The Self-focusing of High Power Hollow Gaussian Laser Beam in Magnetized Plasma by Operating the Relativistic Nonlinearity

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Abstract

This article examines the influence of the order and intensity of a paraxial hollow Gaussian laser beam (HGLB) on the self-focusing phenomenon through magnetized plasma due to the relativistic nonlinearity. The appropriate differential equations of the nonlinear propagation of (HGLB) are derived then they have been solved numerically by designing a Matlab program. It is found that the self-focusing of (HGLB) will be decreased when its order are raised. It is observed that the HGLB is manifesting a soliton wave guide form at the order (n=2) of (HGLB) this happened because the equilibrium between the diffraction term and self-focusing term. as The relationship of laser beam intensity with self-focusing is important where The amount of self-focusing increases with increasing intensity of (HGLB) where $\alpha_0 = 0.9$. The self-focusing is weak to the point that it may not appear, but the phenomenon of natural diffraction may occur as a result of the absence of a relativistic nonlinear interaction if the intensity of the incident laser beam is insufficient for the occurrence of such an interaction inside the magnetised plasma.

Keywords: relativistic nonlinearity, laser beam self-focusing, paraxial riegen, order of laser beam, Hollow Gaussian laser beam

1-Introduction

In all contexts wheather they are experimental or theoretical, researchers have recently interested in producing stimulated positive lenses which are generated by self focusing of laser beam inside plasma[1-3]. The self-focusing phenomenon will work to increase the laser beam intensity and control its uses in a variety of future applications such as the ion-electron accelerators, coherent terahertz radiation and nuclear fusion plasma reactors [4-6]. In the field of laser-plasma nonlinear interactions, the most researcheers have interest in fundamental Gaussian mode (TEM_{00}) of laser beam and only fewer them employ the other laser modes such Hermite- Gaussian laser beam, cosh-Gaussian laser beam elliptic Gaussian laser beam and hollow Gaussian laser beam[7]. It is very known for laser designers that the hollow Gaussian laser mode is containing large energy inside laser cavity due to its large diameter comparing with fundamental Gaussian mode (TEM_{00}). as Because of its ring profile output, hollow laser beams in particular have a higher efficiency and better effect on the treatment of malignant lesions of the trachea, esophageal, and other cavity organs. Other engineering fields can also benefit from hollow laser beams. For instance, it is possible to measure and find underground tunnels using hollow laser beams [8]. the self-focusing of a strong hollow Gaussian laser beam in a collisionless plasma , its influence on the excitation of the electron plasma wave. The cumulative impact of relativistic nonlinearity taken into account in the interaction of the hollow-Gaussian laser beam with a plasma[9]. When a material is subjected to strong laser light, its refractive index changes, which leads to self-focusing. Two dominant processes govern how self-focusing works in laser plasma interactions. One is called the pondermotive nonlinerity which is brought on by redistribution of plasma electrons throug laser beam field profile where electrons will travel from higher laser field region toward lower laser field region .Other nonlinearity of laser plasma interaction is relativistic nonlinearity which is depending on the variation of plasma electron mass without change plasma electron position in ambient plasma[10, 11]..We chose a laser beam with a hollow Gaussian intensity profile [12], which is equivalent to an optical beam with zero intensity in the middle. Based on the extension of the eikonal and nonlinear dielectric constants to r2, where r is the distance from the beam's axis, this work has been done under the WKB and paraxial-ray approximations in the presence of relativistic a non-linearity [13,14].

The article will be organised as follows: The relativistic nonlinear dielectric tensor's final formula is computed in section (2). In section (3), relevant equations for the paraxial Hollow Gaussian laser beam's self-focusing is determined. Sections (4) and (5), respectively, introduce the numerical results with extensive discussion of the overall findings and conclusions.

2- Complex Dielectric Constants for Relativistic Plasma

In collisionless homogeneous plasma with electron density n_0, Consider the propagation of a right circularly polarized (RCP) intense hollow Gaussian beam (HGB) of frequency ω_0 and wave vector k_0 along the external magnetic field B_0 where $B_0 // z-$axis. The electric field vector E_{0+} for such a beam may be expressed in a cylindrical coordinate system with azimuthal symmetry as:

$$\vec{E}_+ = \vec{E}_{0+}\, e^{i(\omega_0 t - k_{0+} z)} \qquad (1)$$

where $\vec{E}_{0+} = \vec{E}_x + \vec{E}_y$ is the electric field amplitude.

