Analysis Of Fuzzy Queueing Models To Optimize The Average System Size In Packet-Switching Network

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Abstract
Single server batch arrival retry queue with varying modes of breakdowns and two stages of restoration is considered under a fuzzy situation. Assuming the arrival, working, retry, breakdown, and repair rates are fuzzy; a mathematical programming method is proposed to construct the membership function for the average system size of the prescribed model. To convert a fuzzy repairable retrial queue into a family of crisp queues, the alpha-cut technique and Zadeh's extension concept are utilised. Trapezoidal fuzzy numbers are used to illustrate the strength of the proposed approach. Ranking fuzzy numbers play a huge role in decision-making under fuzzy conditions. This ranking method is the most reliable, simple to apply, and used to find the defuzzification of the syatem measures. A particular application in packet-switching network is given for a better understanding of the model.

Key words: Fuzzy queue, Server Breakdown, packet switching network, Zadeh's extension principle, alpha cut approach, Trapezoidal fuzzy number

1 Introduction and Relevant Literature

The remarkable extension of queues to the fuzzy world has practical consequences for decision analysis, operations research, computer technology, and abstract theory. Because of its practical applicability in real life, the fuzzy retrial queuing model has recently gained much interest. Fuzzy retrial queues have a variety of real-world applications, including communication networks, production models, financial sectors, tollbooths, and service stations, among others.

When the characteristics of the queuing system, such as the arrival rate and working rate, are identified precisely, effective methods for examining has been devised. However, these parameters may not be given accurately in some situations due to uncontrolled circumstances. In various practical applications, statistical data can be acquired individually; for example, the arrival rate and working rate are better expressed by language phrases such as rapid, modest, or sluggish than by a probability distribution based on statistical theory. This type of indefinite evidence will correctly identify the structure performance measurement.

Zadeh (1965, 1973, 1978) established the idea of fuzziness to deal with imperfect information. The notion of fuzzy set theory is well-known for modeling imprecision or uncertainty from mental processes. Yager (1986) discussed the extension concept and the ranging approach for defuzzification. Unfortunately, their method only supplied simple remedies. In other words, the performance measures' membership functions are not fully explained. Li and Lee (1989) suggested an all-encompassing method for queuing systems in a fuzzy environment. Analytical data for the M/F/1 and FM/FM/1 systems are offered to demonstrate the methodology. Fuzzy queues are far more realistic than the often utilised crisp queues in many real-world scenarios. Kao et al. (1999) employed computational programming to construct the membership functions of four fundamental fuzzy queues with one or two fuzzy variables: M/F/1, F/M/1, F/F/1, and FM/FM/1, where F represents fuzzy time and FM signifies fuzzified exponential time. Zimmermann (2001) highlighted various real-world applications of fuzzy set theory.

In a recent study on fuzzy queueing theory based on possibility theory, Buckley et al . (2001) explore the fundamental findings. Then, he use this to solve two application problems: the first is a machine servicing issue to finding the ideal amount of repair teams, and the second is a queueing decision issue to finding the ideal number of servers).

Chen (2005) demonstrated how to build membership functions for performance indicators in bulk service queuing systems with fuzzy arrival and working rates. Two fuzzy queues that are frequently seen in transportation management are used as examples to show the viability of the suggested technique. Machine interference is a prevalent problem in manufacturing and production activities. Because of uncontrollable reasons, the parameters of the machine interference problem may be ambiguous. Chen (2006) suggested a mathematical programming technique for constructing the membership function of the machine interference system's measure, with the machine breakdown rate and working rate being fuzzy numbers.

Ke and Lin (2006) defined the membership functions of the key variables of a queuing model with an unstable server, in which the customer arrival rate and operation rate, as well as the server failure rate and restoration rate, are all vague values. An efficient approach is offered to locate the best answers at each level of possibility. More information is available for management to utilise as a result of the system attributes being stated and managed by the membership functions. By extending the fuzzy environment, the proposed approach may more accurately represent fuzzy queues with an unstable server, and the analytical findings associated with this queuing model will be informative and relevant for system developers and users.

