to match customer demand, inventory optimization helps firms to minimize these expenditures.

Reduce waste and obsolescence:
Businesses can decrease the amount of excess inventory they hold—i.e., inventory that could become obsolete or expire before it can be sold—by optimizing inventory levels and strengthening demand forecasting. Inventory management can be made more cost-effective and efficient by minimizing waste and obsolescence.

Factors to Consider in Inventory Optimization:
Inventory optimization involves balancing the costs of holding inventory with the costs of stock-outs and lost sales, and there are several factors that need to be considered in order to make effective decisions about inventory management:

1. **Demand Forecasting:** Accurate demand forecasting is critical for effective inventory optimization. This involves predicting future demand for products based on historical data and trends, as well as considering factors such as seasonality, promotional activities, and changes in market conditions.
2. **Lead Time:** Lead time refers to the amount of time it takes for a business to receive an order from a supplier and have it available for sale. This is an important factor to consider in inventory optimization, as it can impact the amount of inventory a business needs to hold in order to meet customer demand.
3. **Safety Stock:** Safety stock is extra inventory that a business holds in order to mitigate the risk of stock-outs. This is an important factor to consider in inventory optimization, as it can impact the amount of inventory a business needs to hold in order to meet customer demand.
4. **Holding Costs:** Holding costs refer to the costs associated with storing and managing inventory, such as storage and handling costs. These costs need to be considered in inventory optimization in order to minimize the total cost of inventory.

5. **Ordering Costs:** Ordering costs refer to the costs associated with placing an order with a supplier, such as shipping and handling costs. These costs need to be considered in inventory optimization in order to minimize the total cost of inventory.
6. **Stock-Out Costs:** Stock-out costs refer to the costs associated with stock-outs, such as lost sales and damage to a business's reputation. These costs need to be considered in inventory optimization in order to minimize the total cost of inventory.
7. **Seasonality:** Seasonality is a factor that can impact demand for certain products, and it is important to consider in inventory optimization in order to ensure that a business has the right products in stock at the right time.

Making wise decisions about inventory management requires taking into account a number of elements, including inventory optimization. Businesses can reduce the overall cost of inventory and increase customer satisfaction and profitability by taking into account variables including demand forecasting, lead time, safety stock, holding costs, ordering costs, stock-out costs, and seasonality.

Techniques for Inventory Optimization:

The reorder point approach, EOQ model, MRP, JIT inventory management, Kanban system, ABC analysis, and multi-echelon inventory optimization are a few methods that can be applied to inventory optimization. Businesses can optimize their inventory levels to reduce overall inventory costs, increase customer happiness, and boost profitability by employing these strategies.

There are some common techniques that can be used for inventory optimization, including:

1. **Reorder Point Method:** The reorder point method involves setting a specific point at which a business needs to reorder inventory to avoid stock-outs. This is calculated based on demand forecasting, lead time, and safety stock.
2. **Economic Order Quantity (EOQ) Model:** The EOQ model is a mathematical formula used to determine the optimal order quantity for inventory that minimizes the total cost of inventory, including holding costs and ordering costs.
3. **ABC Analysis:** ABC analysis is a categorization technique used to prioritize inventory management efforts by dividing inventory into categories based on their importance to a business.

Challenges in Inventory Optimization:

Inventory optimization can present several challenges, including:

- Accurate Demand Forecasting: Accurate demand forecasting is essential for effective inventory optimization, but it can be difficult to predict future demand with complete accuracy.
- Lead Time Variability: Lead time variability, or the time it takes to receive inventory after an order is placed, can make it difficult to determine the optimal inventory levels to maintain.
- Balancing Stock-outs and Overstocking: Balancing the risk of stock-outs with the cost of overstocking is a challenge in inventory optimization, as businesses must find the right balance between having enough inventory to meet demand and avoiding the cost of holding too much inventory.
- Inconsistent Data: Inconsistent data, such as incorrect product information or outdated inventory counts, can make it difficult to effectively optimize inventory levels.
- Lack of Visibility: Lack of visibility into inventory levels and supply chain processes can make it difficult to optimize inventory and make informed decisions about inventory management.
- Seasonal Variations: Seasonal variations in demand, such as holiday shopping or seasonal fluctuations, can make it difficult to determine the right inventory levels to maintain, especially for businesses that sell products that are highly seasonal.
- Unpredictable Market Conditions: Unpredictable market conditions, such as sudden changes in consumer demand, can make it difficult to maintain optimal inventory levels and respond to changes in demand.