The general motion equation of an electron in electromagnetic field is [15]

$$m_0 \gamma \frac{\partial}{\partial t}\vec{\upsilon} = -e\vec{E} - \frac{e}{c}\left(\vec{\upsilon}\times\vec{B}_0\right), \qquad (2)$$

where γ is the relativistic factor , $\vec{\upsilon}$ represents the oscillation velocity gained by laser beam and c the velocity of light.

In the (RCP) laser field, the electron oscillating velocity will modify as

$$\vec{\upsilon}_{0+} = \vec{\upsilon}_x + i\,\vec{\upsilon}_y = \frac{ie\vec{E}_{0+}}{m_0\gamma\omega_0\left(1-\frac{\omega_{ce}}{\gamma\omega_0}\right)}, \qquad (3)$$

where $\omega_{ce} = \frac{eB_0}{m_0 c}$ the cyclotron frequency, $-e$ and m_0 are the charge and rest mass of electron respectively and $\gamma = (1-\frac{\upsilon_{0+}^2}{c^2})^{-\frac{1}{2}}$.

Proposing ($1<\gamma<2$) (Hasson *et al.*) [16], the relativistic factor γ will be

$$\gamma \cong 1 + \frac{e^2}{m_e^2\, c^2\, \omega_0^2}\frac{\vec{A}_{0+}.\vec{A}_{0+}^{*}}{\left(1-\frac{\omega_{ce}}{\omega_0}\right)^2} = 1 + \alpha_+\,\vec{A}_{0+}\,\vec{A}_{0+}^{*}$$

(4)

It is impitent to mention that the relativistic nonlinearity factor $\alpha_+ = \frac{e^2}{2m_0^2c^2\omega_0^2}\cdot\frac{1}{(1-\frac{\omega_{ce}}{\omega_0})^2}$ will become zero at non-relativistic scope ($\gamma = 1$).

Due to the appearence of the relativistic nonlinearity so that the components of the tensor dielectric constants $\underline{\underline{\varepsilon}}$ in plasma medium will be modefied as following

$$\underline{\underline{\varepsilon}} = \begin{vmatrix} 1-\frac{\omega_p^2}{\omega_0^2\,\gamma\left(1-\frac{\omega_{Ce}^2}{\omega_0^2\gamma^2}\right)} & \frac{-i\left(\frac{\omega_p^2}{\omega_0^2\gamma}\right)\left(\frac{\omega_{Ce}}{\omega_0\,\gamma}\right)}{\left(1-\frac{\omega_{Ce}^2}{\omega_0^2\gamma^2}\right)} & 0 \\ \frac{i\left(\frac{\omega_p^2}{\omega_0^2\gamma}\right)\left(\frac{\omega_{Ce}}{\omega_0\,\gamma}\right)}{\left(1-\frac{\omega_{Ce}^2}{\omega_0^2\gamma^2}\right)} & 1-\frac{\omega_p^2}{\omega_0^2\,\gamma\left(1-\frac{\omega_{Ce}^2}{\omega_0^2\gamma^2}\right)} & 0 \\ 0 & 0 & 1-\frac{\omega_p^2}{\omega_0^2\gamma} \end{vmatrix}$$

(5)

The tensor components of the dielectric constants will contribute in the modification of the general wave equation of electromagnetic wave propagating through magnetized plasma to become as

$$\nabla^2\vec{E} - \nabla\left(\vec{\nabla}\cdot\vec{E}\right) + \frac{\omega_0^{\;2}}{c^2}\underline{\underline{\varepsilon}}\cdot\vec{E} = 0, \quad (6)$$

Depending upon the components of the delectric constants (see Eq. 5), the effective dielectric constant (ε_+) of a RCP laser wave (pump) may be given as following

$$\varepsilon_+ = \varepsilon_{xx} - i\varepsilon_{xy} = 1 - \frac{\frac{\omega_p^2}{\omega_0^2\gamma}}{\left(1-\frac{\omega_{Ce}}{\omega_0\gamma}\right)}$$

where $\omega_p = \sqrt{\left(\frac{4\pi n_0 e^2}{m_0}\right)}$ is the electron plasma frequency.