The system features of a queueing model with fuzzy customer arrival, retry, and operative rates were built by Ke et al. (2007). In this case, a fuzzy retrial queue is converted using the -cut technique into a family of traditional crisp retrial queues. A collection of parametric non-linear programmes has been built to characterise the family of crisp retrial queues using the membership functions of the system characteristics. More data is offered for management's usage since the membership functions express and regulate the system features. The fuzzy retrial-queue is more correctly represented by extending this model to the fuzzy nature, and analytical findings are improved.

In order to handle fuzzy threshold-based space priority buffers, Wang et al. (2009) created a discrete-time queueing model and tested its effectiveness in real-world scenarios. The analysis of the pertinent performance metrics, such as the packet loss probability of high-priority traffic and of low-priority traffic, is done using a matrix-analytic technique. According to intuition, the fuzzy threshold produces a reduced packet loss probability for low-priority packets by adapting effectively to various input traffic flow conditions and the packet loss rate criteria of high-priority packets. Kalyanaraman et al. (2010) investigated a retry queueing system with interruptions in a fuzzy environment.

Viswanathan et al. (2015) developed the non-linear parametric programs to give a description about crisp retrial family queues with Coxian 2 vacation. Comparative analysis of both crisp and fuzzy retrial two phase queueing model with Bernoulli vacation and restricted admission was given by Ebenesar Anna Bagyam and Udaya Chandrika (2019).

Fuzzy Markovian queues, in which all of the system constraints are fuzzy numbers, were studied by Chen et al. (2020) for the optimum and equilibrium techniques. Using approaches from fuzzy logic and queuing theory, this work investigates the membership functions of the optimal and equilibrium procedures in both tangible and intangible circumstances. In order to characterise the family of crisp techniques, we build two parametric nonlinear programmes using Zadeh's extension concept and the alpha - cut methodology. Then, in single and multi-server models, the membership functions of the tactics are determined. To estimate the stability approach in the fuzzy intelligence, the grated mean integration approach is also used.

Kannadasan and Padmavathi (2021) discussed about the fuzzy techniques with use of hexagonal fuzzy numbers and the authors have acquired the fuzzy environment with the presence of numerical results. Revathi and Selvakumari (2021) given the priority disciplines of fuzzy retrial models. Fuzzy retrial queue model is applied to most of the telecommunication systems. In this paper, we can find the suitable application of the prescribed model in a packet-switching network. As a result, a single server batch arrival retry queue with varying modes of breakdowns and two stages of restoration would be appropriate for the above-stated problem. The major goal of this article is to analyze the mean system size under fuzzy conditions.

The remaining part of the paper is organised as follows:: The construction of a fuzzy repairable queue is given in Section 2. By considering arrival, service, and repair rates as a trapezoidal fuzzy numbers, numerical results are given in Section 4. Section 5 presents results and discussion based on the numerical illustration. Finally, in the last section, the conclusion has been drawn by highlighting the novelty of the investigation.

2 Construction of Fuzzy Repairable Retrial Queue

2.1 Overview of the Model

Single server batch arrival queue with M modes of breakdown and two stages of repair with retrial is considered under a fuzzy environment. Although the server is running, it is susceptible to one of the M modes of breakdowns. The failing server requires necessary repair based on the kind of breakdown and optional common repair. When a server breaks, it stops serving clients and waits for the necessary repair to begin. The time spent waiting for the server to be repaired is referred to as setup time. As quickly as the necessary restoration is accomplished, the customer may choose common repair with a particular probability p or deny with the complimentary probability. Before the service provider went down, the client who was now receiving service either stayed in the service position with probbaility r or left the amenity area with complementary probability and kept coming back with certain rate. Once repaired, the server either waits for the same client or resumes serving the client who was stopped. This period of waiting is known as reserved time. Until the interrupted consumer exits the system, the server cannot accept new clients. If the server is busy or unavailable, it is referred to as blocked. (Ebenesar Anna Bagyam and Udayachandrika, 2018)

