"A Study Of Some Coincidence And Common Fixed Point Theorem In Fuzzy Metric Spaces"

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ABSTRACT :
The research report in this thesis deals mainly with fixed point theorems and their applications. The F-type fuzzy topological spaces and quasi fuzzy metric spaces are introduced. Also, the variational principle and Christi's fixed point theorem in F-type fuzzy topological spaces are established, the results of which are utilized to obtain a fixed point theorem for Manger probabilistic metric space. The compatible pair of reciprocally continuous mappings is defined and a fixed point theorem in a fuzzy metric space is obtained which generates a fixed point but does not force the map to be continuous. Further, V|/-compatible mapping is introduced in a fuzzy metric space and established the altering distances between the points using certain control functions which differ from the previous works. R-weakly commuting of type (A) and non compatible mappings in fuzzy metric space are introduced which leads to the proof of fixed point theorem without assuming completeness of the space or continuity. In addition, the common fixed point theorem for sequence of self mappings is proved by using the notion of families of functions. The concept of compatibility is introduced in generalized fuzzy metric space and obtained common fixed point theorems for compatible mappings. Further, we prove some relations between compatible mappings and compatible maps of type (a) and (3) in generalized fuzzy metric spaces. Besides, applications of fixed point theorems in various fuzzy differential equations are discussed.

KEY WORD : fixed point theorem , F-type differential equation , fuzzy metric , fuzzy topology mathematics .

INTRODUCTION : The concept of fuzzy set was introduced by L.A. Zadeh in his classical paper. The fuzziness of a symbol lies in the lack of well-defined boundaries of the set of the objects to which this symbol applies. More specifically, let X be a reference set, also called a universe of discourse, covering a definite range of objects. Consider a subset A where transition between membership and non membership is gradual rather than abrupt. A fuzzy subset p of X is a mapping from X into unit interval [0,1]. considered fuzzy sets whose grade of membership lies in complete distributive lattice. If the fuzzy set p takes values '0' or '1', then p is an ordinary subset of X. The most common example of fuzzy sets are natural, for let X be a city and x is a citizen of X. Then ' x is tall in the city' is an imprecise statement. Here 'tall' is a vague description. How tall is tall ? No well-defined boundary exists between being tall and not being tall. Similarly ' x is beautiful' and 'x is ugly', 'x is good' etc, are all vague concepts. We can explain these kinds of concepts in real life situations with the help of fuzzy sets. The theory of fuzzy sets has proved it self to be of importance in pattern recognition problems, both using statical decision theoretic and syntactic approaches. was the one who applied the concept of fuzzy sets to topological spaces and generalized many of the concepts of general topology to what might be called fuzzy topological spaces.
In his paper, C.L. Change defined fuzzy topology on X, as family of fuzzy subsets T of X satisfying the following conditions

$$(a)\ \emptyset, X \in T$$
$$(b)\ if\ A, B \in T\ Then\ A \cap B \in T$$
$$(C)\ if\ A \in T\ for\ each\ i \in I\ then\ \cup A \in T$$

fuzzy pre uniform spaces. M. A. Erceg [10] introduced metric spaces in fuzzy set theory. He proved that a pseudo-quasi metric on a set may be regarded as a distance function between subsets of that set. This equivalent definition is generalized to fuzzy set theory where points need not have Boolean properties and hence in which a natural generalization of a pseudo-quasi metric is unsatisfactory. Additional axioms were introduced which corresponds to pseudo metrics and metrics in fuzzy set theory. defined fuzzy metric and fuzzy pseudo metric on the space of all fuzzy points by defining distance between two fuzzy points as a real number. continued the study of fuzzy pseudo metric spaces and introduced Separation axioms and modified version of fuzzy unit interval and proved Orisons Lemma. He also proved Baier theorem and contraction mapping theorem in Psuede metric spaces. introduced the concept of fuzzy metric space They defined the distance between two points in a fuzzy metric space as a non-negative upper semicontinuous, normal and convex fuzzy number. They studied some properties of fuzzy metric spaces and proved some fixed point theorems proved that some fuzzy metric spaces have completion.

Fuzzy sets are taken up with enthusiasm by engineers, computer scientists and operations researchers, particularly in Japan where fuzzy controllers are now an integral part of many manufacturing devices. A notable reason is the relationship that the fuzzy sets have a multi-valued logic, offering decision possibilities such as 'may be true' and 'may be false', suitably quantified in addition to the traditional dichotomy of true or false. Fixed point theorems in fuzzy mathematics are emerging with vigorous hope and vital trust. The study of Kramosil and Michalek's [17] of fuzzy metric space paved the way for very soothing machinery to develop fixed point theorems for contractive type maps. Applications of fuzzy fixed points emerges in the fields approximation theory, min-max problems, mathematical economics, variational inequalities, eigen value problems and boundary value problems. The objective of the thesis is to study the fixed point theorems in fuzzy metric spaces. To study the fixed point theorems in fuzzy metric spaces, the following types are analysed:

I) methods based on the contraction mappings.
II) methods based on the non expansive mappings.
III) methods based on the pointwise weakly commuting mappings.
IV) methods based on the compatible mappings.

