Quotient-4 Cordial Labeling of Some Unicyclic Graphs and Some Corona of Ladder Graphs

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Abstract
Let G (V, E) be a simple graph of order p and size q. Let $\varphi : V(G) \rightarrow Z_5 - \{0\}$ be a function. For each edge set E (G) define the labeling $\varphi^*: E(G) \rightarrow Z_4$ by $\varphi^*(uv) = \left\lceil \frac{\varphi(u)}{\varphi(v)} \right\rceil$ (mod 4) where $\varphi(u) \geq \varphi(v)$. The function φ is called Quotient-4 cordial labeling of G if $|v_\varphi(i) - v_\varphi(j)| \leq 1$, $1 \leq i, j \leq 4$, $i \neq j$ where $v_\varphi(x)$ denote the number of vertices labeled with x and $|e_\varphi(k) - e_\varphi(l)| \leq 1$, $0 \leq k, l \leq 3, k \neq l$, where $e_\varphi(y)$ denote the number of edges labeled with y. Here some unicyclic graphs such as (C_n; $K_{1,2}$), $C_m(1,2\ldots m)$, ($C_n(2P_m)$) and some corona of ladder graphs such as ($OL(\beta) \odot K_1$), ($CL(\beta) \odot K_1$), ($SL(\beta) \odot K_1$), ($ML(\beta) \odot K_1$), ($CRL(\beta) \odot K_1$), ($PL(\beta) \odot K_1$), ($PCL(\beta) \odot K_1$) and ($HL(\beta) \odot K_1$) proved to be quotient-4 cordial graphs.

Keywords: Cycle, Unicycle, Ladder graph, Corona graph, Quotient-4 cordial labeling, Quotient-4 cordial graph.

1. INTRODUCTION

Here the graphs considered are finite, simple, undirected and non-trivial. Graph theory has a good development in the graph labeling and has a broad range of applications. Refer Gallian [5] for more information. The cordial labeling concept was first introduced by Cahit [2]. H- and H2 –cordial labeling was introduced by Freeda S and Chellathurai R.S [3]. Mean Cordial Labeling was introduced by Albert William, Indira Rajasingh, and S Roy [1]. A graph G is said to be quotient-4 cordial graph if it receives quotient-4 cordial labeling .Let $v_\varphi(i)$ denotes the number of vertices labeled with i and $e_\varphi(k)$ denotes the number of edges labeled with k, $1 \leq i \leq 4$, $0 \leq k \leq 3$.

2. DEFINITIONS

Definition: 2.1[6] Let G (V, E) be a simple graph of order p and size q. Let $\varphi: V(G) \rightarrow Z_5 - \{0\}$ be a function. For each edge set E (G) define the labeling $\varphi^*: E(G) \rightarrow Z_4$ by $\varphi^*(uv) = \left\lceil \frac{\varphi(u)}{\varphi(v)} \right\rceil$ (mod 4) where $\varphi(u) \geq \varphi(v)$. The function φ is called Quotient-4 cordial labeling of G if $|v_\varphi(i) - v_\varphi(j)| \leq 1$, $1 \leq i, j \leq 4$, $i \neq j$ where $v_\varphi(x)$ denote the number of vertices labeled with x and $|e_\varphi(k) - e_\varphi(l)| \leq 1$, $0 \leq k, l \leq 3, k \neq l$, where $e_\varphi(y)$ denote the number of edges labeled with y.

Definition: 2.2 A graph $C_m(1,2,\ldots,m)$ is a graph obtained from a cycle of vertices x_1, $x_2 \ldots x_m$ having cycle of length m by attaching r pendant edges to each of r^{th} vertex. The pendant vertices are labeled as y_1, $y_2 \ldots y_s$, where $s = \frac{m\,(m+1)}{2}$.

Definition: 2.3 A graph (C_n; $K_{1,2}$) is obtained by attaching the root of $K_{1,2}$ at each vertex of a cycle C_n through an edge.

Definition: 2.4 A graph C_n ($2P_m$) is obtained by attaching P_m to the first and $(\frac{n+1}{2})^{th}$ vertices of an odd cycle C_n through an edge or by attaching P_m to the first and $(\frac{n}{2}+1)^{th}$ vertices of an even cycle C_n through an edge.

Definition: 2.5[7] An Open Ladder graph OL (β), $\beta \geq 3$ is obtained from two copies of path $P_{\beta-1}$ with the vertex set V (OL (β)) $= \{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E (OL (β)) $= \{u_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 < \theta < \beta\}$.

Definition: 2.6[7] A Closed Ladder graph CL (β), $\beta \geq 3$ is obtained by combining two copies of path $P_{\beta-1}$ with the vertex set V (CL (β)) $= \{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E (CL (β)) $= \{u_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 \leq \theta \leq \beta\}$.

Definition: 2.7[7] A Slanting Ladder graph SL (β), $\beta \geq 3$ is obtained by combining two copies of path $P_{\beta-1}$ with the vertex set V (SL (β) $= \{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E (SL (β)) $= \{u_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\}$.

Definition: 2.8[7] A Mobius Ladder graph ML (β), $\beta \geq 3$ is obtained by combining two copies of path $P_{\beta-1}$ with the vertex set V (ML (β)) $= \{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E (ML (β)) $= \{u_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 \leq \theta \leq \beta\} \cup \{u_1 v_\beta\} \cup \{v_1 u_\beta\}$.

Definition: 2.9[7] A Circular Ladder graph CRL (β), $\beta \geq 3$ is obtained by combining two copies of path $P_{\beta-1}$ with the vertex set V (CRL (β)) $= \{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E (CRL (β)) $= \{u_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 \leq \theta \leq \beta\} \cup \{u_1 u_\beta\} \cup \{v_1 v_\beta\}$.

Definition: 2.10[7] Consider the Closed Ladder graph CL (β), $\beta \geq 2$, introducing a vertex w_i between the vertices v_θ and $v_{\theta+1}$, for $1 \leq \theta \leq \beta - 1$ resulting a new graph called pentagonal ladder graph denoted by PL (β).

Definition: 2.11[7] Consider the Pentagonal Ladder graph PL (β), $\beta \geq 3$, by connecting the vertices v_1 and v_β by a new vertex w_β and connecting u_1 and u_β by an edge resulting a new graph called pentagonal circular ladder graph denoted by PCL (β).

Definition: 2.12[7] Consider the Closed Ladder graph CL (β), $\beta \geq 2$, by adding a new vertices t_i between the vertices u_θ and $u_{\theta+1}$, for $1 \leq \theta \leq \beta - 1$ and w_θ between the vertices v_θ and $v_{\theta+1}$, for $1 \leq \theta \leq \beta - 1$ resulting a new graph called hexagonal ladder graph denoted by HL (β).

Definition: 2.13[8][9] The corona product of a graph G_1 (with p points) and another graph G_2 is got by taking a copy of G_1 and p copies of G_2 say $G_{2,1}$, $G_{2,2}$, ... $G_{2,p}$ and then join every vertex in $G_{2,i}$ with the i^{th} vertex of G_1. If m numbers of pendent vertices are attached at each vertex of G, then the resultant graph obtained from G is the graph $G \odot mK_1$. When m = 1, $G \odot K_1$ is the corona of G.

3. MAIN RESULT

3.1 Unicyclic Graphs

Theorem: 3.1.1 The graph $G = C_m\,(1, 2 \ldots m)$ is Quotient-4 cordial if $m \geq 3$.

Proof: Let V (G) = { x_r: $1 \le r \le m$ } ∪ {y_s : $1 \le s \le \frac{(m^2+m)}{2}$ } and E (G) = {(x_rx_{r+1}) : $1 \le r \le m-1$} ∪ { x_1 x_m} ∪ { x_ry_s : $1 \le r \le m$, $\frac{r(r-1)}{2}+1 \le s \le \frac{r(r+1)}{2}$}.

Here |V (G) | = |E (G)| = $\frac{m(m+3)}{2}$.

Define φ : V (G) → {1, 2, 3, 4}.

Labeling of x_r's values are given below.

For $1 \le r \le m$.

φ(x_r) = 1.

Labeling of y_s's values are given below.

Case (i): When $m \equiv 0, 5$ (modulo 8) and $m \neq 5$.

φ(y_s) = 1 if $\frac{m(3m+9)}{8}+1 \le s \le \frac{m(m+1)}{2}$.

φ(y_s) = 2 if s ≡ 1 (modulo 3) and $1 \le s \le \frac{m(3m+9)}{8}-2$.

φ(y_s) = 3 if s ≡ 2 (modulo 3) and $2 \le s \le \frac{m(3m+9)}{8}-1$.

φ(y_s) = 4 if s ≡ 0 (modulo 3) and $3 \le s \le \frac{m(3m+9)}{8}$.

Case (ii): When $m \equiv 1, 4$ (mod 8) and $m \neq 4$.

φ(y_s) = 1 if $\frac{3m^2+9m+4}{8} \le s \le \frac{m(m+1)}{2}$.

φ(y_s) = 2 if s ≡ 1 (modulo 3) and $1 \le s \le \frac{3m^2+9m+4}{8}-1$.

φ(y_s) = 3 if s ≡ 2 (modulo 3) and $2 \le s \le \frac{3m^2+9m+4}{8}-3$.

φ(y_s) = 4 if s ≡ 0 (modulo 3) and $3 \le s \le \frac{3m^2+9m+4}{8}-2$.

Case (iii): When $m \equiv 2, 3$ (mod 8) and $m \neq 3$.