2.1 Notations Used

λ - Arrival rate μ - service rate η - retrial rate

ω - failure rate β_1, β_2 - necessary and common reapir rates

γ - setup rate m_1 and m_2 - First two moments of the batch arrival

2.2 Solution Methodology

Assume that the arrival rate (λ), retrial rate (η), service rate (μ), setup rate (γ), essential repair rate (β_1) and common repair rate (β_2) are represented by fuzzy numbers $\tilde{\lambda}$, $\tilde{\eta}$, $\tilde{\mu}$, $\bar{\gamma}$, $\tilde{\beta}_1$ and $\tilde{\beta}_2$ respectively. Let $\varphi_{\tilde{\lambda}}(x)$, $\varphi_{\tilde{\eta}}(y)$, $\varphi_{\tilde{\mu}}(s)$, $\varphi_{\tilde{\gamma}}(v_1)$, $\varphi_{\tilde{\beta}_1}(v_2)$ and $\varphi_{\tilde{\beta}_2}(v_3)$ denote the membership functions of the corresponding parameters. Then we have

$$\tilde{\lambda} = \{(x,\ \varphi_{\tilde{\lambda}}(x)) / x \in X\}$$

$$\tilde{\eta} = \{(y,\ \varphi_{\tilde{\eta}}(y)) / y \in Y\}$$

$$\tilde{\mu} = \{(s,\ \varphi_{\tilde{\mu}}(s)) / s \in S\}$$

$$\tilde{\gamma} = \{(v_1,\ \varphi_{\tilde{\gamma}}(v_1)) / v_1 \in V_1\}$$

$$\tilde{\beta}_1 = \{(v_2,\ \varphi_{\tilde{\beta}_1}(v_2)) / v_2 \in V_2\}$$

and $\tilde{\beta}_2 = \{(v_3, \varphi_{\tilde{\beta}_2}(v_3)) / v_3 \in V_3\}$

where X, Y, S, V_1, V_2 and V_3 are the crisp sets of arrival, retry, service, setup, essential repair and optional repair rates respectively. $L_s(\tilde{\lambda}, \tilde{\eta}, \tilde{\mu}, \bar{\gamma}, \tilde{\beta}_1, \tilde{\beta}_2)$ is fuzzy numbers since $\tilde{\lambda}$, $\tilde{\eta}$, $\tilde{\mu}$, $\bar{\gamma}$, $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are all fuzzy numbers. Based on Zadeh's extension principle (Zadeh, 1978), the membership function $L_s(\tilde{\lambda}, \tilde{\eta}, \tilde{\mu}, \bar{\gamma}, \tilde{\beta}_1, \tilde{\beta}_2)$ is defined as $\varphi_{L_s(\tilde{\lambda}, \tilde{\eta}, \tilde{\mu}, \tilde{\gamma}, \tilde{\beta}_1, \tilde{\beta}_2)}(z) = \varphi_{\tilde{L}_s}(z)$ Hence, $\varphi_{\tilde{L}_s}(z) =$

$$\sup_{\Omega} \min \{\varphi_{\tilde{\lambda}}(x), \varphi_{\tilde{\eta}}(y), \varphi_{\tilde{\mu}}(s), \varphi_{\tilde{\gamma}}(v_1), \varphi_{\tilde{\beta}_1}(v_2), \varphi_{\tilde{\beta}_1}(v_2), \varphi_{\tilde{\beta}_2}(v_3)\} \quad (1)$$

where $\Omega = \{x \in X, y \in Y, s \in S, v_1 \in V_1, v_2 \in V_2, v_3 \in V_3 \;/\; \frac{x\, m_1}{s}[1 + \sum_{i=1}^{M} \omega_i T_4] + (1 - A^*(x))\, m_1 < 1\}$

(2)

$$L_s(x, y, s, v_1, v_2, v_3) = \frac{T_2}{T_1} \frac{x\, m_1}{s} [1 + \sum_{i=1}^{M} \omega_i T_4] \quad (3)$$

$$T_1 = 1 - \frac{x\, m_1}{s} [1 + \sum_{i=1}^{M} \omega_i T_4] + m_1 (1 - A^*(x))$$

$$T_2 = [\frac{x\, m_1^2}{s} (1 + \sum_{i=1}^{M} \omega_i T_4) + \frac{m_2}{2}] (1 - A^*(x)) + \frac{T_3}{2}$$