By Using Eq. (4) the effective dielectric constant ε_+ can be written as following

$$\varepsilon_+ = 1 - \frac{\left(\frac{\omega_p}{\omega_0}\right)^2}{\left(1-\frac{\omega_{Ce}}{\omega_0}\right)} + \frac{\left(\frac{\omega_p}{\omega_0}\right)^2}{\left(1-\frac{\omega_{Ce}}{\omega_0}\right)}\alpha_+\vec{A}_{0+}\vec{A}_{0+}^* \quad (7)$$

It is obvious that the effective dielectric constantε_+ consists of a linear partε_{0+} and a nonlinear part$\emptyset_+(\vec{A}_{0+}\vec{A}_{0+}^*)$, where the latter is appearing as a result of relativistic electron mass increasing. Both parts of the effective dielectric constant ε_+ may written as

$$\varepsilon_{0+} = 1 - \frac{\left(\frac{\omega_p}{\omega_0}\right)^2}{\left(1-\frac{\omega_{ce}}{\omega_0}\right)} \quad (8)$$

$$\phi_+ = \varepsilon_{2+}\vec{A}_{0+}\vec{A}_{0+}^* \quad (9)$$

where

$$\varepsilon_{2+} = \frac{1}{2}\left(\frac{e}{m_0\,c\,\omega_0}\right)^2\frac{\left(\frac{\omega_p}{\omega_0}\right)^2}{\left(1-\frac{\omega_{Ce}}{\omega_0}\right)^4} \quad (10)$$

3. Hollow Gaussian Laser Beam (HGLB) Self-focusing

Inside magnetized plasma, the laser wave ia a transverse wave as long as its field is varying along z-axis of external macnetic field larger than its variation across x-y wave front plane thus no space charge will occure therefore one may write [17]:

$$\vec{\nabla}\cdot\vec{D} = \vec{\nabla}\cdot\left(\varepsilon\vec{E}\right) = 0 \quad (11)$$

Putting the components of dielectric tensor in Eq. (11), one may obtain:

$$\frac{\partial E_z}{\partial z} \cong -\frac{1}{\varepsilon_{zz}}\left[\varepsilon_{xx}\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}\right) + \varepsilon_{xy}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\right] \quad (12)$$

Explioting the(E_x and E_y) components to rewrite the circular polarized electric field amplitude ($A_{0+} = E_x + iE_y$) and introducing the zero approximation,so the Eq.(12) and Eq.(6) may be combained togeather to become as following.

$$\frac{\partial^2 A_{0+}}{\partial z^2} + \frac{1}{2}\left(1 + \frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)A_{0+} + \frac{\omega_0^2}{c^2}(\varepsilon_{0+} + \varepsilon_{2+}A_{0+}A_{0+}^*)A_{0+} = 0 \quad (13)$$

It is important to refer that the product of nonlinear parts with $\frac{\partial^2 A_{0+}}{\partial x^2}$ or $\frac{\partial^2 A_{0+}}{\partial y^2}$ have been neglected [18].

Now by assuming $A'_{0+} = A_{0+}\,e^{i(\omega_0 t - k_{0+}z)}$ and substituting its value in Eq. (13), one can obtain

$$2ik_{0+}\frac{\partial A'_{0+}}{\partial z} = \frac{1}{2}\left(1 + \frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)A'_{0+} + \frac{\omega_0^2}{c^2}(\varepsilon_{2+}A'_{0+}A'^{*}_{0+})A_{0+} \quad (14)$$

Here A'_{0+} represents the complex amplitude of HGLB which its value a 0 is given as following

$$(A'_{0+})_{z=0} = E_{00}\left(\frac{x^2}{2x_0^2}\right)^n e^{-\left(\frac{x^2}{2x_0^2}\right)} \quad (15)$$

Where n=0 for GB and n≥1,2,…… for HGLB.

In two-dimensional Gaussian beam ($\frac{\partial}{\partial y} = 0$) ,by introducing an eikonal $A'_{0+} = A^0_{0+}\,e^{(i\,k_{0+}S_+)}$, where (A^0_{0+}) is the real functions and (S_+) is the phase for the laser beam inside magnetic , so Eq.