φ(y_s) = 1 if $\frac{3m^2+9m+2}{8} \le s \le \frac{m(m+1)}{2}$.

φ(y_s) = 2 if s ≡ 1 (modulo 3) and $1 \le s \le \frac{3m^2+9m+2}{8}-3$.

φ(y_s) = 3 if s ≡ 2 (modulo 3) and $2 \le s \le \frac{3m^2+9m+2}{8}-2$.

φ(y_s) = 4 if s ≡ 0 (modulo 3) and $3 \le s \le \frac{3m^2+9m+2}{8}-1$.

Case (iv): When $m \equiv 6, 7$ (mod 8).

φ(y_s) = 1 if $\frac{3m^2+9m+6}{8} \le s \le \frac{m(m+1)}{2}$.

φ(y_s) = 2 if s ≡ 1 (modulo 3) and $1 \le s \le \frac{3m^2+9m+6}{8}-2$.

φ(y_s) = 3 if s ≡ 2 (modulo 3) and $2 \le s \le \frac{3m^2+9m+6}{8}-1$.

φ(y_s) = 4 if s ≡ 0 (modulo 3) and $3 \le s \le \frac{3m^2+9m+6}{8}-3$.

Case (v): When $m = 3, 5$.

φ(y_s) = 2 if s ≡ 1 (modulo 3) and $1 \le s \le \frac{m(m+1)}{2}-2$.

φ(y_s) = 3 if s ≡ 2 (modulo 3) and $2 \le s \le \frac{m(m+1)}{2}-1$.

φ(y_s) = 4 if s ≡ 0 (modulo 3) and $3 \le s \le \frac{m(m+1)}{2}$.

Case (vi): When $m = 4$.

φ(y_s) = 2 if s ≡ 1 (modulo 3) and $1 \le s \le \frac{m(m+1)}{2}$.

φ(y_s) = 3 if s ≡ 2 (modulo 3) and $2 \le s \le \frac{m(m+1)}{2}-2$.

φ(y_s) = 4 if s ≡ 0 (modulo 3) and $3 \le s \le \frac{m(m+1)}{2}-1$.

By the result of above labeling we could see that, {φ (x_rx_{r+1}) : $1 \le r \le m-1$},{ φ (x_1 x_m)}, { φ (x_ry_s) : $1 \le r \le m$, $\frac{r(r-1)}{2}+1 \le s \le \frac{r(r+1)}{2}$} ∈ Z_4 and also for all i≠j ∈ [1, 4], |v_φ (i) – v_φ (j)| ≤ 1 and $k \neq l$ ∈ [0, 3], |e_φ (k) – e_φ (l)| ≤ 1. Thus by definition 2.1, the graph C_m (1, 2…m), $m \geq 3$ is Quotient-4 cordial labeling.

Theorem 3.1.2: A graph (C_n; $K_{1,2}$) is quotient-4 cordial if $n \geq 3$.
Proof: Let G be a (C_n; $K_{1,2}$) graph. Let V (G) = $\{a_\alpha : 1 \leq \alpha \leq n\} \cup \{b_\alpha : 1 \leq \alpha \leq n\} \cup \{c_\beta : 1 \leq \beta \leq 2n\}$ and E (G) = $\{a_\alpha a_{\alpha+1} : 1 \leq \alpha \leq n-1\} \cup \{a_1a_n\} \cup \{a_\alpha b_\alpha : 1 \leq \alpha \leq n\} \cup \{b_\alpha c_\beta : 1 \leq \alpha \leq n, 2\alpha - 1 \leq \beta \leq 2\alpha\}$.
Here $|V(G)| = |E(G)| = 4n$.
Define φ : V (G) → {1, 2, 3, 4}.

Labeling of a_α's values are given below.
For $1 \leq \alpha \leq n$.
$\varphi(a_\alpha) = 4$.

Labeling of b_α's values are given below.
For $1 \leq \alpha \leq n$.
$\varphi(b_\alpha) = 1$.

Labeling of c_β's values are given below.
For $1 \leq \beta \leq 2n$.
$\varphi(c_\beta) = 2$ if $\beta \equiv 1$ (modulo 2).
$\varphi(c_\beta) = 3$ if $\beta \equiv 0$ (modulo 2).

By the result of above labeling we could see that, $\{\varphi(a_\alpha a_{\alpha+1}) : 1 \leq \alpha \leq n-1\}, \{\varphi(a_1a_n)\}, \{\varphi(a_\alpha b_\alpha) : 1 \leq \alpha \leq n\}, \{\varphi(b_\alpha c_\beta) : 1 \leq \alpha \leq n, 2\alpha - 1 \leq \beta \leq 2\alpha\} \in Z_4$ and also for all i≠j ∈ [1, 4], $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and , $k \neq l$ ∈ [0, 3], $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Thus by definition 2.1, the graph (C_n; $K_{1,2}$), $n \geq 3$ is Quotient-4 cordial labeling.

Theorem 3.1.3: A graph C_n ($2P_m$) is quotient-4 cordial if n is odd and $n \geq 3$.
Proof: Let V (G) = $\{a_\alpha : 1 \leq \alpha \leq n\} \cup \{b_\beta : 1 \leq \beta \leq m\} \cup \{c_\omega : 1 \leq \omega \leq m\}$ and E (G) = $\{a_\alpha a_{\alpha+1} : 1 \leq \alpha \leq n-1\} \cup \{a_1a_n\} \cup \{b_\beta b_{\beta+1} : 1 \leq \beta \leq m-1\} \cup \{a_1b_1\} \cup \{c_\omega c_{\omega+1} : 1 \leq \omega \leq m-1\} \cup \{a_{\frac{n+1}{2}} c_1\}$.
Here $|V(G)| = n + 2m$, $|E(G)| = n + 2m$.
Define φ : V (G) → {1, 2, 3, 4}.

Labeling of a_α's are given below.
Case 1: When $n \equiv 1, 5, 9, 13$ (modulo 16).
For $1 \leq \alpha \leq n$.
$\varphi(a_\alpha) = 1$ if $\alpha \equiv 0, 3$ (modulo 8).
$\varphi(a_\alpha) = 2$ if $\alpha \equiv 5, 6$ (modulo 8).
$\varphi(a_\alpha) = 3$ if $\alpha \equiv 1, 2$ (modulo 8).
$\varphi(a_\alpha) = 4$ if $\alpha \equiv 4, 7$ (modulo 8).

Case 2: When $n \equiv 3, 11$ (modulo 16).
For $1 \leq \alpha \leq n-2$, the labeling of a_α values are same as case 1.
$\varphi(a_n) = 4$, $\varphi(a_{n-1}) = 1$.

Case 3: When $n \equiv 7, 15$ (modulo 16).
For $1 \leq \alpha \leq n-1$.
$\varphi(a_n) = 1$, the labeling of a_α values are same as case 1.

Labeling of b_β's are given below.

Case 1: When $n \equiv 1$ (mod 16) and $m \equiv 0, 1, 2, 3, 4, 5, 6, 7$ (modulo 8).
For $1 \leq \beta \leq m$.
$\varphi(b_\beta) = 1$ if $\beta \equiv 1, 3$ (modulo 4).
$\varphi(b_\beta) = 3$ if $\beta \equiv 0$ (modulo 4).
$\varphi(b_\beta) = 4$ if $\beta \equiv 2$ (modulo 4).

Case 2: When $n \equiv 3$ (mod 16) and $m \equiv 0, 1, 2, 3, 4, 5, 6, 7$ (modulo 8).
For $1 \leq \beta \leq m$.
$\varphi(b_\beta) = 1$ if $\beta \equiv 2, 5, 7$ (modulo 8).
$\varphi(b_\beta) = 3$ if $\beta \equiv 0, 1$ (modulo 8).
$\varphi(b_\beta) = 4$ if $\beta \equiv 3, 4, 6$ (modulo 8).

Case 3: When $n \equiv 5$ (modulo 16).
Sub Case 3.1: When $m \equiv 0, 1, 3, 4, 7$ (modulo 8).
For $1 \leq \beta \leq m$.
$\varphi(b_\beta) = 1$ if $\beta \equiv 1$ (modulo 8).
$\varphi(b_\beta) = 2$ if $\beta \equiv 3, 4, 6, 7$ (modulo 8).
$\varphi(b_\beta) = 3$ if $\beta \equiv 0, 2, 5$ (modulo 8).

Sub Case 3.2: When $m \equiv 2, 5$ (modulo 8).
For $1 \leq \beta \leq m - 1$, the labeling of b_β values are same as sub case 3.1.
$\varphi(b_m) = 2$.

Sub Case 3.3: When $m \equiv 6$ (modulo 8).
For $1 \leq \beta \leq m - 2$, the labeling of b_β values are same as sub case 3.1.
$\varphi(b_{m-1}) = 2$, $\varphi(b_m) = 3$.

Case 4: When $n \equiv 7$ (modulo 16).
Sub Case 4.1: When $m \equiv 0, 1, 2, 3, 4, 6, 7$ (modulo 8).
For $1 \leq \beta \leq m$.
$\varphi(b_\beta) = 1$ if $\beta \equiv 3, 5, 7$ (modulo 8).
$\varphi(b_\beta) = 3$ if $\beta \equiv 0, 4$ (modulo 8).
$\varphi(b_\beta) = 4$ if $\beta \equiv 1, 2, 6$ (modulo 8).