$$T_3 = x^2 m_1^2 [\frac{2}{s^2} (1 + \sum_{i=1}^{M} \omega_i T_4)^2 + \frac{2}{s} \sum_{i=1}^{M} \omega_i [\frac{\bar{r}}{\tau}(\frac{1}{\tau} + \frac{1}{v_1} + \frac{1}{v_2} + \frac{p}{v_3}) + \frac{1}{v_1 v_2} + \frac{1}{v_1^2} + \frac{1}{v_2^2} + \frac{p}{v_3^2} + \frac{p}{v_1 v_3} + \frac{p}{v_2 v_3}]] + \frac{x\, m_2}{s} [1 + \sum_{i=1}^{M} \omega_i T_4]$$

$$T_4 = \frac{1}{v_1} + \frac{1}{v_2} + \frac{p}{v_3} + \frac{\bar{r}}{\tau}$$

and $A^*(x) = \frac{y}{y + x}$

The crisp intervals for the α-cuts of $\tilde{\lambda}$, $\tilde{\eta}$, $\tilde{\mu}$, $\tilde{\gamma}$, $\tilde{\beta}_1$ and $\tilde{\beta}_2$ and upper and lower bound of the invervals are tabulated as below:

Table 1 Crisp Intervals, Upper and Lower Bounds

	Crisp Intervals	Upper Bound	Lower Bound
λ(α)	$[\min_{x \in X} \{x / \varphi_{\tilde{\lambda}}(x) \geq \alpha\}, \max_{x \in X} \{x / \varphi_{\tilde{\lambda}}(x) \geq \alpha\}]$	$\max \varphi_{\tilde{\lambda}}^{-1}(\alpha) = x_\alpha^U$	$\min \varphi_{\tilde{\lambda}}^{-1}(\alpha) = x_\alpha^L$
η(α)	$[\min_{y \in Y} \{y / \varphi_{\tilde{\eta}}(y) \geq \alpha\}, \max_{y \in Y} \{y / \varphi_{\tilde{\eta}}(y) \geq \alpha\}]$	$\max \varphi_{\tilde{\eta}}^{-1}(\alpha) = y_\alpha^U$	$\min \varphi_{\tilde{\eta}}^{-1}(\alpha) = y_\alpha^L$
μ(α)	$[\min_{s \in S} \{s / \varphi_{\tilde{\mu}}(s) \geq \alpha\}, \max_{s \in S} \{s / \varphi_{\tilde{\mu}}(s) \geq \alpha\}]$	$\max \varphi_{\tilde{\mu}}^{-1}(\alpha) = s_\alpha^U$	$\min \varphi_{\tilde{\mu}}^{-1}(\alpha) = s_\alpha^L$
γ(α)	$[\min_{v_1 \in V_1} \{v_1 / \varphi_{\tilde{\gamma}}(v_1) \geq \alpha\}, \max_{v_1 \in V_1} \{v_1 / \varphi_{\tilde{\gamma}}(v_1) \geq \alpha\}]$	$\max \varphi_{\gamma}^{-1}(\alpha) = v_{1\alpha}^U$,	$\min \varphi_{\tilde{\gamma}}^{-1}(\alpha) = v_{1\alpha}^L$
$\beta_1(\alpha)$	$[\min_{v_2 \in V_2} \{v_2 / \varphi_{\tilde{\beta}_1}(v_2) \geq \alpha\}, \max_{v_2 \in V_2} \{v_2 / \varphi_{\tilde{\beta}_1}(v_2) \geq \alpha\}]$	$\max \varphi_{\tilde{\beta}_1}^{-1}(\alpha) = v_{2\alpha}^U$	$\min \varphi_{\tilde{\beta}_1}^{-1}(\alpha) = v_{2\alpha}^L$
$\beta_2(\alpha)$	$[\min_{v_3 \in V_3} \{v_3 / \varphi_{\tilde{\beta}_2}(v_3) \geq \alpha\}, \max_{v_3 \in V_3} \{v_3 / \varphi_{\tilde{\beta}_2}(v_3) \geq \alpha\}]$	$\max \varphi_{\tilde{\beta}_2}^{-1}(\alpha) = v_{3\alpha}^U$	$\min \varphi_{\tilde{\beta}_2}^{-1}(\alpha) = v_{3\alpha}^L$