To overcome these challenges and achieve effective inventory optimization, businesses must adopt a strategic and data-driven approach, invest in technology and resources, and continuously monitor and adjust their inventory management processes.

Technology Used for Inventory Optimization:

There are several technologies that can be used to support inventory optimization, including:

Inventory Management Software: Inventory management software helps businesses optimize inventory levels by providing real-time data on inventory levels, sales, and demand, as well as offering tools for forecasting demand and calculating optimal inventory levels.

Artificial Intelligence (AI) and Machine Learning (ML) Technologies: AI and ML technologies can be used to analyze data from various sources, including sales data, customer behavior, and supply chain data, to make more accurate demand forecasts and optimize inventory levels.

Radio Frequency Identification (RFID) Systems: RFID systems use wireless technology to track inventory in real-time, providing valuable insights into inventory levels and movement, enabling organizations to make informed decisions about inventory management.

By ***adopting these technologies, businesses can automate manual processes, access real-time data, and make informed decisions*** about inventory management, enabling them to optimize inventory levels and achieve better results. However, it's important to keep in mind that technology alone won't guarantee success, as effective inventory optimization requires a combination of technology, process improvement, and data-driven decision making.

Step by Step Process to Optimize Inventory Levels:

Here is a step-by-step process to optimize inventory levels:

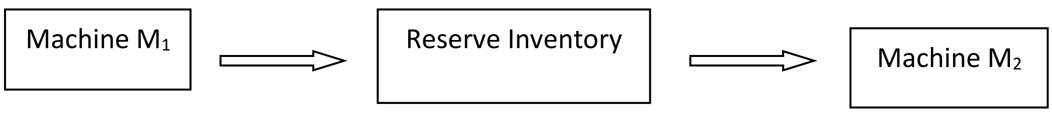
1. **Assess Current Inventory Management Processes:** Begin by evaluating current inventory management processes, including ordering and receiving processes, inventory tracking and control, and safety stock levels.
2. **Gather and Analyze Data:** Collect data on sales, demand, and inventory levels to gain insights into current inventory levels and demand patterns. This data can be used to create accurate demand forecasts and calculate optimal inventory levels.
3. **Develop a Demand Forecasting Model:** Use the data collected to develop a demand forecasting model that can accurately predict future demand based on historical data, market trends, and other relevant factors.
4. **Determine Optimal Inventory Levels:** Use the demand forecasting model and inventory data to determine the optimal inventory levels that will balance the need to maintain high service levels with the cost of inventory.
5. **Implement Inventory Control Processes:** Implement inventory control processes to maintain the optimal inventory levels determined, including regular reviews of inventory levels, automated reordering processes, and safety stock levels.
6. **Continuously Monitor and Adjust Inventory Levels:** Continuously monitor and adjust inventory levels based on changes in demand, market conditions, and other relevant factors, and make any necessary changes to the inventory management processes.

Optimizing inventory levels requires a data-driven and continuous improvement approach, along with the implementation of technology to automate manual processes, access real-time data, and make more accurate demand forecasts. By following this process, businesses can improve their inventory management processes, balance the cost of inventory with the need to maintain high service levels, and achieve better results.

The goal of inventory optimization is to strike a balance between having enough inventory to meet customer demand, while minimizing the cost and risk associated with holding too much inventory. By leveraging the right tools, techniques, and strategies, businesses can achieve this balance and achieve long-term success in a highly competitive marketplace.

SUITABLE INVENTORY MODELS USED IN INVENTORY OPTIMIZATION

The following diagram explains the system.



Notations:

h : Cost per unit time of holding one unit of reserve inventory

d : Cost per unit time of idle time of machine M_2

μ : Mean time interval between successive breakdowns of machine M_1 , assuming exponential distributions of inter-arrival times.

t : Continuous random variable denoting the repair time of M_1 with probability density function g(.) and CDF G (.).

r : Content consumption rate per unit time of machine M_2

S : Reserve inventory between M_1 and M_2

$\hat{S}$: Optimum reserve inventory

T : Random variable denoting the idle time of M_2

RESULTS

Model I:

If T is a random variable denoting idle time of M_2 then it is given by

$$\begin{cases} 0 & if\, t \le \frac{S}{r} \\ t - \frac{S}{r} & if\, t > \frac{S}{r} \end{cases}$$

Hence the expected total cost of inventory holding and idle time of M_2 per unit of time is given by

$$E(C) = hS + \frac{d}{\mu} E(T)$$

$$\Rightarrow E(C) = hS + \frac{d}{\mu}\int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)g(t)dt$$

The optimal reserve size $\hat{S}$can be obtained by solving the equation$\frac{dE(c)}{dS} = 0$

The expression for optimal reserve inventory is given by G$\left[\frac{\hat{S}}{r}\right] = 1 - \left[\frac{r\mu h}{d}\right]$

This is the basic model discussed in [1].