Sub Case 4.2: When $m \equiv 5$ (modulo 8).
For $1 \leq \beta \leq m - 2$, the labeling of b_β values are same as sub case 4.1.
$\varphi(b_m) = \varphi(b_{m-1}) = 2$.

Case 5: When n $\equiv$ 9 (modulo 16) and $m \equiv 0, 1, 2, 3, 4, 5, 6, 7$ (modulo 8).
For $1 \leq \beta \leq m$.
$\varphi(b_\beta) = 1$ if $\beta \equiv 1, 3, 5, 7$ (modulo 8).
$\varphi(b_\beta) = 3$ if $\beta \equiv 0, 4$ (modulo 8).
$\varphi(b_\beta) = 4$ if $\beta \equiv 2, 6$ (modulo 8).

Case 6: When $n \equiv 11$ (modulo 16).
Sub Case 6.1: $m \equiv 0, 1, 2, 3, 4, 5$ (modulo 8).
For $1 \leq \beta \leq m$.
$\varphi(b_\beta) = 1$ if $\beta \equiv 1, 4, 7$ (modulo 8).
$\varphi(b_\beta) = 3$ if $\beta \equiv 0, 3$ (modulo 8).
$\varphi(b_\beta) = 4$ if $\beta \equiv 2, 5, 6$ (modulo 8).

Sub Case 6.2: $m \equiv 6, 7$ (modulo 8).
For $1 \leq \beta \leq m - 1$, the labeling of b_β values are same as sub case 6.1.
$\varphi(b_m) = 2$.

Case 7: When $n \equiv 13$ (modulo 16).
Sub Case7.1: When $m \equiv 0, 1, 3, 4, 7$ (modulo 8).
For $1 \leq \beta \leq m$.
$\varphi(b_\beta) = 1$ if $\beta \equiv 1$ (modulo 8).
$\varphi(b_\beta) = 2$ if $\beta \equiv 3, 4, 6, 7$ (modulo 8).
$\varphi(b_\beta) = 3$ if $\beta \equiv 0, 2, 5$ (modulo 8).

Sub Case 7.2: When $m \equiv 2, 5$ (modulo 8).
For $1 \leq \beta \leq m - 1$, the labeling of b_β values are same as sub case 7.1.
$\varphi(b_m) = 2$.

Sub Case 7.3: When $m \equiv 6$ (modulo 8).
For $1 \leq \beta \leq m - 2$, the labeling of b_β values are same as sub case 7.1.
$\varphi(b_{m-1}) = 2$, $\varphi(b_m) = 4$.

Case 8: When $n \equiv 15$ (modulo 16).

Sub Case 8.1: $m \equiv 0, 1, 2, 6, 7$ (modulo 8).
For $1 \leq \beta \leq m$.
$\varphi(b_\beta) = 1$ if $\beta \equiv 1, 3$ (modulo 8).
$\varphi(b_\beta) = 2$ if $\beta \equiv 6, 7$ (modulo 8).
$\varphi(b_\beta) = 3$ if $\beta \equiv 0$ (modulo 8).
$\varphi(b_\beta) = 4$ if $\beta \equiv 2, 4, 5$ (modulo 8).

Sub Case 8.2: When $m \equiv 3, 4$ (modulo 8).
For $1 \leq \beta \leq m-1$, the labeling of b_β values are same as sub case 8.1.
$\varphi(b_m) = 3$.

Sub Case 8.3: When $m \equiv 5$ (modulo 8).
For $1 \leq \beta \leq m-2$, the labeling of b_β values are same as sub case 8.1.
$\varphi(b_{m-1}) = 3$, $\varphi(b_m) = 4$.

Labeling of c_ω's are given below.

Case 1: When $n \equiv 1$ (modulo 16)
Sub Case 1.1: When $m \equiv 0, 1, 2, 3, 4, 5, 7$ (modulo 8).
For $1 \leq \omega \leq m$.
$\varphi(c_\omega) = 2$ if $\omega \equiv 1, 2, 5, 6$ (modulo 8).
$\varphi(c_\omega) = 3$ if $\omega \equiv 0, 7$ (modulo 8).
$\varphi(c_\omega) = 4$ if $\omega \equiv 3, 4$ (modulo 8).

Sub Case 1.2: When $m \equiv 6$ (modulo 8).
For $1 \leq \omega \leq m-1$, the labeling of c_ω values are same as sub case 1.1.
$\varphi(c_m) = 3$.

Case 2: When $n \equiv 3$ (mod 16).
Sub Case 2.1: $m \equiv 0, 1, 2, 3, 4, 6, 7$ (modulo 8).

For $1 \leq \omega \leq m$.
$\varphi(c_\omega) = 1$ if $\omega \equiv 4$ (modulo 8).
$\varphi(c_\omega) = 2$ if $\omega \equiv 1, 2, 6, 7$ (modulo 8).
$\varphi(c_\omega) = 3$ if $\omega \equiv 3, 5$ (modulo 8).
$\varphi(c_\omega) = 4$ if $\omega \equiv 0$ (modulo 8).

Sub Case 2.2: When $m \equiv 5$ (modulo 8).
For $1 \leq \omega \leq m-1$, the labeling of c_ω values are same as sub case 2.1
$\varphi(c_m) = 2$.

Case 3: When $n \equiv 5$ (modulo 16).
Sub Case 3.1: When $m \equiv 0, 1, 2, 3, 4, 7$ (modulo 8).
For $1 \leq \omega \leq m$.
$\varphi(c_\omega) = 1$ if $\omega \equiv 0, 3, 5$ (modulo 8).
$\varphi(c_\omega) = 3$ if $\omega \equiv 4$ (modulo 8).
$\varphi(c_\omega) = 4$ if $\omega \equiv 1, 2, 6, 7$ (modulo 8).

Sub Case 3.2: When $m \equiv 5$ (modulo 8).
For $1 \leq \omega \leq m-1$, the labeling of c_ω values are same as sub case 3.1
$\varphi(c_m) = 4$.

Sub Case 3.3: When $m \equiv 6$ (modulo 8).
For $1 \leq \omega \leq m-2$, the labeling of c_ω values are same as sub case 3.1
$\varphi(c_{m-1}) = 4$, $\varphi(c_m) = 1$.

Case 4: When $n \equiv 7$ (modulo 16).
Sub Case 4.1: When $m \equiv 0, 1, 2, 3, 4$ (modulo 8).
For $1 \leq \omega \leq m$.
$\varphi(c_\omega) = 1$ if $\omega \equiv 1$ (modulo 8).
$\varphi(c_\omega) = 2$ if $\omega \equiv 3, 4, 5, 6$ (modulo 8).
$\varphi(c_\omega) = 3$ if $\omega \equiv 2, 7$ (modulo 8).
$\varphi(c_\omega) = 4$ if $\omega \equiv 0$ (modulo 8).

Sub Case 4.2: When $m \equiv 5, 7$ (modulo 8).
For $1 \leq \omega \leq m-3$, the labeling of c_ω values are same as sub case 4.1
$\varphi(c_{m-2}) = 1$, $\varphi(c_{m-1}) = 4$, $\varphi(c_m) = 3$.

Sub Case 4.3: When $m \equiv 6$ (modulo 8).
For $1 \leq \omega \leq m-1$, the labeling of c_ω values are same as sub case 4.1
$\varphi(c_m) = 3$.

Sub Case 4.4: When $m \equiv 7$ (modulo 8).
For $1 \leq \omega \leq m-2$, the labeling of c_ω values are same as sub case 4.1.

φ (c_{m-1}) =4, φ (c_m) = 3.

Case 5: When $n \equiv 9$ (modulo 16).
Sub Case 5.1: When $m \equiv 0, 1, 2, 3, 4, 7$ (modulo 8).
For $1 \leq \omega \leq m$.
φ (c_ω) = 2 if $\omega \equiv$ 2, 3, 4, 5 (modulo 8).
φ (c_ω) = 3 if $\omega \equiv$ 0, 7 (modulo 8).
φ (c_ω) = 4 if $\omega \equiv$ 1, 6 (modulo 8).

Sub Case 5.2: When $m \equiv 5, 6$ (modulo 8).
For $1 \leq \omega \leq m - 1$, the labeling of c_ω values are same as sub case 5.1.
φ (c_m) = 3.

Case 6: When $n \equiv 11$ $(mod\ 16)$ and $m \equiv 0, 1, 2, 3, 4, 5, 6, 7$ (modulo 8).
For $1 \leq \omega \leq m$.
φ (c_ω) = 1 if $\omega \equiv$ 5 (modulo 8).
φ (c_ω) = 2 if $\omega \equiv$ 0, 1, 2, 3 (modulo 8).
φ (c_ω) = 3 if $\omega \equiv$ 4, 7 (modulo 8).
φ (c_ω) = 4 if $\omega \equiv$ 6 (modulo 8).

Case 7: When $n \equiv 13$ (modulo 16) and $m \equiv 0, 1, 2, 3, 4, 5, 6, 7$ (modulo 8).
For $1 \leq \omega \leq m$.
φ (c_ω) = 1 if $\omega \equiv$ 2, 4, 6 (modulo 8).
φ (c_ω) = 3 if $\omega \equiv$ 5 (modulo 8).
φ (c_ω) = 4 if $\omega \equiv$ 0, 1, 3, 7 (modulo 8).

Case 8: When $n \equiv 15$ (modulo 16).
Sub Case 8.1: When $m \equiv 0, 1, 2, 3, 4, 6, 7$ (modulo 8).