Further, the application of fixed point theorems are applied in various fuzzy differential equations.
Now it is well recognized that this system embraces upon complexities and uncertainties in various physical schemes. Abstract coincidence and fixed point equation are of immense importance in interpreting various physical formulations. A recent attempt of to formulate the well known Banach contraction principle in fuzzy metric spaces in the sense of appears to provide a very soothing platform for developing Banach type fixed point theory. following Grabiec\s approach, have obtained Banach type fixed point theorems in fuzzy metric spaces. Following Grabiec's approach to Banach fixed point theorems in fuzzy metric spaces obtained some new coincidence theorems for a family of mappings on an arbitrary set with values in a fuzzy ,v metric space and derives a few general fixed point theorems for a family of mappings on a fuzzy metric space. These fixed point theorems are applied to obtain common solutions of fixed point type equations on product spaces. a fixed point theorem for a contractive type fuzzy map. Further, extended the same. Also generalized the result. obtained some common fixed point theorems for compatible maps of type(/3) on fuzzy metric space. proved a common fixed point theorem for a sequence of mappings in a fuzzy metric space. This results offers a generalization of theorem. Further, introduced /?-weakly commuting maps in fuzzy metric: space and proved common fixed point theorems., proved the common fixed point theorems for a pair of self-maps in a fuzzy metric space. This result offers a generalization of findings. The (ixed point theory for fuzzy mappings in complete metric spaces has periodically received certain attention (see [8, 12. 13, 11, 9, 20]). On the other hand, the theory of quasi-uniform, quasi-metric spaces has been considered and applied in the last years, by many authors, studied of hyperspaces, function spaces, fuzzy topology, fixed point theory, theoretical computer science etc. (see [3, 9]). introduced the concept of compatibility in fuzzy metric space and use it to prove common fixed point theorems for four compatible mappings which generalized the results of [4, 5]. Applications of fixed point theorems in fuzzy differential equations have been discussed. In the light of the above, some significant results have been obtained on the following topics:

*Variational principle of fixed point theorems in certain fuzzy topological spaces.
* Fixed points for compatible mappings in fuzzy metric space.
*Common fixed points for noncompatible mappings in fuzzy metric space.
* Fixed point theorems for a sequence of mappings in fuzzy metric space.
* Common fixed points of compatible maps in generalized fuzzy metric space. Applications of fixed point theorems are
* Existence and uniqueness of the fuzzy solution for a nonlinear fuzzy integrodifferential equation.
* Existence and uniqueness of the fuzzy solution for the nonlinear fuzzy Volterra integrodifferential equations.
* Existence and uniqueness of the fuzzy solution for a nonlinear fuzzy neutral functional differential equation.

1.1 Common fixed point theorems for weakly contractive and weakly caristi conditions in Cone metric spaces:
generalized the concept of metric spaces, replacing the set of real numbers by an ordered Banach space defining in this way a cone metric space. The metric space with distance was introduced by composed these concepts together and introduced cone metric space with w-distance and proved few fixed point theorems. In this section, we introduce cone G-metric space and prove fixed point theorems of weakly contractive and weakly Christi.

1.1.1. Lemma. Let X be a Cone G-metric space with metric G, let p be a w-distance on X and let f be a function from X into E that $0 < f(x)$
for any $x \in X$. Then a function q from $X \times X \times X$ into E given by

$$q(x,y,z) = f(x) + f(y) + p(x,y,z)$$

for each $(x,y,z) \in X \times X \times X$ is also a w-distance.

Proof: (i) $q(x,y,z) > 0$.

(ii) For every $x, y, z, \in$ X, $q(x,y,z)$

$$= f(x) + f(y) + p(x,y,z) < f(x) + f(a) + p(x,a,a) + f(y) + f(a) + p(a,y,z),$$
$$= q(x,a,a) + q(a,y,z).$$

(iii) It is obvious that the function f s lower semi-continuous.
(iv) Let $a \in E$ with 0 « a be fixed then since p is w-distance on X , there exists (3<E with 0 «(3 such that $p(x,a,a)$«{3, $q(a,y,z)$« $G(x,y,z)$ «a.

So assume $q(x,a,a)$«$(3, q(a,y,z)$«$(3$ $we\ have$

$$p(x,a,a) < f(x)+f(a)+p(x,a,a) = q(x,a,a)« p,$$
$$p(a,y,z) < f(a)+f(y)+p(a,y,z) = q(a,y,z)« p.$$
$$So\ p(x,a,a)«p, p(a,y,z) «p\ and\ it\ imply\ G(x,y,z)«a.$$

1.1.2. Lemma [11] There is not normal cone with normal constant M<1.

1.1.3. Proposition [11] For each k>l, there is a normal cone with normal constant M>k. [17] introduced cone metric space with w-distance and proved the following fixed point theorems :

1.1.4. Theorem. Let (X, d) be a complete cone metric space with w-distance p. Let P be a normal cone on X. Suppose a mapping T :X→ X satisfy the contractive condition P$(T_x, T_y) * k\, p(x,y)$ for all x, X, where ke [0 ,1) is a constant. Then, T has a unique fixed point in X. For each $x \in X$, the iterative sequence $\{T_n$ (x)} f converges to the fixed point.