To build the membership function $\varphi_{\tilde{L}_s}(z)$, α-cuts approach is used. $(L_s)_\alpha^L$ and $(L_s)_\alpha^U$ are the lower bound and the upper bound of the α-cuts of $\varphi_{\tilde{L}_s}(z)$ respectively. Thus, $(L_s)_\alpha^L = \min_{\Omega} L_s(x, y, s, v_1, v_2, v_3)$ and $(L_s)_\alpha^U = \max_{\Omega} L_s(x, y, s, v_1, v_2, v_3)$, subject to $x_\alpha^L \leq x \leq x_\alpha^U$, $y_\alpha^L \leq y \leq y_\alpha^U$, $s_\alpha^L \leq s \leq s_\alpha^U$, $v_{1\alpha}^L \leq v_1 \leq v_{1\alpha}^U$, $v_{2\alpha}^L \leq v_2 \leq v_{2\alpha}^U$ and $v_{3\alpha}^L \leq v_3 \leq v_{3\alpha}^U$. The membership function $\varphi_{\tilde{L}_s}(z)$ can be defined as follows:

$\varphi_{\tilde{L}_s}(z) = L(z), \text{if } (L_s)_{\alpha=0}^L \leq z \leq (L_s)_{\alpha=1}^L$

$\varphi_{\tilde{L}_s}(z) = 1, \text{if } (L_s)_{\alpha=1}^L \leq z \leq (L_s)_{\alpha=1}^U$ and

$\varphi_{\tilde{L}_s}(z) = R(z), \text{if } (L_s)_{\alpha=1}^U \leq z \leq (L_s)_{\alpha=0}^U$

where L(z) = inverse of $(L_s)_\alpha^L$ = Left shape function and R(z) = inverse of $L_q)_\alpha^U$ = Right shape function respectively.

2.3 Defuzzification - Ranking Method

For defuzzify the fuzzy value of mean system size into a crisp one Yager ranking index method is used.

The formula for Yager ranking index is $\frac{1}{2}\int_0^1 [(L_s)_\alpha^L + (L_s)_\alpha^U]\, d\alpha$,

Where $[(L_s)_\alpha^L, (L_s)_\alpha^U]$ is the α-cuts of $\tilde{L}_s$.

3 Numerical Illustration

Assume that arrival rate, retry rate, working rate, setup rate, essential repair rate and optional repair rates are trapezoidal fuzzy number such that $\tilde{\lambda}$ = [0.5, 1, 1.5, 2], $\tilde{\eta}$ = [25, 26, 27, 28], $\tilde{\mu}$ = [28, 29, 30, 31], $\tilde{\gamma}$ = [5, 6, 7, 8], $\tilde{\beta}_1$ = [15, 16, 17, 18] and $\tilde{\beta}_2$ = [17, 18, 19, 20] with the fixed values M = 4, ω_i = 2.5, m_1 = 2, m_2 = 6, p_i = 0.6, r_i = 0.5 and τ_i = 5 (i = 1, 2, 3, 4).

The values of $(L_s)_\alpha^L$ and $(L_s)_\alpha^U$ are obtained as

$(L_s)_\alpha^L$ = [(α + 1) (– 2 α^8 + 241 α^7 – 9153 α^6 + 26506 α^5 + 7631928 α^4 – 239135100 α^3 + 3300827064 α^2 – 22186570720 α + 59110464000)] / [10 (α – 8) (α – 18) (α – 20) (5 α^5 – 515 α^4 + 19061 α^3 – 308961 α^2 + 2091714 α – 4244640)]

and

$(L_s)_\alpha^U$ = [– (α – 4) (– 2 α^8 – 193 α^7 – 4596 α^6 + 95723 α^5 + 7010268 α^4 + 149453025 α^3 + 1557960000 α^2 + 8015423125 α + 16420016250)] / [10 (α + 5) (α + 15) (α + 17) (5 α^5 + 440 α^4 + 13331 α^3 + 163872 α^2 + 699000 α + 276000)]

Because of complexity in finding the closed form expression of devloping we analyze $\varphi_{\tilde{L}_s}(z)$ numerically. The numerical result of the membership function $\varphi_{\tilde{L}_s}(z)$ for different values of α is shown in Fig. 1. Moreover, $\varphi_{\tilde{L}_s}(z)$ appears a trapezoidal like structure as the arrival rate, retry rate, working rate, setup rate and repair rate (essential and optional) are trapezoidal fuzzy members. Figure 2 reports the α-cuts of expected system size (LL- Lower bound and LU - Upper bound) for different α-values.