For $1 \leq \omega \leq m$.
φ (c_ω) = 1 if $\omega \equiv$ 0, 5 (modulo8).
φ (c_ω) = 2 if $\omega \equiv$ 2, 3 (modulo 8).
φ (c_ω) = 3 if $\omega \equiv$ 4, 6, 7 (mod 8).
φ (c_ω) = 4 if $\omega \equiv$ 1 (modulo 8).

Sub Case 8.2: When $m \equiv 5$ (modulo 8).
For $1 \leq \omega \leq m - 1$, the labeling of c_ω values are same as sub case 8.1.
φ (c_m) = 3.

By the result of above labeling we could see that, {φ (a_α $a_{\alpha+1}$) : $1 \leq \alpha \leq n-1$}, {φ (a_1a_n)}, { φ ($b_\beta b_{\beta+1}$) : $1 \leq \beta \leq m-1$}, {φ (a_1b_1)}, { φ ($c_\omega c_{\omega+1}$) : $1 \leq \omega \leq m-1$}, {φ ($a_{\frac{n+1}{2}}$ c_1)} $\in Z_4$ and also for all i≠j ∈ [1, 4], |v_φ (i) – v_φ (j)| ≤ 1 and $k \neq l$ ∈ [0, 3], |e_φ (k) – e_φ (l)| ≤ 1. Thus by definition 2.1, the graph C_n ($2P_m$), $n \geq 3$ is Quotient-4 cordial labeling.

Theorem 3.1.4: A graph C_n ($2P_m$) is quotient-4 cordial if n is even and $n \geq 4$.
Proof: Let V (G) = {a_α : $1 \leq \alpha \leq n$} ∪ {b_β : $1 \leq \beta \leq m$} ∪ { c_ω : $1 \leq \omega \leq m$} and E (G) = {a_α $a_{\alpha+1}$: $1 \leq \alpha \leq n-1$} ∪ {a_1a_n} ∪ {$b_\beta b_{\beta+1}$: $1 \leq \beta \leq m-1$} ∪ {a_1b_1} ∪ { c_ω $c_{\omega+1}$: $1 \leq \omega \leq m-1$} ∪ {$a_{(n+1)/2}c_1$ }.
Here $|V\ (G)| = n + 2m$, $|E\ (G)| = n + 2m$.
Define φ : V (G) → {1, 2, 3, 4}.

Labeling of a_α values are given below.
Case 1: When $n \equiv 0$ (modulo 8) and $m \not\equiv 2$(modulo 8).
For $1 \leq \alpha \leq n$
φ (a_α) = 1 if $\alpha \equiv$ 1, 4 (modulo 8).
φ (a_α) = 2 if $\alpha \equiv$ 6, 7 (modulo 8).
φ (a_α) = 3 if $\alpha \equiv$ 0, 5 (modulo 8).
φ (a_α) = 4 if $\alpha \equiv$ 2, 3 (modulo 8).

Sub Case 1.1: Labeling of b_β.
Sub Case 1.1.1: When $m \equiv 0, 1, 3, 4, 5, 6, 7$ (modulo 8).
For $1 \leq \beta \leq m$.
φ (b_β) = 1 if β ≡ 0, 3, 5 (modulo 8).
φ (b_β) = 3 if β ≡ 4, 6, 7 (modulo 8).

$\varphi(b_\beta) = 4$ if $\beta \equiv 1, 2$ (modulo 8).

Sub Case 1.2: Labeling of c_ω.

Sub Case 1.2.1: $m \equiv 0, 1, 7$ (modulo 8).

For $1 \leq \omega \leq m$.

$\varphi(c_\omega) = 1$, if $\omega \equiv 7$ (modulo 8).

$\varphi(c_\omega) = 2$, if $\omega \equiv 1, 3, 4, 5$ (modulo 8).

$\varphi(c_\omega) = 3$, if $\omega \equiv 2$ (modulo 8).

$\varphi(c_\omega) = 4$, if $\omega \equiv 0, 6$ (modulo 8).

Sub Case 1.2.2: $m \equiv 3$ (modulo 8).

For $1 \leq \omega \leq m-1$, the labeling of c_ω values are same as sub case 1.2.1.

$\varphi(c_m) = 1$.

Sub Case 1.2.3: $m \equiv 4$ (modulo 8).

For $1 \leq \omega \leq m-3$, the labeling of c_ω values are same as sub case 1.2.1.

$\varphi(c_m) = 1$, $\varphi(c_{m-1}) = 3$, $\varphi(c_{m-2}) = 2$.

Sub Case 1.2.4: $m \equiv 5$ (modulo 8).

For $1 \leq \omega \leq m-3$, the labeling of c_ω values are same as sub case 1.2.1.

$\varphi(c_m) = \varphi(c_{m-1}) = 2$, $\varphi(c_{m-2}) = 3$.

Sub Case 1.2.5: $m \equiv 6$ (modulo 8).

For $1 \leq \omega \leq m-5$, the labeling of c_ω values are same as sub case 1.2.1.

$\varphi(c_m) = 1$, $\varphi(c_{m-1}) = 4$, $\varphi(c_{m-2}) = 3$, $\varphi(c_{m-3}) = \varphi(c_{m-4}) = 2$.

Case 2: When $n \equiv 0$ (modulo 8) and $m \equiv 2$ (modulo 8).

For $1 \leq \alpha \leq n-7$.

$\varphi(a_\alpha) = 1$ if $\alpha \equiv 2, 5$ (modulo 8).

$\varphi(a_\alpha) = 2$ if $\alpha \equiv 0, 7$ (modulo 8).

$\varphi(a_\alpha) = 3$ if $\alpha \equiv 3, 4$ (modulo 8).

$\varphi(a_\alpha) = 4$ if $\alpha \equiv 1, 6$ (modulo 8).

$\varphi(a_n) = 4$, $\varphi(a_{n-1}) = \varphi(a_{n-4}) = 1$, $\varphi(a_{n-2}) = \varphi(a_{n-3}) = \varphi(a_{n-5}) = 3$, $\varphi(a_{n-6}) = 2$.

Sub Case 2.1: When $m \equiv 2$ (modulo 8).

For $1 \leq \beta \leq m$.

$\varphi(b_\beta) = 1$ if $\beta \equiv 1, 4, 7$ (modulo 8).

$\varphi(b_\beta) = 3$ if $\beta \equiv 5, 6$ (modulo 8).

$\varphi(b_\beta) = 4$ if $\beta \equiv 0, 2, 3$ (modulo 8).

Sub Case 2.2: $m \equiv 2$ (modulo 8).

For $1 \leq \omega \leq m$.

$\varphi(c_\omega) = 1$ if $\omega \equiv 7$ (modulo 8).

$\varphi(c_\omega) = 2$ if $\omega \equiv 1, 2, 4, 5$ (modulo 8).

$\varphi(c_\omega) = 3$ if $\omega \equiv 0, 6$ (modulo 8).

$\varphi(c_\omega) = 4$ if $\omega \equiv 3$ (modulo 8).

Case 3: When $n \equiv 2$ (modulo 8).

For $1 \leq \alpha \leq n-2$.

$\varphi(a_\alpha) = 1$ if $\alpha \equiv 2, 5$ (modulo 8).

$\varphi(a_\alpha) = 2$ if $\alpha \equiv 0, 7$ (modulo 8).

$\varphi(a_\alpha) = 3$ if $\alpha \equiv 3, 4$ (modulo8).

$\varphi(a_\alpha) = 4$ if $\alpha \equiv 1, 6$ (modulo8).

$\varphi(a_n) = 1$, $\varphi(a_{n-1}) = 3$.

Sub Case 3.1: Labeling of b_β.

Sub Case 3.1.1: When $m \equiv 0, 1, 2, 3, 4, 6$ (modulo 8).

For $1 \leq \beta \leq m$.

$\varphi(b_\beta) = 1$, if $\beta \equiv 2, 5, 7$ (modulo 8).

$\varphi(b_\beta) = 3$, if $\beta \equiv 3, 4, 6$ (modulo 8).

$\varphi(b_\beta) = 4$, if $\beta \equiv 0, 1$ (modulo 8).

Sub Case 3.1.2: $m \equiv 5$ (modulo 8).

For $1 \leq \beta \leq m-2$, the labeling of b_β values are same as sub case 3.1.1.

$\varphi(b_m) = 4$, $\varphi(b_{m-1}) = 1$.

Sub Case 3.1.3: $m \equiv 7$ (modulo 8).

For $1 \leq \beta \leq m-1$, the labeling of b_β values are same as sub case 3.1.1.

$\varphi(b_m) = 4$.

Sub Case 3.2: Labeling of c_ω.

Sub Case 3.2.1: $m \equiv 0, 1, 2, 3, 4, 6, 7$ (modulo 8).

For $1 \leq \omega \leq m$

φ (c_ω) = 1 if $\omega \equiv 4$ (modulo 8).

φ (c_ω) = 2 if $\omega \equiv 1, 2, 6, 7$ (modulo 8).

φ (c_ω) = 3 if $\omega \equiv 0$ (modulo 8).

φ (c_ω) = 4 if $\omega \equiv 3, 5$ (modulo 8).

Sub Case 3.2.2: $m \equiv 5$ (modulo 8).

For $1 \leq \omega \leq m - 3$, the labeling of c_ω values are same as sub case 3.2.1.

φ (c_m) = 4, φ (c_{m-1}) = 3, φ (c_{m-2}) = 2.

