An Overview of Linear Transformation

Abhay dhadwal[1, a)] , Ravinder Kumar poonia[2, b)]

[1]Post graduate student, Department of Mathematics, UIS, Chandigarh University, Gharuan, Mohali, Punjab, 140301, India
[2]Associate Professor, Department of Mathematics,
UIS, Chandigarh University, Gharuan, Mohali, Punjab,
140301, India
[a)] Corresponding author: dhadwalabhay@gmail.com
[b)] Rkpooniae7260@gmail.com

Abstract
Linear transformations play an important role within the sector of algebra. In this paper we will be covering different parts of the linear transformations starting from its definition to kernels and examples. Yet, when we want to proceed or change the image in any way like rotating it about a point on the screen, we require a function to evaluate its original position for each of the original vectors. While, a vector could be used to specify, a certain type of motion actual vectors themselves are essentially static, unchanging objects. These transformations can be defined on finite or infinite spaces so there have been different types of linear transformations. It's known by different names such as linear maps or mapping or vector space homomorphism. The functions satisfying the property under vector addition and scalar multiplications are termed as linear transformation. A writing review that directly connects to the content of this section is provided, along with headings for additional research and didactic proposals

KeyWords: - Linear transformation, kernel, image, range, vector space, Linear transformation characteristics ,Theorem of Nullity for Rank and Matrix representation.

INTRODUCTION

The intent of this paper is to discuss about the linear transformations, its definition, algebraic classification, examples and features. In algebra, a linear transformation will be defined as a map from one to another vector space. These transformations can be defined only if it satisfies the two properties, (vector addition and scalar multiplication) The linear transformation sometimes also known as the vector space homomorphism, the linear map or the linear mapping. The initiation of the speculation of system of linear equations was done by Rene Descartes in 1637 . He has described mappings in this that retain the linear structure of many vectors space's much as how the length of vector parametrizes the line. The function is called linear because it preserves the linear combinations, also the linear mappings give the result as a line. The range for any linear transformation can be seen as endomorphism if it comes to be same as the domain vector space. Also, it can be considered as automorphism if it is invertible. These transformations play a vital role not only within the branch of algebra of mathematics but also in the real life as well. These are important because they preserve the structure of every vector space in which these transformations are defined. If both vector spaces are specified over the same field, then these transformations can also be defined. The kernel and image, both are the subspaces of the range of the defined linear transformation .

PRELIMINARIES

"Definition":

Allow A and B be the vector space above the identical field Q. Then the mapping Q: $A \rightarrow B$ is known as linear transformation if it for any two vectors a, b ∈ A and any scalar c ∈ Q, the below two axioms needs to be satisfied:

(1) $Q(a + b) = Q(a) + Q(b)$ ...… 1

(2)Q (c a) = c Q(a) ……2

Condition 1 and 2 are equivalent to sup.

Q (αa + βb) = α Q(a) + β Q(b)

Note: a) condition (1)is called Additive property of T and condition (2) is called homogenous property of T.

For any vectors $a_1 \dots\dots\dots., a_n$ V and scalars $c_1 \dots\dots\dots., c_n$ K, the following equations hold due to the associativity of the addition operation indicated as +.

$c_1 f(a_1) + \dots\dots + c_n f = c_1 f(a_1) + \dots\dots + c_n f = c_1 f(a_1) + \dots\dots + c_n f = c_1 f(a_1) + \dots\dots + c_n f\ (a_n)$ 4].

As a result, a linear map is one in which linear combinations are preserved[5].

It follows that f(0b) =0a by denoting the zero elements of the vector spaces A and B with the letters 0b and 0a, respectively. In the equation for degree 1 homogeneity, let c = 0 and an A be the variables:

F(0b) = f(0b) = 0f(b) = 0a F(0b) = f(0b) = 0f(b) = 0a F(0b)

Another definition:

A linear transformation S is a mapping from one vector space A to one more vector space B. [5].