(14) may be seperated into real and imaginary parts, as following [19]:

$$2\frac{\partial S_+}{\partial z}+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\partial S_+}{\partial z}\right)^2+\frac{1}{2\,k_{0+}^2\,A_{0+}^0}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{\partial^2 A_{0+}^0}{\partial x^2}-\frac{\varepsilon_{2+}}{\varepsilon_{0+}}(A_{0+}^0)^2=0 \qquad (16a)$$

$$\frac{\partial (A_{0+}^0)^2}{\partial z}+\frac{1}{2}(A_{0+}^0)^2\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{\partial^2 S_+}{\partial x^2}+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{\partial S_+}{\partial x}\frac{\partial (A_{0+}^0)^2}{\partial x}=0$$

(16b)

The initial hollow Gaussian beam for (z>0) with initial beam radius x_0 is given by

$$(A_{0+}^0)^2=\frac{E_{00}^2}{2^{2n}f_{0+}^2}\left(\frac{x}{x_0 f_{0+}}\right)^{4n}e^{-\left(\frac{x}{x_0 f_{0+}}\right)^2}$$

(17)

where $f_{0+}(z)$ is the beamwidth parameter which refer to the normalization of laser beam spot size.

Now for detrmine the position of maximum intensity in HGLB,one may use the

$$\eta=\left[\left({}^{x}/_{x_{0f_{0+}}}\right)-\sqrt{2n}\right]$$

Rewritting Eq. (16a) and Eq. (16b) in term of η, so:-

$$\frac{\partial}{\partial x}=\frac{1}{x_0 f_{0+}}\frac{\partial}{\partial \eta} \qquad (18a)$$

$$\frac{\partial}{\partial z}=\frac{\partial}{\partial z}\frac{\sqrt{2n}+\eta}{f_{0+}}\frac{df_{0+}}{dz}\frac{\partial}{\partial \eta}$$

(18b) Using Eqs.(18a) and (18b) so that real part[Eq. (16a)] will be expressed as

$$2\left[\frac{\partial S_+}{\partial z}-\frac{(\sqrt{2n}+\eta)}{f_{0+}}\frac{df_{0+}}{dz}\frac{\partial S_+}{\partial \eta}\right]+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{A_{0+}^2}{x_0^2 f_{0+}^2}\left[\frac{\partial^2 S_+}{\partial \eta^2}+\frac{1}{(\sqrt{2n}+\eta)}\frac{\partial S_+}{\partial \eta}\right]$$
$$=\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{1}{k_{0+}^2 A_{0+}x_0^2 f_{0+}^2}\left(\frac{\partial^2 A_{0+}}{\partial \eta^2}+\frac{1}{(\sqrt{2n}+\eta)}\frac{\partial A_{0+}}{\partial \eta}\right)+\frac{\varepsilon_{+2}}{\varepsilon_{0+}}\frac{A_{0+}^2}{\left(1-\frac{\omega_c}{\omega}\right)^2} \qquad (19a)$$

Also$\left(\frac{\partial A_{0+}^2}{\partial z}-\frac{\sqrt{2n}+\eta}{f_{0+}}\frac{df_{0+}}{dz}\frac{\partial A_{0+}^2}{\partial \eta}\right)+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{A_{0+}^2}{x_0^2 f_{0+}^2}\left[\frac{\partial^2 S_+}{\partial \eta^2}+\frac{1}{(\sqrt{2n}+\eta)}\frac{\partial S_+}{\partial \eta}\right]$

$$+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{1}{x_0^2 f_{0+}^2}\frac{\partial A_{0+}^2}{\partial \eta}\frac{\partial S_+}{\partial \eta}=0$$

(19b)

Now,for magnetized plasma (longitudinal magnetized field), one may redrive

The Eq. (19b) to obtain as:-

$$\frac{2}{f_{0+}}\frac{df_{0+}}{dz}=\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{2\beta_+}{x_0^2 f_{0+}^2}$$

$$\therefore \beta_+(z)=2\left(1+\frac{\epsilon_{0+}}{\epsilon_{zz}}\right)^{-1}\frac{1}{f_{0+}}$$

$$\frac{d\beta_+}{dz}=\frac{2x_0^2}{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)}\left(\frac{df_{0+}}{dz}\right)^2+\frac{2x_0^2 f_{0+}}{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)}\frac{d^2 f_{0+}}{dz^2}$$

(20b)