Case 4: When $n \equiv 4$ (modulo 8).

For $1 \leq \alpha \leq n - 3$.

φ (a_α) = 1 if $\alpha \equiv 0, 3$ (modulo 8).

φ (a_α) = 2 if $\alpha \equiv 5, 6$ (modulo 8).

φ (a_α) = 3 if $\alpha \equiv 4, 7$ (modulo 8).

φ (a_α) = 4 if $\alpha \equiv 1, 2$ (modulo 8).

φ (a_n) = φ (a_{n-2}) = 1, φ (a_{n-1}) = 3.

Sub Case 4.1: Labeling of b_β.

Sub Case 4.1.1: $m \equiv 0, 1, 2, 3, 7$ (modulo 8).

For $1 \leq \beta \leq m$.

φ (b_β) = 1, if $\beta \equiv 3, 5, 7$ (modulo 8).

φ (b_β) = 3, if $\beta \equiv 2, 4$ (modulo 8).

φ (b_β) = 4, if $\beta \equiv 0, 1, 6$ (modulo 8).

Sub Case 4.1.2: $m \equiv 4$ (modulo 8).

For $1 \leq \beta \leq m - 1$, the labeling of b_β values are same as sub case 4.1.1.

φ (b_m) = 4.

Sub Case 4.1.3: $m \equiv 5$ (modulo 8).

For $1 \leq \beta \leq m - 2$, the labeling of b_β values are same as sub case 4.1.1.

φ (b_m) = 3, φ (b_{m-1}) = 4.

Sub Case 4.1.4: $m \equiv 6$ (modulo 8).

For $1 \leq \beta \leq m - 3$, the labeling of b_β values are same as sub case 4.1.1.

φ (b_m) = 3, φ (b_{m-1}) = 1, φ (b_{m-2}) = 4.

Sub Case 4.2: Labeling of c_ω.

Sub Case 4.2.1: $m \equiv 1, 2, 3, 5, 6$ (modulo8).

For $1 \leq \omega \leq m$.

φ (c_ω) = 1 if $\omega \equiv 7$ (modulo 8).

φ (c_ω) = 2 if $\omega \equiv 1, 2, 3, 4$ (modulo 8).

φ (c_ω) = 3 if $\omega \equiv 0, 5$ (modulo 8).

φ (c_ω) = 4 if $\omega \equiv 6$ (modulo 8).

Sub Case 4.2.2: $m \equiv 0$ (modulo 8).

For $1 \leq \omega \leq m - 3$, the labeling of c_ω values are same as sub case 4.2.1.

φ (c_m) = 2, φ (c_{m-1}) = 4, φ (c_{m-2}) = 3.

Sub Case 4.2.3: $m \equiv 4$ (modulo 8).

For $1 \leq \omega \leq m - 1$, the labeling of c_ω values are same as case 4.2.1.

φ (c_m) = 3.

Sub Case 4.2.4: $m \equiv 7$ (modulo 8).

For $1 \leq \omega \leq m - 1$, the labeling of c_ω values are same as case 4.2.1.

φ (c_m) = 2.

Case 5: When $n \equiv 6$ (modulo 8).

For $1 \leq i \leq n - 2$.

φ (a_α)= 1 if i ≡ 0, 3 (modulo 8).

φ (a_α)= 2 if i ≡ 5, 6 (modulo 8).

φ (a_α)= 3 if i ≡ 1, 2 (modulo 8).

φ (a_α)= 4 if i ≡ 4, 7 (modulo 8).

φ (a_n) = 1, φ (a_{n-1}) = 4.

Sub Case 5.1: Labeling of b_β.

Sub Case 5.1.1: $m \equiv 3, 4, 5, 6$ (modulo 8).

For $1 \leq \beta \leq m$.

φ (b_β) = 1 if $\beta \equiv 1, 6$ (modulo 8).

φ (b_β) = 2 if β ≡ 3, 4 (modulo 8).
φ (b_β) = 3 if β ≡ 0, 5, 7 (modulo 8).
φ (b_β) = 4 if β ≡ 2 (modulo 8).

Sub Case 5.1.2: $m \equiv 0, 1$ (modulo 8).
For $1 \le \beta \le m-1$, the labeling of b_β values are same as sub case 5.1.1.
φ (b_m) = 2.

Sub Case 5.1.3: $m \equiv 2$ (modulo 8).
For $1 \le \beta \le m-2$, the labeling of b_β values are same as sub case 5.1.1.
φ (b_m) = 3, φ (b_{m-1}) = 2.

Sub Case 5.1.4: $m \equiv 7$ (modulo 8).
For $1 \le \beta \le m-2$, the labeling of b_β values are same as sub case 5.1.1.
φ (b_m) = 4, φ (b_{m-1}) = 1.

Sub Case 5.2: Labeling of c_ω.
Sub Case 5.2.1: $m \equiv 0, 1, 2, 3, 4, 5, 6$ (modulo 8).
For $1 \le \omega \le m$.
φ (c_ω) = 1 if ω ≡ 4, 7 (modulo 8).
φ (c_ω) = 2 if ω ≡ 1, 2 (modulo 8).
φ (c_ω) = 3 if ω ≡ 3 (modulo 8).
φ (c_ω) = 4 if ω ≡ 0, 5, 6 (modulo8).

Sub Case 5.2.2: $m \equiv 7$ (modulo 8).
For $1 \le \omega \le m-4$, the labeling of c_ω values are same as sub case 5.2.1.
φ (c_m) = 2, φ (c_{m-1}) = 4, φ (c_{m-2}) = 1, φ (c_{m-3}) = 3.

By the result of above labeling we could see that, {φ (a_α $a_{\alpha+1}$) : $1 \le \alpha \le n-1$},{ φ (a_1a_n)}, { φ ($b_\beta b_{\beta+1}$) : $1 \le \beta \le m-1$}, { φ (a_1b_1)}, { φ (c_ω $c_{\omega+1}$) : $1 \le \omega \le m-1$}, {φ ($a_{\frac{n+1}{2}}$ c_1)} ∈ Z_4 and also for all i≠j ∈ [1, 4], |v_φ (i) – v_φ (j)| ≤ 1 and $k \ne l$ ∈ [0, 3], |e_φ (k) – e_φ (l)| ≤ 1. Thus by definition 2.1, the graph C_n ($2P_m$), $n \ge 4$ is Quotient-4 cordial labeling.

3.2. Corona of Ladder Graphs

Theorem: 3.2.1 The graph (OL (β) ⊙ K_1) is quotient-4 cordial if $\beta \ge 3$.

Proof: Let G be a (OL (β) ⊙ K_1) graph.
V (G) = $\{x_\theta, u_\theta, v_\theta, y_\theta : 1 \le \theta \le \beta\}$.
E (G) = $\{(x_\theta u_\theta), (v_\theta y_\theta) : 1 \le \theta \le \beta\} \cup \{(u_\theta u_{\theta+1}), (v_\theta v_{\theta+1}) : 1 \le \theta \le \beta - 1\} \cup \{u_\theta v_\theta : 2 \le \theta \le \beta - 1\}$.
Here $|V(G)| = 4\beta$, $|E(G)| = 5\beta - 4$.
Define the function φ : V (G) → {1, 2, 3, 4} as follows.

The values of x_θ are labeled as follows.
For $1 \le \theta \le \beta$.
φ (x_θ) = 2 if θ ≡ 1 (modulo 2).
φ (x_θ) = 4 if θ ≡ 0 (modulo 2).

The values of u_θ are labeled as follows.
Case (i): When β ≡ 0 (modulo 2).
For $1 \le \theta \le \beta$.
φ (u_θ) = 1 if θ ≡ 1 (modulo 2).
φ (u_θ) = 4 if θ ≡ 0 (modulo 2).

Case (ii): When β ≡ 1 (modulo 2).
For $1 \le \theta \le \beta - 1$, the labeling of u_θ values are same as case (i).
φ (u_β) = 4.

The values of v_θ are labeled as follows.
Case (i): When $\beta \equiv 0, 2, 3$ (modulo 4).
For $1 \le \theta \le \beta$.
φ (v_θ) = 1 if θ ≡ 2 (modulo 4).
φ (v_θ) = 3 if θ ≡ 0, 1, 3 (modulo 4).

Case (ii): When β ≡ 1 (modulo 4).

For $1 \leq \theta \leq \beta - 2$, the labeling of v_θ values are same as case (i).
φ (v_β) = 3, φ ($v_{\beta-1}$) = 1.

The values of y_θ are labeled as follows.
Case (i): When $\beta \equiv 0, 1, 2$ (modulo 4).
For $1 \leq \theta \leq \beta$.
φ (y_θ) = 1 if $\theta \equiv 0$ (modulo 4).
φ (y_θ) = 2 if $\theta \equiv 2$, 3 (modulo 4).
φ (y_θ) = 3 if $\theta \equiv 1$ (modulo 4).

Case (ii): When $\beta \equiv 3$ (modulo 4).
For $1 \leq \theta \leq \beta - 1$, the labeling of y_θ values are same as case (i).
φ (y_β) = 1.