S: A -- B, where m and n are vector spaces

X: the domain of S

Y: the co-domain of S

A mapping S is termed as a linear transformation if it satisfies the subsequent two axioms:

1) $S\ (m+n) = S(m) + S(n), \forall$ m, n ∈A

2) $S(cm) = c\ T(m), \forall$ c ∈ R

2.1 Algebraic classification of Linear transformation:

Let us consider A and B be the vector space above a field K, and X: A → B be a linear map [6].

a) Monomorphism:

If X meets the following conditions, it is said to be injective or monomorphism:

1. X is one-one.
2. Ker X = {0_V}
3. dim (Ker X) = 0
4. X is left-invertible, which means that the identity map on V is described by a linear map S: W V.

b) Epimorphism: Epimorphism is a term that refers to a

If X meets the following criteria, it is said to be surjective or epimorphism:

1. X is onto
2. coKer X = {0_w }
3. X is right-invertible, which means that the identity map on V is described by a linear map S: W V.

c)Isomorphism:

If X is both right-invertible and left-invertible, it is said to be an isomorphism.

3. A linear transformation's kernel S :

The set of all the vectors in X whose image under the linear transformation S: XY is zero is known as the kernel of the linear transformation if X(F) and Y(F) are two vector spaces. Ker (S) or N are used to indicate it (S).

T(x) = 0Y; T(x) =N(T) = x X; T(x)=N(T) =x X

Example:1 Verify the accuracy of a linear transformation. S $(X_1, X_2) = (X_1 - X_2, X_1 + 2X_2)$. [9]

Solution: Let x = (a_1, a_2) and y = (b_1, b_2)

Then, vector addition property,

S (x+ y) = S $(m_1 + n, m_2 + n_2) = \ ((m_1 + n_1) - (m_2 + n_2), (m_1 + n_1) + 2(m_2 + n_2))$

$= ((m_1 - m_2) + (n_1 - n_2), (m_1 + 2m_2) + (n_1 + 2n_2))$

$= \ (m_1 - m_2, m_1 + 2m_2) + (n_1 - n_2, n_1 + 2n_2)$

= S(x) + T(y)

Scalar multiplication:

cx = c (m_1, m_2) = (cm_1, cm_2)

S (x) = S (cm_1, cm_2) = $(cm_1 - cm_2, cm_1, 2cm_2)$

= $c(m_1 - m_2, m_1 + 2m_2)$

= c S (x)

Since it satisfies both the properties, therefore, T is linear transformation.

Example:2 Demonstrate that the linear transformation S:$R^2 \to R^2$elucidate by S (x, y) = (0, -x) is a linear transformation.

Proof: let u = (u_1, u_2) and v =$(v_1, v_2)R^2$ be any real numbers, and (x, y) be any real numbers

Therefore xu +yv = x(u_1, u_2) +y(v_1, v_2)= $(xu_1 + yv_1, xu_2 + yv_2)$belongs to R^2

Now S(xu+ yv) = $S(xu_1 + yv_1, xu_2 + yv_2)$ = $(0, -(xu_1 + yv_1))$

= $(0, -xu_1) + (0 - yv_1)$ = $xS(u_1, u_2) + yS(v_1, v_2)$

= $xS(u) + xS(v)$

Therefore, the given transformation is a linear transformation .

Example of Functions that are not linear transformations:

1. f(x) = sinx

sin $(x_1 + x_2) \neq$ sin (x_1) + sin (x_2)

$sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right) \neq$ sin ($\frac{\pi}{2}$) + sin ($\frac{\pi}{3}$)

This implies that f(x) = sinx is not a linear transformation.

2. $\mathrm{f(x)} = \mathrm{x}^2$

$(\mathrm{x}_1 + \mathrm{x}_2) \neq \mathrm{x}_1^2 + \mathrm{x}_2^2$

This convey that $\mathrm{f(x)} = \mathrm{x}^2$ is not a linear transformation.

3. $f(x) = x + 1$

It is not a linear transformation because this function does not fulfil both vector addition and scalar multiplication.

