Rainbow Dominator Chromatic Number of Extended Jewel Graph, Trees and Firecracker Graph

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Abstract
Rainbow vertex coloring and rainbow dominator chromatic number of graphs have been developing rapidly in recent times in the design of mathematical modelling. This was followed by Rainbow dominator coloring which has attracted many researchers in graph theory. In this paper, a study on rainbow dominator coloring of extended jewel graph, fire cracker graph, olive tree, coconut tree and banana graph is undertaken. Rainbow dominator chromatic number $\chi_{rd}(G)$ is also determined for these graphs. Few illustrations are also shown.

Keywords — Coloring, Rainbow dominator coloring, rainbow dominator chromatic number

Introduction

Assigning of colors to vertices subject to certain conditions is called graph coloring which is a special case of graph labelling. It finds its application in scheduling [5], image processing [7], data mining [1] etc. For different coloring patterns, many parameters were introduced and analyzed [1][5][16][17][18][19]. The minimum number of colors required to color the graph is called chromatic number. In the year 2008, Chatrand, John and Mckeon introduced Rainbow edge coloring [4]. A new concept of Rainbow vertex coloring was introduced by Krivelevich and Yuster in 2010[9]. Kulkarni Sunita Jagannatharao, S. K. Rajendra and R. Murali studied Rainbow dominator coloring for standard graphs namely path, prism graph and wheel graph in 2021[11]. In this paper, we find the rainbow dominator chromatic number for extended jewel graph, fire cracker graph, olive tree graph and banana graph.

Definition 1: Proper Coloring [11]
A proper coloring of a graph G is an assignment of colors to the vertices of the graph such that no two adjacent vertices have the same color and the chromatic number $\chi(G)$ of the graph is the minimum number of colors needed in a proper coloring of G.

Definition 2: Dominator Coloring[11]
A dominator coloring of a graph is a proper coloring such that each vertex dominates every vertex in at least one color class consisting of vertices with the same color/.

The chromatic number of a graph is the minimum number of colors needed in a dominator coloring of G

3: Rainbow Dominator Coloring[11]

A rainbow dominator coloring of a graph G is a proper rainbow coloring of the graph G ,in which every vertex of G dominates every vertex of some color class.The minimum number of color classes in the graph G is called the rainbow dominator chromatic number and is denoted by χ_{rd} (G).

Definition 4: Rainbow Connection Number [9]

In a connected edge colored graph G, if any two vertices are connected by a rainbow path which is a path whose edges have distinct colors. The minimum number of colors required to make the graph rainbow connected is called Rainbow connection number.

Definition 5: Extended jewel graph [14].

The extended jewel graph, EJ_n^* is obtained from jewel graph without prime edge J_n^* by appending arbitrary vertices in J_n^* such a way that they all are connected to vertex x and vertex y.

Definition 6: Banana tree graph [10].

A banana tree Bt (n,k) is a graph obtained by connecting a single leaf from n distinct copies of a k star graph with a single vertex distinct from the star graphs.

Definition 7: Firecracker tree [2]

A firecracker graph FC(m, n) obtained from the concatenation of star $K_{1,n}$ (n fixed) by linking one leaf from each.

Definition 8: Olive tree [6]

Olive tree OT_k is a rooted tree consisting of k branches where the i^{th} branch is a path of length i.

Definition 9: Coconut tree [12]

A coconut tree CT(m,n) is the graph ,for all positive integer n and m ≥ 2 is obtained from the path $P_{n=}$ by appending 'n' new pendant edges at an end vertex of P_m.

Main results

Theorem 1: The rainbow dominator chromatic number of an Extended jewel graph

E (J_n^*) is χ_{rd} E(J_n^*) = 4.

Let J_n^* be any jewel graph without prime edge. Let $V(J_n^*) = \{x, y, u, v, v_i\}/ 1 \le i \le n$ and $E(J_n^*)$= {ux,vx,uy,vy, uv_i, vvi/ $1 \le i \le n$}.

Let G = E(J_n^*) be any extended jewel graph obtained from jewel graph without prime edge. J_n^* by appending J_n^*such a way that all J_n^* are connected to vertex x and vertex y. The vertices are colored using the following procedure.