By the result of above labeling we could see that, { φ(($x_\theta u_\theta$), ($v_\theta y_\theta$)): $1 \leq \theta \leq \beta$}, {φ (($u_\theta u_{\theta+1}$), ($v_\theta v_{\theta+1}$)) : $1 \leq \theta \leq \beta - 1$}, { φ ($u_\theta v_\theta$) : $2 \leq \theta \leq \beta - 1$} ∈ Z_4 and also for all i≠j ∈ [1, 4], |v $_\varphi$ (i) – v $_\varphi$ (j)| ≤ 1 and $k \neq l$ ∈ [0, 3], |e $_\varphi$ (k) – e $_\varphi$ (l)| ≤ 1. Thus by definition 2.1, the graph (OL (β) ⊙ K_1), $\beta \geq 3$ is Quotient-4 cordial labeling.

Theorem: 3.2.2 The graph (CL (β) ⊙ K_1) is quotient-4 cordial if $\beta \geq 2$.
Proof: Let G be a (CL (β) ⊙ K_1) graph.
V (G) = {$x_\theta, u_\theta, v_\theta, y_\theta$: $1 \leq \theta \leq \beta$}.
E (G) = {{($x_\theta u_\theta$), ($v_\theta y_\theta$), ($u_\theta v_\theta$): $1 \leq \theta \leq \beta$} ∪ {($u_\theta u_{\theta+1}$), ($v_\theta v_{\theta+1}$) : $1 \leq \theta \leq \beta - 1$}}.
Here $|V(G)| = 4\beta$, $|E(G)| = 5\beta - 2$.
Define the function φ : V (G) → {1, 2, 3, 4} as follows.

The values of x_θ are labeled as follows.
Case (i): When $\beta \equiv 0$ (modulo 2).
For $1 \leq \theta \leq \beta$.
φ (x_θ) = 2 if $\theta \equiv 0$ (modulo 2).
φ (x_θ) = 4 if $\theta \equiv 1$ (modulo 2).

Case (ii): When $\beta \equiv 1$ (modulo 2).
For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).
φ (x_β) = 2.

The values of u_θ are labeled as follows.
For $1 \leq \theta \leq \beta$.
φ (u_θ) = 1 if $\theta \equiv 0$ (modulo 2).
φ (u_θ) = 4 if $\theta \equiv 1$ (modulo 2).

The values of v_θ are labeled as follows.
For $1 \leq \theta \leq \beta$.
φ (v_θ) = 1 if $\theta \equiv 1$ (modulo 4).
φ (v_θ) = 3 if $\theta \equiv 0$, 2, 3 (modulo 4).

The values of y_θ are labeled as follows.
Case (i): When $\beta \equiv 0, 2, 3$ (modulo 4).
For $1 \leq \theta \leq \beta$.
φ (y_θ) = 1 if $\theta \equiv 3$ (modulo 4).
φ (y_θ) = 2 if $\theta \equiv 0$, 1 (modulo 4).
φ (y_θ) = 3 if $\theta \equiv 2$ (modulo 4).

Case (ii): When $\beta \equiv 1$ (modulo 4).
For $1 \leq \theta \leq \beta - 1$, the labeling of y_θ values are same as case (i).
φ (y_β) = 3.

By the result of above labeling we could see that, { φ (($x_\theta u_\theta$), ($v_\theta y_\theta$), ($u_\theta v_\theta$)): $1 \leq \theta \leq \beta$}, { φ (($u_\theta u_{\theta+1}$), ($v_\theta v_{\theta+1}$)) : $1 \leq \theta \leq \beta - 1$} ∈ Z_4 and also for all i≠j ∈ [1, 4], |v $_\varphi$ (i) – v $_\varphi$ (j)| ≤ 1 and $k \neq l$ ∈ [0, 3], |e $_\varphi$ (k) – e $_\varphi$ (l)| ≤ 1. Thus by definition 2.1, the graph (CL (β) ⊙ K_1), $\beta \geq 2$ is Quotient-4 cordial labeling.

Theorem: 3.2.3 The graph (SL (β) ⊙ K_1) is quotient-4 cordial if $\beta \geq 2$.

Proof: Let G be a (SL (β) ⨀ K1) graph.
V (G) = $\{x_{\theta,} u_{\theta,} v_\theta, y_\theta: 1 \le \theta \le \beta\}$.
E (G) = $\{(x_\theta u_\theta), (v_\theta y_\theta) : 1 \le \theta \le \beta\} \cup \{(u_\theta u_{\theta+1}), (v_\theta v_{\theta+1}), (u_\theta v_{\theta+1}) : 1 \le \theta \le \beta - 1\}$.
Here $|V(G)| = 4\beta$, $|E(G)| = 5\beta - 3$.
Define the function φ : V (G) → {1, 2, 3, 4} as follows.

The values of x_θ are labeled as follows.
Case (i): When $\beta \equiv 0, 2, 3$ (modulo 4).
For $1 \le \theta \le \beta$.
φ (x_θ) = 2 if θ ≡ 0 (modulo 2).
φ (x_θ) = 3 if θ ≡ 1 (modulo 2).

Case (ii): When $\beta \equiv 1$ (modulo 4).
For $1 \le \theta \le \beta - 1$, the labeling of x_θ values are same as case (i).
φ (x_β) = 1.

The values of u_θ are labeled as follows.
Case (i): When $\beta \equiv 0, 1, 2$ (modulo 4).
For $1 \le \theta \le \beta$.
φ (u_θ) = 1 if $\theta \equiv 0$ (modulo 2).
φ (u_θ) = 3 if $\theta \equiv 1$ (modulo 2).

Case (ii): When $\beta \equiv 3$ (modulo 4).
For $1 \le \theta \le \beta - 1$, the labeling of u_θ values are same as case (i).
φ (u_β) = 1.

The values of v_θ are labeled as follows.
For $1 \le \theta \le \beta$.
φ (v_θ) = 1 if θ ≡ 2 (modulo 4).
φ (v_θ) = 4 if θ ≡ 0, 1, 3 (modulo 4).

The values of y_θ are labeled as follows.
Case (i): When $\beta \equiv 0, 2, 3$ (modulo 4).
For $1 \le \theta \le \beta$.
φ (y_θ) = 1 if θ ≡ 0 (modulo 4).
φ (y_θ) = 2 if θ ≡ 2, 3 (modulo 4).
φ (y_θ) = 4 if θ ≡ 1 (modulo 4).

Case (ii): When $\beta \equiv 1$ (modulo 4).
For $1 \le \theta \le \beta - 1$, the labeling of y_θ values are same as case (i).
φ (y_β) = 2.

By the result of above labeling we could see that, { φ $((x_\theta u_\theta), (v_\theta y_\theta))$: $1 \le \theta \le \beta\}$, { φ $((u_\theta u_{\theta+1}), (v_\theta v_{\theta+1}), (u_\theta v_{\theta+1}))$: $1 \le \theta \le \beta - 1\} \in Z_4$ and also for all i≠j ∈ [1, 4], |v φ (i) – v φ (j)| ≤ 1 and $k \ne l$ ∈ [0, 3], |e φ (k) – e φ (l)| ≤ 1. Thus by definition 2.1, the graph (SL (β) ⨀ K1), $\beta \ge 2$ is Quotient-4 cordial labeling.

Theorem: 3.2.4 The graph (ML (β) ⨀ K1) is quotient-4 cordial if $\beta \ge 3$.
Proof: Let G be a (ML (β) ⨀ K1) graph.
V (G) = $\{x_{\theta,} u_{\theta,} v_\theta, y_\theta: 1 \le \theta \le \beta\}$.
E (G) = $\{(x_\theta u_\theta), (v_\theta y_\theta), (u_\theta v_\theta) : 1 \le \theta \le \beta\} \cup \{(u_\theta u_{\theta+1}), (v_\theta v_{\theta+1}) : 1 \le \theta \le \beta - 1\} \cup (u_1 v_\beta) \cup (v_1 u_\beta)$.
Here $|V(G)| = 4\beta$, $|E(G)| = 5\beta$.
Define the function φ : V (G) → {1, 2, 3, 4} as follows.

The values of x_θ are labeled as follows.
Case (i): When $\beta \equiv 0, 1$ (modulo 4).
For $1 \le \theta \le \beta$.
φ (x_θ) = 1 if θ ≡ 3 (modulo 4).
φ (x_θ) = 2 if θ ≡ 0, 1 (modulo 4).
φ (x_θ) = 4 if θ ≡ 2 (modulo 4).

Case (ii): When $\beta \equiv 2$ (modulo4).
For $1 \le \theta \le \beta - 1$, the labeling of x_θ values are same as case (i).
φ (x_β) = 3.

Case (iii): When $\beta \equiv 3$ (modulo4).
For $1 \leq \theta \leq \beta - 1$, the labeling of x_θ values are same as case (i).
φ (x_β) = 2.

The values of u_θ are labeled as follows.
Case (i): When $\beta \equiv 0, 1, 3$ (modulo 4).
For $1 \leq \theta \leq \beta$.
φ (u_θ) = 1 if θ ≡ 3 (modulo 4).
φ (u_θ) = 3 if θ ≡ 0, 1 (modulo 4).
φ (u_θ) = 4 if θ ≡ 2 (modulo 4).

Case (ii): When $\beta \equiv 2$ (modulo 4).
For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).
φ (u_β) = 1.

The values of v_θ are labeled as follows.
Case (i): When $\beta \equiv 0$ (modulo 4).
For $1 \leq \theta \leq \beta$.
φ (v_θ) = 1 if θ ≡ 0, 2 (modulo 4).
φ (v_θ) = 3 if θ ≡ 1 (modulo 4).
φ (v_θ) = 4 if θ ≡ 3 (modulo 4).
Case (ii): When $\beta \equiv 1, 3$ (modulo 4).
For $1 \leq \theta \leq \beta - 1$, the labeling of v_θ values are same as case (i).
φ (v_β) = 1.