Zero Transformation [1]:

$S: A \to \mathrm{B}$ $\mathrm{S(v)} = 0\ \forall\ \mathrm{a} \in \mathrm{A}$

Identity Transformation [1]:

$S: A \to \mathrm{B}$ $\mathrm{S\ (a) = a}, \quad \forall\ \mathrm{a} \in \mathrm{A}$

The characteristics of linear transformations [3]:

If $T: V \to W$ is a linear transformation from V(F) to W(F). Then

$S: V \to X$ is a linear transformation from V(F) to W(X). then , and $a, b \in V$

1.S(0) = 0

2.S (-a) = - S (a)

3.S (b-a) = S (b) –S (a)

4.If $\mathrm{a} = \mathrm{c_1 a_1} + \mathrm{c_2 a_2} + \cdots + \mathrm{c_n a_n}$. Then ($S(a) = S(\mathrm{c_1 a_1} + \mathrm{c_2 a_2} + \cdots + \mathrm{c_n a_n})$

4. Rank and Nullity of Linear Transformation

RANK : If V (F) and W (F) are vector spaces and T: V W be an L.T., then the dimension of the range space of T is known as the rank of T. (T)

Therefore, (T) = dim (Range T)

Nullity: If T: V →W is an L.T., and V (F) and W (F) are vector spaces, then T's nullity is its null spaces' dimension, and it is represented by the symbol v (T)

So, v (T) = dim (Null space of T)

Range: When T: V →W is a linear transformation and V (F) and W (F) are vector spaces, the image set of V under T is either R (T) or T (V), i.e., Range T = T (v) | v V.

Rang Space is another name for Range T. (A vector space is R (T)) [10]

RANK - NULLITY THEOREM OR SYLVESTER'S LAW OF NULLITY

If both V and W are vector spaces and T is a linear transformation, then V W. Consider the V to have n dimensions. If V is a finite - dimensional space, then Rank T + Nullity T = n Rank (T) + Nullity (T) = dim V. Furthermore, V R (T) and N (T) have finite dimensions.

Important points:

1.A linear transformation is known for its operation preserving property.

2.A linear transformation A linear operator is one that transforms a vector space into itself.

Example of Linear Transformation and bases:[11]

Q1 Let $\left(S\colon R \overset{3}{\to} R^2\right)$be a linear transformation such that S (1, 0, 0) = (2, -1, 4),

S (0, 1, 0) = (1, 5, -2) , S (0, 0, 1) = (0, 3, 1), Find S (2, -2, -1).

Solution:

(2, -2, -1) = 2(1, 0, 0) -2(0, 1, 0) - 1(0, 0, 1)

S (2, -2, -1) = 2S (1, 0, 0) -2S (0, 1, 0) – 1S (0, 0, 1) [because given transformation is a linear transformation)

= 2(2, -1, 4) - 2(1, 5, -2) - 1(0, 3, 1)

=(4,-2,8) - (2,10,-10) - (0,3,1) = (2,-15,17) .

Q2 Let $\left(S\colon R \overset{3}{\to} R^2\right)$be a linear transformation such that S(1,0,0)=(3,-2,1) ;

S(0,1,0)= (2,1,-1) ; S(0,0,1) =(-2,-2,1) , find S(1,2,3)

Solution:

(1,2,3) = 1(1,0,0,) +2(0,0,1) +3(0,0,1)

S(1,2,3) = 1S(1,0,0) +2S(0,1,0) + 3S(0,0,1) [because given transformation is a linear transformation)

=1 (3,-2,1) +2 (2,1,-1) +3 (-2,-2,1)

=(3,-2,1) +(4,2,-2) +(-6,-6,3) = (1,-6,2)

5. The Matrix of a Linear Transformation:

For a vector x in the domain of T, given matrix of a linear transformation is one where T(x)=Ax. This implies that multiplication by this matrix while applying the transformation T to a vector is equivalent. Such a matrix, which is specific to the transformation, can be found for any linear transformation T from Rn to Rm for fixed values of n and m.

CONCLUSION

The property of a function that satisfies the vector addition and scalar multiplication of the vector spaces above a given field F is known as the linear transformation, also known as the linear map or vector space homomorphism [1]. In this study, we discuss numerous linear transformation properties, starting with the image and ending with the transformation kernel. These transformations have been divided into different categories according to their algebraic properties, these are monomorphism, epimorphism and isomorphism. These transformations are very important not only in the linear algebra branch of mathematics but also in the real life. One of the main uses of these transformations is in the machine learning application. These transformations are used in the rotation, 2D and 3D object translation and scaling the linear transformations can be used to change the shape of things. They're also employed as a mechanism for representing change, such as in calculus, where derivatives are used, or in relativity, where they're used to keep track of the local reference frame alternations .

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