Let us color the vertices u, x, y as C_1, C_2, C_1. The vertices v, w, z are colored as C_2, C_3, C_4. U_1,U_2,U_3,U_4 are colored C_1..U_1,U_2,…U_n dominates vertex {z}. U dominates {x,v}. y dominates {z}. x dominates {w}. w dominates {z}. v dominates {w} z dominates {w}. Thus, every vertex will dominate at least one-color class. And for every pair of vertices there exists a rainbow path. The rainbow dominator chromatic number for the extended jewel graph is χ_{rd} **E(** J_n^*) = 4.

Dominating set	Dominated color classess
U_1, U_2,…,U_n	C_4
U	C_2
V	C_3
W	C_4
X	C_3
Y	C_4
Z	C_3

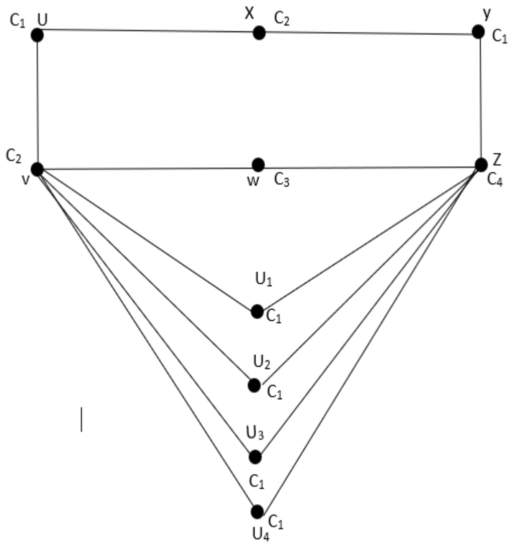

Fig 1: Extended jewel graph

Thus, we have determined the rainbow dominator chromatic number of extended jewel graph.

Theorem 2: The rainbow dominator chromatic number of an olive tree T_4 is $\chi_{rd}(T_4)$ = 8.

Let Olive tree T_4 have 4 branches with the vertices $\{V_0,V_1,\dots,V_{10}\}$. We color the vertices using the following procedure.

Let vertex V_0 be colored C_1, V_2 be colored C_2, V_1,V_6 and V_{10} be colored C_3.V_7,V_8,V_9 be colored C_6,C_7,C_8 respectively. V_1,V_2 and V_7 dominates $\{V_0\}$.V_4 dominates $\{V_4\}$. V_5 dominates $\{V_4\}$. V_6 dominates $\{V_5\}$. V_8 dominates $\{V_7\}$. V_8 dominates $\{V_7\}$. V_{10} dominates $\{V_9\}$. Thus, every vertex will dominate at least one-color class. And for every pair of vertices there exists a rainbow path.

Dominating set	Color class.	Dominating set	Color class.
V_1. V_2	C_1	V_7	C_1
V_3	C_2	V_8	C_6
V_4	C_4	V_9	C_7
V_5	C_4	V_{10}	C_8
V_6	C_5		

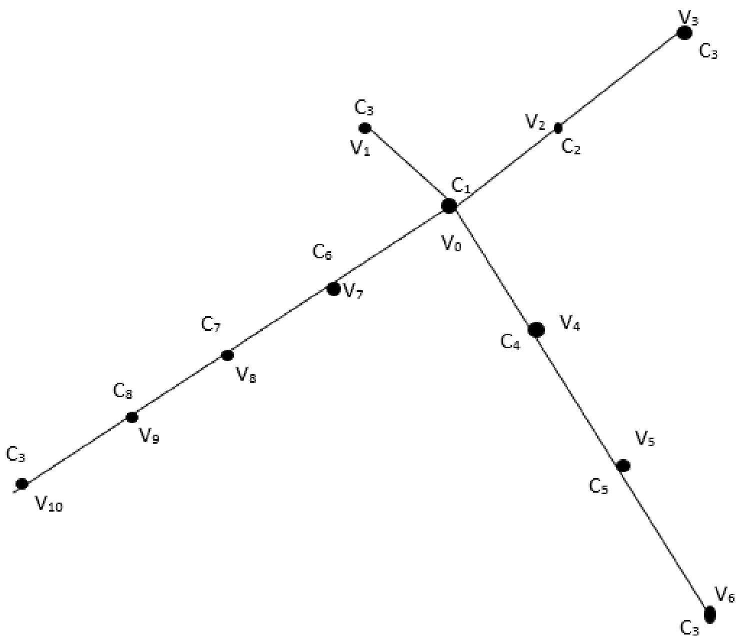

Fig 2: Olive tree

Thus, the rainbow dominator chromatic number for Olive tree is determined.