Case (iii): When $\beta \equiv 2$ (modulo 4).
For $1 \leq \theta \leq \beta - 2$, the labeling of v_θ values are same as case (i).
φ (v_β) = 1, φ ($v_{\beta-1}$) = 4.

The values of y_θ are labeled as follows.
Case (i): When $\beta \equiv 0, 3$ (modulo 4).
For $1 \leq \theta \leq \beta$.
φ (y_θ) = 2 if θ ≡ 0, 1 (modulo 4).
φ (y_θ) = 3 if θ ≡ 2 (modulo 4).
φ (y_θ) = 4 if θ ≡ 3 (modulo 4).

Case (ii): When $\beta \equiv 1$ (modulo 4).
For $1 \leq \theta \leq \beta - 1$, the labeling of y_θ values are same as case (i).
φ (y_β) = 4.

Case (ii): When $\beta \equiv 2$ (modulo 4).
For $1 \leq \theta \leq \beta - 2$, the labeling of y_θ values are same as case (i).
φ (y_β) = 2, φ ($v_{\beta-1}$) = 4.

By the result of above labeling we could see that, { φ $((x_\theta u_\theta),\ (v_\theta y_\theta), (u_\theta v_\theta))$: $1 \leq \theta \leq \beta\}$, { φ $((u_\theta u_{\theta+1}), (v_\theta v_{\theta+1}))$: $1 \leq \theta \leq \beta - 1\}$, {φ $(u_1 v_\beta)$}, {φ $(v_1 u_\beta)$} ∈ Z_4 and also for all i≠j ∈ [1, 4], |v $_φ$ (i) – v $_φ$ (j)| ≤ 1 and $k \neq l$ ∈ [0, 3], |e $_φ$ (k) – e $_φ$ (l)| ≤ 1. Thus by definition 2.1, the graph (ML (β) ⊙ K_1), $\beta \geq 3$ is Quotient-4 cordial labeling.

Theorem: 3.2.5 The graph (CRL (β) ⊙ K_1) is quotient-4 cordial if $\beta \geq 2$.
Proof: Let G be a (CRL (β) ⊙ K_1) graph.
V (G) = $\{x_{\theta,} u_{\theta,} v_\theta, y_\theta : 1 \leq \theta \leq \beta\}$.
E (G) = $\{(x_\theta u_\theta), (v_\theta y_\theta), (u_\theta v_\theta) : 1 \leq \theta \leq \beta\} \cup \{(u_\theta u_{\theta+1}),\ (v_\theta v_{\theta+1}) : 1 \leq \theta \leq \beta - 1\} \cup (u_1 u_\beta) \cup (v_1 v_\beta)$.
Here $|V(G)| = 4\beta$, $|E(G)| = 5\beta$.
Define the function φ : V (G) → {1, 2, 3, 4} as follows.

The values of x_θ are labeled as follows.
Case (i): When $\beta \equiv 0$ (modulo 2).
For $1 \leq \theta \leq \beta$.
φ (x_θ) = 2 if θ ≡ 0 (modulo 2).
φ (x_θ) = 3 if θ ≡ 1 (modulo 2).

Case (ii): When $\beta \equiv 1$ (modulo2).

For $1 \leq \theta \leq \beta - 1$, the labeling of x_θ values are same as case (i).
φ (x_β) = 2.

The values of u_θ are labeled as follows.
For $1 \leq \theta \leq \beta$.
φ (u_θ) = 1 if $\theta \equiv 0$ (modulo 2).
φ (u_θ) = 3 if $\theta \equiv 1$ (modulo 2).

The values of v_θ are labeled as follows.
For $1 \leq \theta \leq \beta$.
φ (v_θ) = 1 if $\theta \equiv 1$ (modulo 4).
φ (v_θ) = 4 if $\theta \equiv 0, 2, 3$ (modulo 4).

The values of y_θ are labeled as follows.
Case (i): When $\beta \equiv 0, 2, 3$ (modulo 4).
For $1 \leq \theta \leq \beta$.
φ (y_θ) = 1 if $\theta \equiv 3$ (modulo 4).
φ (y_θ) = 2 if $\theta \equiv 0, 1$ (modulo 4).
φ (y_θ) = 4 if $\theta \equiv 2$ (modulo 4).

Case (ii): When $\beta \equiv 1$ (modulo 4).
For $1 \leq \theta \leq \beta - 1$, the labeling of y_θ values are same as case (i).
φ (y_β) = 4.

By the result of above labeling we could see that, { φ $((x_\theta u_\theta), (v_\theta y_\theta), (u_\theta v_\theta))$: $1 \leq \theta \leq \beta$}, { φ $((u_\theta u_{\theta+1}), (v_\theta v_{\theta+1}))$: $1 \leq \theta \leq \beta - 1$}, {φ $(u_1 u_\beta)$}, {φ $(v_1 v_\beta)$} ∈ Z_4 and also for all i≠j ∈ [1, 4], |v $_\varphi$ (i) – v $_\varphi$ (j)| ≤ 1 and $k \neq l$ ∈ [0, 3], |e $_\varphi$ (k) – e $_\varphi$ (l)| ≤ 1. Thus by definition 2.1, the graph (CRL (β) ⊙ K_1), $\beta \geq 2$ is Quotient-4 cordial labeling.

Theorem: 3.2.6 The graph (PL (β) ⊙ K_1) is quotient-4 cordial if $\beta \geq 3$.
Proof: Let G be a (PL (β) ⊙ K_1) graph.

V (G) = $\{x_\theta, u_\theta, v_\theta : 1 \leq \theta \leq \beta\} \cup \{w_\theta : 1 \leq \theta \leq \beta - 1\} \cup \{y_\theta : 1 \leq \theta \leq 2\beta - 1\}$.
E (G) = $\{(x_\theta u_\theta), (v_\theta y_{2\theta-1}), (u_\theta v_\theta) : 1 \leq \theta \leq \beta\} \cup \{(u_\theta u_{\theta+1}), (v_\theta w_\theta), (w_\theta v_{\theta+1}), (w_\theta y_{2\theta}) : 1 \leq \theta \leq \beta - 1\}$.
Here $|V(G)| = 6\beta - 2$, $|E(G)| = 7\beta - 4$.
Define the function φ : V (G) → {1, 2, 3, 4} as follows.

The values of x_θ are labeled as follows.
Case (i): When $\beta \equiv 0$ (modulo 2).
For $1 \leq \theta \leq \beta$.
φ (x_θ) = 2 if $\theta \equiv 2$ (modulo 4) and $\theta = 4$.
φ (x_θ) = 4 if $\theta \equiv 0, 1, 3$ (modulo 4) and $\theta \neq 4$.

Case (ii): When $\beta \equiv 1$ (modulo 2).
For $1 \leq \theta \leq \beta$.
φ (x_θ) = 2 if $\theta \equiv 0$ (modulo 2) .
φ (x_θ) = 4 if $\theta \equiv 1$ (modulo 2) .

The values of u_θ are labeled as follows.
For $1 \leq \theta \leq \beta$.
φ (u_θ) = 1 if $\theta \equiv 0$ (modulo 2).
φ (u_θ) = 4 if $\theta \equiv 1$ (modulo 2).

The values of v_θ are labeled as follows.
For $1 \leq \theta \leq \beta$.
φ (v_θ) = 1 if $\theta \equiv 1$ (modulo 2).
φ (v_θ) = 3 if $\theta \equiv 0$ (modulo 2).

The values of w_θ are labeled as follows.
For $1 \leq \theta \leq \beta - 1$.
φ (w_θ) = 3 if $\theta \equiv 0, 1$ (modulo 2).

The values of y_θ are labeled as follows.
Case (i): When $\beta \equiv 0$ (modulo 2).
For $1 \leq \theta \leq 2\beta - 1$.

φ (y_θ) = 1 if $\theta \equiv$ 3, 5 (modulo 8), $\theta \neq 5$ and $\theta = 1$.
φ (y_θ) = 2 if $\theta \equiv$ 0, 1, 4, 6, 7 (modulo 8) and $\theta \neq 1$.
φ (y_θ) = 4 if $\theta \equiv$ 2 (modulo 8) and $\theta =$ 5.

Case (ii): When $\beta \equiv 1$ (modulo 2).
For $1 \leq \theta \leq 2\beta - 1$.
φ (y_θ) = 1 if $\theta \equiv$ 1, 2 (modulo 8) and $\theta \neq 1$.
φ (y_θ) = 2 if $\theta \equiv$ 0, 3, 4, 7 (modulo 8) and $\theta = 1$.
φ (y_θ) = 3 if θ = 5.
φ (y_θ) = 4 if $\theta \equiv$ 5, 6 (modulo 8) and $\theta \neq 5$.

By the result of above labeling we could see that, { φ $((x_\theta u_\theta),\ (v_\theta y_{2\theta-1}), (u_\theta v_\theta))$: $1 \leq \theta \leq \beta$}, { φ $((u_\theta u_{\theta+1}), (v_\theta w_\theta), (w_\theta v_{\theta+1}), (w_\theta y_{2\theta}))$: $1 \leq \theta \leq \beta -$ 1} ∈ Z_4 and also for all i≠j ∈ [1, 4], $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and $k \neq l$ ∈ [0, 3], $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Thus by definition 2.1, the graph (PL (β) ⊙ K_1), $\beta \geq 3$ is Quotient-4 cordial labeling.