Model II :

In this Model, it is assumed that the repair time of machine M_1 is a random variable and undergoes a parametric change. That is the probability density function of the repair time in gamma distribution and it takes Parametric change after the truncation point X_0

ie.,
$$\begin{cases} g(\theta;\beta,t) if t \le x_0 \\ g(\theta^*;\beta,t) if t > x_0 \end{cases}$$

$$\begin{cases} g(\theta;\beta,t) = \frac{\theta^\beta}{\Gamma(\beta)}e^{-\theta t}t^{\beta-1} if t \le x_0 \\ g(\theta^*;\beta,t) = \frac{\theta^\beta}{\Gamma(\beta)}\theta^* e^{-\theta^* t}e^{x_0(\theta^*-\theta)} if t > x_0 \end{cases}$$

Thus, it can be shown that the distribution of repair time satisfies the so called SCBZ property as discussed [8] (1.4)

Ifx_0 is a random variable denoting that truncation point and it is distributed as exponential with parameter λ,then the probability density function of repair time can be written as

$f(t) = g(\theta,\beta;t)P[t \le x_0]$+$g(\theta^*,\beta;t)P[t > x_0]$

$$f(t) = g(\theta,\beta;t)e^{-\lambda t}dt + \lambda\int_0^t g(\theta^*,\beta;t)\,\lambda e^{-\lambda x_0}dx_0 dx_0$$

It may be observed that the random variable 'T' defined in equation (1) also undergoes a parametric change and the average idle time of M_2 is.

$$E(T) = \int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)f(t)dt$$

$$= \int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)g(\theta;\beta,t)e^{-\lambda t}dt + \int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)\left[\int_0^t g(\theta^*;\beta,t)\lambda e^{-\lambda x_0}dx_0\right]dt$$

Thus the expected total cost

$$E(C) = hS + \frac{d}{\mu}\left[\int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)g(\theta;\beta,t)e^{-\lambda t}dt + \int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)\left[\int_0^t g(\theta^*;\beta,t)\lambda e^{-\lambda x_0}dx_0\right]dt\right]$$

$$= hS + \frac{d}{\mu}\left[\int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)\frac{\theta^\beta}{\Gamma(\beta)}e^{-\theta t}e^{-\lambda t}\,dt + \int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)\left[\int_0^t e^{x_0(\theta^*-\theta)}\theta^* e^{-\theta^* t}\lambda e^{-\lambda x_0}dx_0\right]dt\right]$$

$$= hS + \frac{d}{\mu}\left[\frac{\theta^\beta}{\Gamma(\beta)}\int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)e^{-(\lambda+\theta)t}t^{\beta-1}dt \quad + \int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)\left[\int_0^t (e^{x_0(\theta^*-\theta)}\theta^* e^{-\theta^* t}\lambda e^{-\lambda x_0}dx_0)\right]dt\right]$$

$$= hS + \frac{d}{\mu}\left[\frac{\theta^\beta}{\Gamma(\beta)}\int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)e^{-(\lambda+\theta)t}t^{\beta-1}dt \quad + \frac{\lambda\theta^*}{(\lambda+\theta-\theta^*)}\int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)e^{-\theta^* t}\left[1 - e^{-i(\lambda+\theta-\theta^*)}\right]dt\right]$$

$$= hS + \frac{d}{\mu}\left[\frac{\theta^\beta}{\Gamma(\beta)}I_1 + \frac{\lambda\theta^*}{(\lambda+\theta-\theta^*)}I_2\right]$$

Where $I_1 = \int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)e^{-(\lambda+\theta)t}t^{\beta-1}dt$

$$I_2 = \int_{\frac{S}{r}}^{\infty}\left(t - \frac{S}{r}\right)e^{-\theta^* t}\left[1 - e^{-t(\lambda+\theta-\theta^*)}\right]dt$$

$$\frac{d}{dS}E(C) = 0$$

$$\frac{d}{dS}E(C) = 0 \Rightarrow h + \left(\frac{d\theta^{\beta}}{\mu\Gamma(\beta)}\right)\left(\frac{d}{dS}I_1\right) + \left(\frac{\lambda}{\mu}\frac{\lambda\theta^*}{(\lambda+\theta-\theta^*)}\right)\frac{d}{dS}I_2 = 0$$