Theorem 3: The rainbow dominator chromatic number of a banana tree is Bt (3,4) is

χ_{rd} **Bt(3,4)** = 8.

A banana tree comprises of n distinct copies of k star graph with a single vertex distinct from star graphs.

Let the banana tree Bt (3,4) have V_2,V_5,V_6,V_7,V_8 as vertices of first k(1,4), $V_3,V_9,V_{10},V_{11},V_{12}$ as vertices of second k(1,4) and $V_4,V_{13},V_{14},V_{15},V_{16}$ as vertices of third k(1,4). The vertices are colored using the following procedure.

Let the vertex V_1 be colored C_1. V_2, V_3, V_4 be colored C_2, C_3, C_4 respectively. V_5, V_9, V_{13} be colored C_5, C_6, C_7. V_6, V_7, V_8, V_{10}, V_{11}, V_{12}, V_{14}, V_{15}, V_{16} be colored C_8.

V_1 dominates $\{V_2\}$. V_2 dominates $\{V_1\}$. V_3 dominates $\{V_1\}$. V_4 dominates $\{V_1\}$. V_5 dominates $\{V_2\}$. V_6,V_7,V_8 dominates $\{V_5\}$. V_9 dominates $\{V_3\}$. V_{10},V_{11},V_{12} dominates $\{V_9\}$

V_{13} dominates $\{V_4\}$and V_{14},V_{15},V_{16} dominates $\{V_9\}$. Thus, every vertex will dominate at least one-color class. And for every pair of vertices there exists a rainbow path.

Dominating set	Color class.	Dominating set	Color class.
V_1	C_2	V_6, V_7, V_8	C_5
V_2	C_1	V_9	C_3
V_3	C_1	V_{10}, V_{11}, V_{12}	C_6
V_4	C_1	V_{13}	C_4
V_5	C_2	V_{14}, V_{15}, V_{16}	C_7

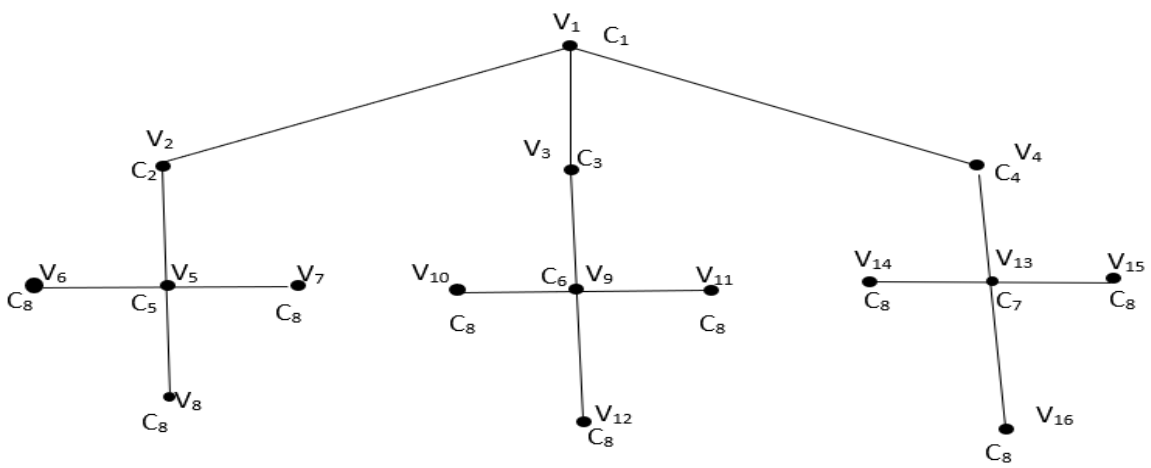

Fig 3: Banana tree

Thus, we have determined the rainbow dominator chromatic number of banana tree.

Theorem 4: The rainbow dominator chromatic number of a coconut tree CT (m,n) is

χ_{rd} **CT (m,n)** = m.

The coconut tree graph comprises of a path P_m and a star graph $k_{1,n}$.

Let $V_1,V_2\ldots.V_m$ be the vertices of the path P_m and $V_{m+1},V_{m+2}\ldots.V_{m+n}$ be the vertices of star $K_{1,n}$. To assign proper colors to the graph, the following procedure is followed.

Assign colors C_1 to V_1.C_2, C_3…C_m to vertices V_2,V_3… V_m of path P_m.Assign color C_1 to the pendant vertices of star $K_{1,n}$.

Thus, every vertex will dominate at least one-color class. And for every pair of vertices there exists a rainbow path.