1.2 Some results in fuzzy metric space :we establish some common fixed point and coincidence point results for a pair of non-linear mapping in 2-Banach space ,which mainly generalize the results of [12,14]

Theorem 1.2.1: Let X be a 2-Banach space with dim $X > 2$ and T be a continuous self mapping of X. Suppose that for any $u \in X$ there exists a function

$$\phi_u: [0,\infty)\ such\ that\ \parallel x - T_x, u \parallel \leq \phi_u(x) - \phi_u(Tx)$$

For all $x \in X$ then T has a fixed point in X

Theorem1.2.2: Let T be a self mapping of a 2 Banach space X

(dim $X > 2$) such that

$$\parallel Tx - T_y, u \parallel \leq h \parallel x - y, u \parallel\ for\ all\ x,y,z \in X$$
$$\text{where h is a constant in } (0,1)\text{. then T has a unique fixed point in X}$$

For all

Theorem 1.2.3 : Let E be a nonempty closed subset of a 2-Banach space X (with dim $X > 2$) and T be a self mapping of E such that for all x ,y in E and u in X,

$$\parallel Tx - T_y, u \parallel \leq a \parallel x - y, u \parallel + b\big(\parallel x - Tx, u \parallel + \parallel y - T_y, u \parallel\big) +$$
$$c\big(\parallel x - T_y, u \parallel + \parallel y - T_x, u \parallel\big)$$
$$for\ all\ x,y,z \in X$$
$$\text{where a, b, c are all strickly non negative constant with } a + 2b + 2c$$
$$\leq 1 \text{ then T has a unique fixed point z in Eand for } x \text{ in E}$$
$$T^n x \to z$$

Our first generalization goes as follows

Theorem 1.2.4 : Let S and T be two continuous self mappings of a 2-Banach space X. Suppose that for any u in X, there exists a function $«: $[0,\infty)$ —»$[0,\infty)$ such that

$$(i)\ \parallel Sx - TS_y, u \parallel \leq \phi_u(Sx) - \phi_u(TS_y)$$
$$(ii)\ \parallel Tx - ST_y, u \parallel \leq \phi_u(Tx) - \phi_u(ST_y)$$
$$for\ all\ x,y, \in X$$

Then T and S have a common fixed point

Proof : for a given $x_0 \in X$ we define a sequence recursively as

$x_{2n+1} = Sx_{2n}$, $x_{2n+2} = Tx_{2n}, n = 0,1 \ldots\ldots.$

We get for all $u \in X\ and\ n = 0,1 \ldots$

$$0 \leq \parallel x_{2n+1} - x_{2n+2}, u \parallel$$
$$0 \leq \parallel Sx_{2n} - Tx_{2n+1}, u \parallel$$
$$0 \leq \parallel Sx_{2n} - Tx_{2n}, u \parallel$$
$$0 \leq \phi_u(Sx_{2n}) - \phi_u(TSx_{2n})$$
$$= \phi_u(x_{2n+1}) - \phi_u(x_{2n+2})$$
$$\text{and } 0 \leq \parallel x_{2n} - x_{2n+1}, u \parallel$$
$$0 \leq \parallel x_{2n} - x_{2n+1}, u \parallel$$
$$= \parallel Tx_{2n-1} - Sx_{2n}, u \parallel$$
$$= \parallel Tx_{2n-1} - STx_{2n-1}, u \parallel$$
$$\leq \phi_u(Tx_{2n-1}) - \phi_u(STx_{2n-1})$$
$$\leq \phi_u(x_{2n}) - \phi_u(x_{2n+1})$$

We find that $\{\phi_u(x_n)\}$ is a monotonic decreasing sequence of real numbers and therefore there exists a number t_u such that $\phi_u(x_n) \to t_u\ as\ n \to \infty$ for all $u \in X$ further for any positive integer m and n with m>n and all $u \in X$ we have

$$\parallel x_n - x_m, u \parallel$$
$$\leq \parallel x_n - x_{n+1}, u \parallel + \parallel x_{n+1} - x_{n+2}, u \parallel \cdots + \parallel x_{m-1} - x_m, u \parallel$$
$$\leq \phi_u(x_n) - \phi_u(x_{n+1}) + \phi_u(x_{n+1}) - \phi_u(x_{n+2}) + \cdots .. \phi_u(x_{m-1}) - \phi_u(x_m)$$
$$= \phi_u(x_n) - \phi_u(x_m) \to 0\ as\ m,n \to \infty$$

Which implies that $\{x_n\}$ is a Cauchy sequence in X as such there exist a point z $\in X$ such that that $\{x_n\} \to z$ now continuity of S and T gives that

Tz = Sz = z

Thus S and T have common fixed point in X

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