It can seen that

$$\frac{d}{dS}I_1 = \frac{d}{dS}\int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)e^{-(\lambda+\theta)t}t^{\beta-1}dt$$

$$= \left(-\frac{1}{r}\right)\int_{\frac{S}{r}}^{\infty} e^{-(\lambda+\theta)t}t^{\beta-1}dt$$

$$= \left(-\frac{1}{r}\right)[\int_{0}^{\infty} e^{-(\lambda+\theta)t}t^{\beta-1}dt - \int_{0}^{\frac{S}{r}} e^{-(\lambda+\theta)t}t^{\beta-1}dt]$$

$$= \left(-\frac{1}{r}\right)\left[\frac{1}{(\lambda+\theta)^{\beta}}\int_{0}^{\infty} e^{-(\lambda+\theta)t}(\lambda+\theta)^{\beta}\,t^{\beta-1}dt - \int_{0}^{\frac{S}{r}} e^{-(\lambda+\theta)t}\,t^{\beta-1}dt\right] = \left(-\frac{1}{r}\right)\left[\frac{1}{(\lambda+\theta)^{\beta}}\Gamma(\beta) - \int_{0}^{\frac{S}{r}} e^{-(\lambda+\theta)t}t^{\beta-1}dt\right]$$

$$= \left(-\frac{1}{r}\right)\left[\frac{1}{(\lambda+\theta)^{\beta}}\Gamma(\beta) - \sum_{j=1}^{\beta}\frac{\prod_{i=1}^{j-1}(\beta-i)}{(\lambda+\theta)^{j}}e^{-(\lambda+\theta)\frac{S}{r}}\left(\frac{S}{r}\right)^{\beta-j}\right]$$

$$\frac{d}{dS}I_1 = \left(-\frac{1}{r}\right)\left[\frac{\Gamma(\beta)}{(\lambda+\theta)^{\beta}} + \sum_{j=1}^{\beta}\frac{\prod_{j=1}^{j-1}(\beta-i)}{(\lambda+\theta)^{j}}e^{-(\lambda+\theta)\frac{S}{r}}\left(\frac{S}{r}\right)^{\beta-j}\right]$$

$I_2$is considered as it is

$$\frac{d}{dS}I_2 = \frac{d}{dS}\int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)e^{-\theta^*}\left[1-e^{-t(\lambda+\theta-\theta^*)}\right]dt$$

$$= 0 - \frac{1}{r}f\left(\frac{S}{r},S\right) + \int_{\frac{S}{r}}^{\infty}\left(-\frac{1}{r}\right)\left[e^{-\theta^* t} - e^{-t(\lambda+\theta)}\right]dt$$

$$= -\frac{1}{r}\left\{\left[\frac{e^{-\theta^* t}}{-\theta^*}\right] - \left[\frac{e^{-t(\lambda+\theta)}}{-(\lambda+\theta)}\right]_{\frac{S}{r}}^{\infty}\right\}$$

$$= -\frac{1}{r}\left\{\left[\frac{e^{-\theta^*\frac{S}{r}}}{\theta^*}\right] + \left[0 - \frac{e^{-\left(\frac{S}{r}\right)(\lambda+\theta)}}{(\lambda+\theta)}\right]\right\}$$

$$= -\frac{1}{r}\left\{\frac{e^{\theta^*\frac{S}{r}}}{\theta^*} - \frac{e^{-\frac{S}{r}(\lambda+\theta)}}{(\lambda+\theta)}\right\}$$

$$\frac{d}{ds}E(C) = 0 \Rightarrow h - \left(\frac{d\theta^{\beta}}{\mu\Gamma(\beta)}\right)\left[\frac{\Gamma(\beta)}{(\lambda+\theta)^{\beta}} + \sum_{j=1}^{\beta}\frac{\prod_{i=1}^{j-1}(\beta-i)}{(\lambda+\theta)^{j}}e^{-(\lambda+\theta)\frac{S}{r}}\left(\frac{S}{r}\right)^{\beta-j}\right]$$

$$-\left(\frac{d\lambda\theta^*}{\mu r(\lambda+\theta-\theta^*)}\right)\left\{\frac{e^{-\theta^*\frac{S}{r}}}{\theta^*} - \frac{e^{-\left(\frac{S}{r}\right)(\lambda+\theta)}}{(\lambda+\theta)}\right\} = 0$$