Theorem: 3.2.7 The graph (PCL (β) ⊙ K_1) is quotient-4 cordial if $\beta \geq 3$.
Proof: Let G be a (PCL (β) ⊙ K_1) graph.
V (G) = $\{x_\theta, u_\theta, v_\theta, w_\theta : 1 \leq \theta \leq \beta\} \cup \{y_\theta : 1 \leq \theta \leq 2\beta\}$.
E (G) = $\{(x_\theta u_\theta), (u_\theta v_\theta), (v_\theta w_\theta), (v_\theta y_{2\theta-1}), (w_\theta y_{2\theta}) : 1 \leq \theta \leq \beta\} \cup \{(u_\theta u_{\theta+1}), (w_\theta v_{\theta+1}) : 1 \leq \theta \leq \beta - 1\} \cup \{u_1 u_\beta\} \cup \{v_1 w_\beta\}$.
Here $|V(G)| = 6\beta$, $|E(G)| = 7\beta$.
Define the function φ : V (G) → {1, 2, 3, 4} as follows.

The values of x_θ are labeled as follows.
For $1 \leq \theta \leq \beta$.
φ (x_θ) = 2 if $\theta \equiv 0$ (modulo 2).
φ (x_θ) = 4 if $\theta \equiv 1$ (modulo 2).

The values of u_θ are labeled as follows.
For $1 \leq \theta \leq \beta$.
φ (u_θ) = 1 if $\theta \equiv 0$ (modulo 2).
φ (u_θ) = 4 if $\theta \equiv 1$ (modulo 2).

The values of v_θ are labeled as follows.
For $1 \leq \theta \leq \beta$.
φ (v_θ) = 1 if $\theta \equiv 1$ (modulo 2).
φ (v_θ) = 3 if $\theta \equiv 0$ (modulo 2).

The values of w_θ are labeled as follows.
For $1 \leq \theta \leq \beta$.
φ (w_θ) = 3 if $\theta \equiv$ 0, 1 (modulo 2).

The values of y_θ are labeled as follows.
Case (i): When $\beta \equiv 0$ (modulo 4).
For $1 \leq \theta \leq 2\beta$.
φ (y_θ) = 1 if $\theta \equiv$ 5, 6 (modulo 8) .
φ (y_θ) = 2 if $\theta \equiv$ 0, 2, 3, 4 (modulo 8).
φ (y_θ) = 4 if $\theta \equiv$ 1, 7 (modulo 8).

Case (ii): When $\beta \equiv 1$ (modulo 4).
For $1 \leq \theta \leq 2\beta - 2$, the labeling of y_θ values are same as case (i).
φ $(y_{2\beta})$ = φ $(y_{2\beta-1})$ = 2.

Case (iii): When $\beta \equiv 2$ (modulo 4).
For $1 \leq \theta \leq 2\beta - 1$, the labeling of y_θ values are same as case (i).
φ $(y_{2\beta})$ = 1.

Case (iv): When $\beta \equiv 3$ (modulo 4).
For $1 \leq \theta \leq 2\beta - 1$, the labeling of y_θ values are same as case (i).

φ ($y_{2\beta}$) = 2.

By the result of above labeling we could see that,
{ φ $((x_\theta u_\theta),(v_\theta y_{2\theta-1}),(u_\theta v_\theta),(v_\theta w_\theta),(w_\theta y_{2\theta}))$: $1 \leq \theta \leq \beta$}, { φ $((u_\theta u_{\theta+1}),(w_\theta v_{\theta+1}))$: $1 \leq \theta \leq \beta - 1$}, {φ($u_1 u_\beta$)}, {φ($v_1 w_\beta$)} $\in Z_4$ and also for all i≠j ∈ [1, 4], |v $_φ$ (i) – v $_φ$ (j)| ≤ 1 and $k \neq l$ ∈ [0, 3], |e $_φ$ (k) – e $_φ$ (l)| ≤ 1. Thus by definition 2.1, the graph (PCL (β) ⊙ K_1), $\beta \geq 3$ is Quotient-4 cordial labeling.

Theorem: 3.2.8 The graph (HL (β) ⊙ K_1) is quotient-4 cordial if $\beta \geq 3$.

Proof: Let G be a (HL (β) ⊙ K_1) graph.

V (G) = $\{u_\theta, v_\theta: 1 \leq \theta \leq \beta\}$ ∪ $\{t_\theta, w_\theta : 1 \leq \theta \leq \beta - 1\}$ ∪ $\{x_\theta, y_\theta: 1 \leq \theta \leq 2\beta - 1\}$.

E (G) = $\{(u_\theta v_\theta),(x_{2\theta-1}u_\theta), (v_\theta y_{2\theta-1}) : 1 \leq \theta \leq \beta\}$ ∪ $\{(x_{2\theta}t_\theta), (u_\theta t_\theta), (t_\theta u_{\theta+1}), (v_\theta w_\theta), (w_\theta v_{\theta+1}),(w_\theta y_{2\theta}) : 1 \leq \theta \leq \beta - 1\}$.

Here $|V(G)| = 8\beta - 4$, $|E(G)| = 9\beta - 6$.

Define the function φ : V (G) → {1, 2, 3, 4} as follows.

The values of x_θ are labeled as follows.

Case (i): When $\beta \equiv 0$ (modulo 2).

For $1 \leq \theta \leq 2\beta - 1$.

φ (x_θ) = 1 if $\theta \equiv$ 4, 5 (modulo 8), θ = 6 and $\theta \neq 5$.

φ (x_θ) = 2 if $\theta \equiv$ 0, 1, 7 (modulo 8), θ = 3, 5 and $\theta \neq 7$.

φ (x_θ) = 3 if $\theta \equiv$ 2 (modulo 8) and $\theta \neq$ 2.

φ (x_θ) = 4 if $\theta \equiv$ 3, 6 (modulo 4), θ = 2, 7 and $\theta \neq$ 3, 6.

Case (ii): When $\beta \equiv 1$ (modulo 2).

For $1 \leq \theta \leq 2\beta - 1$.

φ (x_θ) = 1 if $\theta \equiv$ 2, 6 (modulo 8), θ = 4 and $\theta \neq 2$.

φ (x_θ) = 2 if $\theta \equiv$ 0, 1, 3, 4, 5 (modulo 8) and $\theta \neq 4$.

φ (x_θ) = 4 if $\theta \equiv$ 7(modulo 4) and θ = 2.

The values of t_θ are labeled as follows.

For $1 \leq \theta \leq \beta - 1$.

φ (t_θ) = 4 if $\theta \equiv$ 0, 1 (modulo 2).

The values of u_θ are labeled as follows.

For $1 \leq \theta \leq \beta$.

φ (u_θ) = 1 if $\theta \equiv$ 0 (modulo 2).

φ (u_θ) = 4 if $\theta \equiv$ 1 (modulo 2).

The values of v_θ are labeled as follows.

For $1 \leq \theta \leq \beta$.

φ (v_θ) = 1 if $\theta \equiv$ 1 (modulo 2).

φ (v_θ) = 3 if $\theta \equiv$ 0 (modulo 2).

The values of w_θ are labeled as follows.

For $1 \leq \theta \leq \beta - 1$.

φ (w_θ) = 3 if $\theta \equiv$ 0, 1 (modulo 2).

The values of y_θ are labeled as follows.

Case (i): When $\beta \equiv 0$ (modulo 2).

For $1 \leq \theta \leq 2\beta - 1$.

φ (y_θ) = 1 if $\theta \equiv$ 2, 3 (modulo 8) and $\theta \neq 2$.

φ (y_θ) = 2 if $\theta \equiv$ 0, 1, 4, 6, 7 (modulo 8) θ = 2 and $\theta \neq 7$.

φ (y_θ) = 3 if $\theta \equiv$ 5 (modulo 8) and θ = 7.

Case (ii): When $\beta \equiv 1$ (modulo 2).

For $1 \leq \theta \leq 2\beta - 1$.

φ (y_θ) = 1 if $\theta \equiv$ 2, 6 (modulo 8), θ = 3 and $\theta \neq 2$.

$\varphi(y_\theta) = 2$ if $\theta \equiv 0, 1, 4$ (modulo 8), $\theta = 2$ and $\theta \neq 1$.
$\varphi(y_\theta) = 3$ if $\theta \equiv 5, 7$ (modulo 8) and $\theta = 1$.
$\varphi(y_\theta) = 4$ if $\theta \equiv 3$ (modulo 8) and $\theta \neq 3$.

By the result of above labeling we could see that, $\{\varphi\ ((x_{2\theta-1}u_\theta), (v_\theta y_{2\theta-1}), (u_\theta v_\theta)): 1 \leq \theta \leq \beta\}$, $\{\varphi\ ((x_{2\theta}t_\theta), (u_\theta t_\theta), (t_\theta u_{\theta+1}), (v_\theta w_\theta), (w_\theta v_{\theta+1}), (w_\theta y_{2\theta})): 1 \leq \theta \leq \beta - 1\} \in Z_4$ and also for all i≠j ∈ [1, 4], $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and $k \neq l \in [0, 3]$, $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Thus, by definition 2.1, the graph (HL (β) ⊙ K_1), $\beta \geq 3$ is Quotient-4 cordial labeling.

4.CONCLUSION

In this paper, it is proved that some unicyclic graphs and some corona of ladder graphs which admits quotient-4 cordial. The existence of quotient-4 cordial labeling of different families of graphs will be the future work.

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