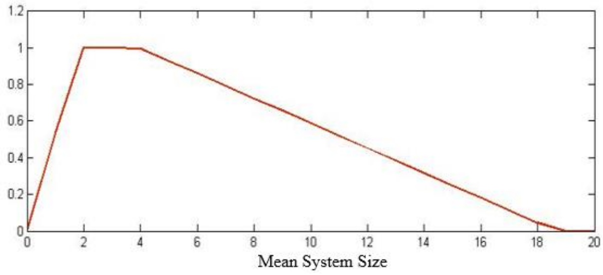

Fig. 1 Mean System Size's Membership Function

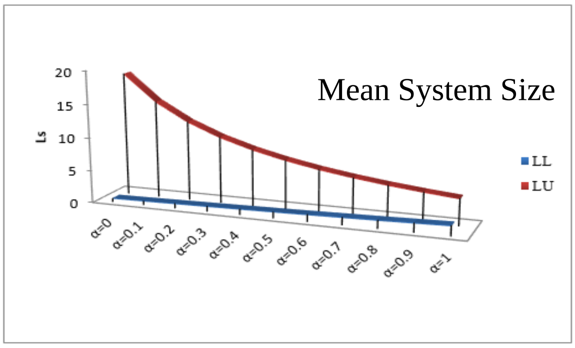

Fig. 2 α-cut values of Mean System Size

4 Discussion on the Results

The range of the system length for possibility level alpha = 1 is about [1.4479, 3.9387], suggesting that it is certainly feasible that the expected number of consumers in the system falls between 1.4479, 3.9387, however this is uncertain. The range of the system length for possibility level alpha = 0 is around [0.4835, 18.6644] at the other extreme. This range suggests that the mean system size will never fall below 0.4835 or increase above 18.6644. 4.6877 is the predicted system size when utilising the Yager ranking index approach. The information mentioned above will be very beneficial when developing a queueing system.

5 Practical Application of the model

Our retrial queue has potential uses in a packet-switching network, where messages are split into IP packets before being transmitted, in addition to its theoretical appeal. For instance, packet-switching technologies constitute the foundation of the majority of contemporary WAN protocols, such as TCP/IP, X.25, and Frame Relay. A router is a connected device in a packet-switching network that is used to transfer packets from a host server to a remote host. To transfer a package from a source host to a destination, the base station must first transmit the package to the router to which it is linked before sending the package to the remote host.

Suppose that packages arrive at the source host through a Poisson mechanism. All packages received by the host are immediately sent to its router. If the router is available, the package is accepted and sent right away, with the assumption that the transmission time will be evenly split. If not, the router rejects the package owing to current malfunctions or MTU (Maximum Transmission Unit) restrictions on the TCP/IP network path. According to FCFS, in this scenario, the blocked package must be retransmitted at a later time and is kept in the source host's buffer (referred to as the orbit). Furthermore, the router may malfunction whether it is idle or when sending packets, due to outside assaults or other technical issues.

We assume that until a packet arrives at the router, the network administrator in charge of fault management always conducts certain auxiliary activities while the router is idle and is always on service when the router is active. If a router fails while transmitting a packet, the network administrator can immediately repair it, and the router will resume broadcasting the paused packet as fast as the repair is completed. If the router fails while it is inactive, the restoration may not begin until the next packet from an outside source comes or until the orbit when the network manager emerges and immediately begins the router repair operation. The delayed period refers to the span between the epoch of the passive failure and the epoch of the next packet arriving. Once the repair for the passive failure is complete, the packet that came even during delayed interval can sometimes be transmitted immediately.

6 Conclusion and Future Work

The well-established classical retrial queuing systems, which are difficult but usually make assumptions that are too far from reality. But by using fuzzy retrial queue it describe realistic scenarios. Fuzzy retrial queues are more precise and perfect than conventional lineups. The model's output from this study may be used to analyze the variables impacting packet-switching networks. In the subsequent research, we will deal with generalizing alternative ways to assist decision-makers in deciding the path for modification in repairable retrial queues.

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