$$\frac{h\mu r(\lambda+\theta-\theta^*)}{d} = \frac{\theta^{\beta}(\lambda+\theta-\theta^*)}{\Gamma(\beta)}\left[\frac{\Gamma(\beta)}{(\lambda+\theta)^{\beta}} + \sum_{j=1}^{\beta}\frac{\prod_{i=1}^{j-1}(\beta-i)}{(\lambda+\theta)^{j}}e^{-(\lambda+\theta)\frac{S}{r}}\left(\frac{S}{r}\right)^{\beta-j}\right] - \lambda\theta^*\left\{\frac{e^{-\theta^*\frac{S}{r}}}{\theta^*} - \frac{e^{-\left(\frac{S}{r}\right)(\lambda+\theta)}}{(\lambda+\theta)}\right\}$$

The value of S which satisfies the equation(1.4) for fixed values of $h,\mu,r,d,\lambda,\theta,\theta^*$and $\beta = 2$ or 3 optimal value of reserve inventory $\hat{\dot{S}}$can be obtained numerically. Select one parameter as varies but others fixed.

NUMERICAL ILLUSTRATION

If $\beta = 1$, the results reduce to results of [6]. The variations in the values of $\hat{\dot{S}}$ consequent to the changes in $h,\mu,r,d,\lambda,\theta,\theta^*$has been studied by taking numerical illustrations. The tables and the corresponding curves are given.

Case (i): The values of the constants are fixed arbitrary, $h = 5, \mu = 2, r = 30, \theta^* = 7, \lambda = 5$ and $d = 3000$ and the optimal inventory$\hat{\dot{S}}$for various values of θ.

θ	3	4	5	6
Ŝ	13.74366	12.5004	11.4756	10.6111

Case (ii): The values of the constants are fixed arbitrary, h = 5, μ = 2, r = 30, λ= 4, λ = 5 and d = 3000 and the optimal reserve inventory Ŝ for various values of θ *

θ *	4	5	6	7
Ŝ	17.2694	14.9895	13.5209	12.5004

Case (iii): The values of the constants are fixed arbitrary, h = 5, μ = 2, r = 30, θ= 4, θ *= 5, d = 3000 and the optimal reserve inventory Ŝ for various values of λ

λ	3	4	5	6
Ŝ	15.4903	15.2044	14.9895	14.8243

Case (iv): The values of the constants are fixed arbitrary, h = 4, r = 30, θ= 1.5, θ *= 3, λ = 5, d = 3200 and the optimal reserve inventory Ŝ for various values of μ

μ	2	2.5	3	3.5
Ŝ	29.3715	27.1103	25.2560	23.6821

Case (v): The values of the constants are fixed arbitrary, h = 4, μ = 2, θ = 4, θ *= 5, λ = 3, d = 3200 and the optimal reserve inventory Ŝ for various values of r

r	30	35	40	45
Ŝ	15.8992	17.4082	18.7609	19.9761

Case (vi): The values of the constants are fixed itrary, μ = 2, r = 30, θ = 1.5, θ *= 3, λ = 3, d = 3000 and the optimal reserve inventory Ŝ for various values of r

h	5	7	10	12
Ŝ	28.6889	25.0507	21.1146	19.0610

Case (vii): The values of the constants are fixed arbitrary, h = 5, μ = 2, r = 30, θ = 1.5, θ *= 3, λ = 5, and the optimal reserve inventory Ŝ for various values of d.

d	3000	3200	3500	3700
Ŝ	26.4547	27.1103	28.0193	28.5823

CONCLUSIONS

From the figures it could be seen that as the value of carrying cost 'h' increases, Ŝ decreases and suggest smallest inventory, If the idle time cost 'd' increases, increases which is quit justifiable.

If the rate of consumption of M_2 increases, the Ŝ also increases and it suggest a larger inventory. As the value of μ, parameter of the distribution of the inter arrival times between successive breakdowns of M_1 increases, then the average number of breakdowns per unit time decreases. Hence there is a decreases in the value of Ŝ and it is quit plausible.

As the value of θ increases, the parameter of the repair time distribution of M_1 increases then the average time to repair the machine M_1 decreases. Thus the repair time of machine M_1 is shorter and hence the optimal reserve inventory Ŝ decreases. A similar behavior in Ŝ is experienced when θ* increases.

Achieving a balance between meeting consumer demand and minimizing the expense and risk of having too much inventory on hand is the aim of inventory optimization. Businesses can attain long-term success in a fiercely competitive market by striking this balance and utilizing the appropriate tools, tactics, and strategies.

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