Also∴ $\frac{2(\sqrt{2n}+\eta)^2 x_0^2 f_{0+}}{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)}\frac{d^2 f_{0+}}{dz^2}=$

$$\frac{1}{2}\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)}{k_{0+}^2 x_0^2 f_{0+}^2}\left[\frac{4n^2}{(\sqrt{2n}+\eta)^2}-\frac{4n(\sqrt{2n}+\eta)}{(\sqrt{2n}+\eta)}+2n+2\eta\sqrt{2n}+\eta^2+\frac{2n}{(\sqrt{2n}+\eta)^2}-1\right]+\frac{\varepsilon_{+2}}{\varepsilon_{0+}}\frac{A_{0+}^2}{\left(1-\frac{\omega_c}{\omega}\right)^2}\frac{E_{00+}^2}{2^{2n}f_{0+}^2}\left(\sqrt{2n}+\eta\right)^{4n}e^{-(\sqrt{2n}+\eta)^2}$$

(20a) By taking the terms which related with η^2 ,so: - $\frac{2\eta^2 x_0^2 f_{0+}}{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)}\frac{d^2 f_{0+}}{dz^2}=$

$$\frac{1}{2}\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\eta^2}{k_{0+}^2 x_0^2 f_{0+}^2}+\frac{\varepsilon_{+2}}{\varepsilon_{0+}}\frac{1}{\left(1-\frac{\omega_c}{\omega}\right)^2}\frac{E_{00+}^2}{2^{2n}f_{0+}^2}\left(\sqrt{2n}+\eta\right)^{4n}e^{-(\sqrt{2n}+\eta)^2} \qquad (21)$$

$$\frac{2x_0^2 f_{0+}}{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)}\frac{d^2 f_{0+}}{dz^2}=\frac{1}{2}\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)}{k_{0+}^2 x_0^2 f_{0+}^2}-4\frac{\varepsilon_{+2}}{\varepsilon_{0+}}\frac{1}{\left(1-\frac{\omega_c}{\omega}\right)^2}\frac{E_{00+}^2}{2^{2n}f_{0+}^2}\left[-2(2n)^{2n}e^{-2n}\right](2)^{2n}$$

(22)

$$\therefore \frac{d^2 f_{0+}}{dz^2}=\frac{1}{4}\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)^2}{k_{0+}^2 x_0^4 f_{0+}^3}-\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)}{\left(1-\frac{\omega_c}{\omega}\right)^2}\left[\frac{\varepsilon_{+2}E_{00+}^2}{\varepsilon_{0+}}\right]\frac{1}{x_0^2 f_{0+}^2}\left[(n)^{2n}e^{-2n}\right]$$

(23)

$$\frac{d^2 f_{0+}}{d\xi^2}=\frac{1}{4}\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)^2}{f_{0+}^3}-\frac{\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)}{\left(1-\frac{\omega_c}{\omega}\right)^2}\left[\frac{\varepsilon_{+2}E_{00+}^2}{\varepsilon_{0+}}\right]\frac{k_{0+}^2 x_0^2}{f_{0+}^2}\left[(n)^{2n}e^{-2n}\right]$$

(24)

Where $\xi={}^{z}/_{k_{0+}x_0^2}$ and $z^2=\xi^2 k_{0+}^2 x_0^4$

Equation 24 explains the self- foucsing in case paraxial,it is composed of two terms,the first term this equation mentioned to the nutral diffraction term.the sconnd term from right side this equation referred to self-foucsing term.

3-Discussion of numerical results

This study is based on the nonlinear interaction between the pump wave from a carbon dioxide (CO2) pulsed laser's Hollow Gaussian mode and the plasma of hydrogen. The last equation (24) have been numerically solved using the following set of experimentally determined parameters:

1. The wavelength of Carbon Dioxide (CO2) ($\lambda = 10.6\ \mu m$).
2. The angular frequency of Carbon Dioxide (CO2) pulsed laser ($\omega = 1.778 \times 10^{14}\ {rad}/_{sec}$).
3. The frequency of Carbon Dioxide (CO2) pulsed laser f= $\frac{\omega}{2\pi} = 2.83 \times 10^{13} HZ$.
4. Potential normlized vector has the laser strength parameters $\alpha_0 = \frac{eE_{00}}{m_e \omega_0 c} = 2.73 \times 10^{11}\ {V}/_{cm}$ and the initial laser beam intensities are $I = 10^{17}\ {W}/_{cm^2}$
5. Magnetic field B= $2 \times 10^4 Gass$
6. The order of Hollow Gaussian Laser Beam (HGLB) n= $0 - 3$