Dominating set	Color class.	Dominating set	Color class.
V_1	C_2	V_4	C_5
V_2	C_3	V_6	C_5
V_3	C_4	V_7, V_8, V_9, V_{10}, V_{11}	C_6

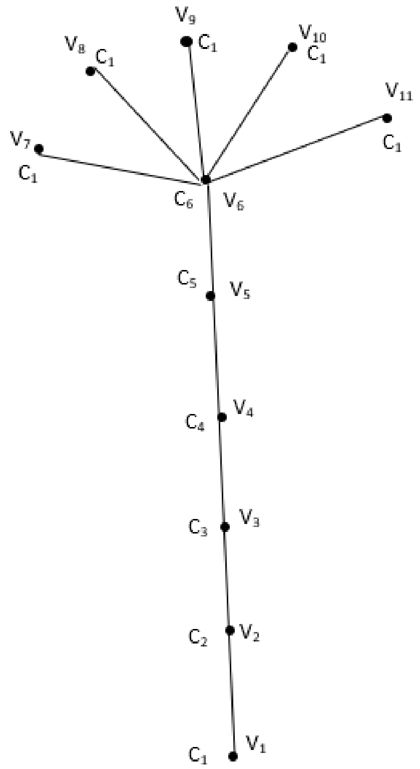

Fig 4: Coconut tree

Thus, we have determined the rainbow dominator chromatic number of coconut tree.

Theorem 5: The rainbow dominator chromatic number of a fire cracker is FC (3,5) is

χ_{rd} [FC] = 7.

The fire cracker graph FC (3,5) comprises of three $k_{1,5}$ stars. The vertices are colored using the following procedure.

Let the vertices V_1,V_2,V_3 be colored C_1,C_2 and C_3 respectively. V_4, V_5 ,V_6 be colored C_4,C_5 and C_6 respectively. V_7, V_8, V_9 are colored C_7. V_{10}, V_{11}, V_{12} are colored C_7.

V_{13}, V_{14}, V_{15} are colored C_7. Thus, every vertex will dominate at least one-color class. And for every pair of vertices there exists a rainbow path.

Dominating set	Color class.	Dominating set	Color class.
V_1	C_2, C_4	V_6	C_3
V_2	C_1, C_3, C_5	V_7, V_8, V_9	C_4
V_3	C_2, C_6	V_{10}, V_{11}, V_{12}	C_5
V_4	C_1	V_{13}, V_{14}, V_{15}	C_6
V_5	C_2		

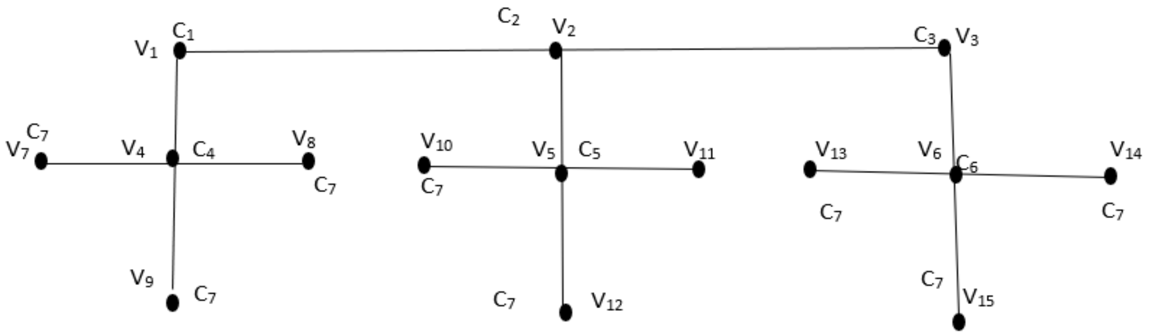

Fig 5: Fire cracker

Thus, rainbow dominator chromatic number of fire cracker graph is determined.

Conclusion:

In this paper, rainbow dominator coloring of connected and undirected finite graphs is determined. The rainbow dominator chromatic number for graphs like extended jewel graph, banana tree, olive tree, coconut tree and fire cracker which is denoted as χ_{rd} are determined. The basic parameters for this rainbow dominator coloring is the existing concepts, rainbow path and dominator coloring of every vertex. It is proved that every pair of vertices has a rainbow path and also satisfies the condition that every vertex dominates at- least one color class. There is further scope for determining rainbow dominator chromatic number of more number of connected finite graphs.

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