- Fig. (1) explain When n=0, in this case, the self-focusing of (HGLB) equation turns into the self-focusing of (GL) equation. Here, too, it will induce a positive lens.
- When n=1 Self-focusing occurs where the self-sorption limit overcomes the normal diffraction limit as a hollow positive lens induced(Ring positive lens).
- When n=2 The Self-focusing of (HGLB) will occur, there is a balance in the mentioned equation, and the beam will spread inside the plasma, approximately in the form of (Soliton).
- The higher order of (HGLB), the more difficult the Self-focusing process. This can be explained by the fact that (relativistic non-linearity) is weak in the case of increasing the order of (HGLB) , where the natural diffraction limit (which represents the first term of the Self-focusing equation) prevails. This is shown by the figure(1) when n =3.

In Figure (2), one may observe the decisive influence of the intensity of the hollow gaussian laser Beam($\alpha_0 = 0.7, 0.8, 0.9$) on the self-focusing of the (HGLB) . At low magnitudes of the laser beam intensity, the natural diffraction term will overcome the self-focusing term of the laser beam (see equation 24). By increasing the intensity values of the laser beam appropriately, so one can record a high increase in the self-focusing of (HGLB).

4. Conclusions

In this study one may conclude that the self-focusing of (HGLB) is more wreaking as long as the order (n) of (HGLB) is increased this is because most of the plasma electrons will move away from the center of the laser beam as the order increases, and thus they will contribute less to the self-focusing process. Figure (1) is showing that for Gaussian laser beam (n=0), the self-focusing process is very strong with comparing with the (HGLB)(i. e. n>0).

It is important to mention that the laser beam self-focusing is increased when the initial laser beam intensity is increased (see figure 2). This may be understood because that the raising laser intensity will give rise more oscillating velocity of the plasma electrons which leading to rise the electrons mass to contribute in the self-focusing phenomenon.

Acknowledgment

This work was partially supported by the Ministry of Higher Education and Scientific Research, Government of Iraq. The authors thank the Department of Physics, Faculty of Science, University of Kufa for valuable help.

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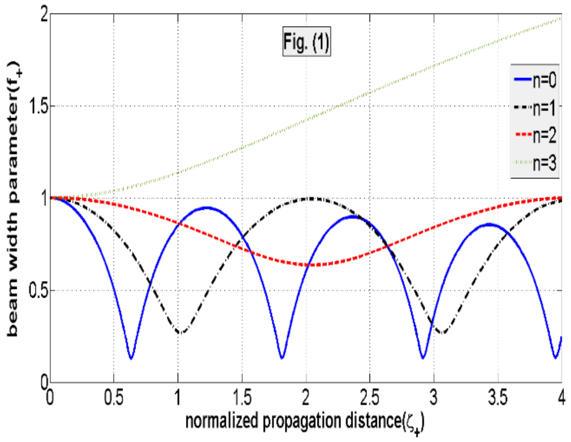
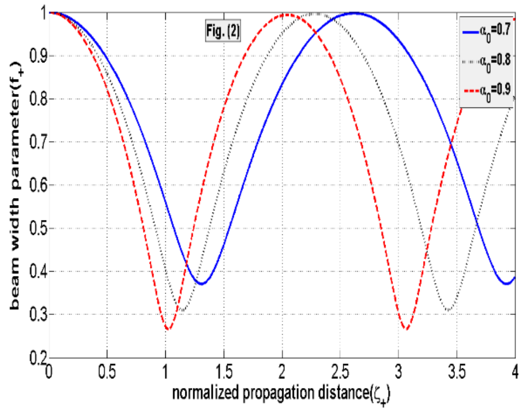
Figure Caption:

Fig. (1): (Color online) Variation of laser beam width parameter (f_0) with normalized distance $\left(\zeta = \frac{z}{k_0 x_0^2}\right)$ at different values of the hollow Gaussian laser beam order (n= 1, 2, 3) and Gaussian laser beam (n=0).

Fig(2). (Color online) Variation of laser beam width parameter (f_0) with normalized distance $\left(\zeta = \frac{z}{k_0 x_0^2}\right)$ at different values of the laser strength parameters($\alpha_0 = 0.7,0.8,0.9$) in paraxial region.





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