Applications of Double Iman Transform to Boundary Value Problem

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Abstract

In this paper, we apply the method of the double Iman Transform for solving one-dimensional boundary value problems. Through this method, the boundary value problem is solved without converting it into an ordinary differential equation; therefore, there is no need to find the complete solution of an ordinary differential equation. This is the biggest advantage of this method. The main focus of this paper is to develop the method of the double Iman transform to solve initial andboundary value problems in applied mathematics.

Keywords: *Boundary Value Problem, Double Iman Transform, Inverse Iman Transform.*

1. Introduction

In mathematics, in the field of differential equations, a boundary value problem is a differential equation together with a set of additional constraints, called the boundary conditions. A solution to a boundary value problem is a solution to the differential equation that also satisfies the boundary conditions.Boundary value problems arise in several branches of physics, as any physical differential equation will have them. Problems involving the wave equation, such as the determination of normal modes, are often stated as boundary value problems. To be useful in applications, a boundary value problem should be well posed. This means that given the input to the problem, there exists a unique solution that depends continuously on the input. Much theoretical work in the field of partial differential equations is devoted to proving that boundary value problems arising from scientific and engineering applications are in fact well-posed. Integral transforms are extensively used in solving boundary value problems and integral equations. The problem related to a partial differential equation can be solved by using a special integral transform Thus, many authors solved the boundary value problems by using asingle Laplace In this study, we use the

Double Iman Transform to solve the Wave and Heat equation, which is a one-dimensional

boundary value problem. Henceforth, the different problems of boundary value are solved without converting them into ordinary differential equations, and there is no need to find a complete solution. So this method is very reliableand convenient for solving boundary value problems. The scheme is put to the test by referring to two different example. Some example of Iman Transform from:

S.no	$f(t)$	$I\{f(t)\}$
1	1	$\frac{1}{v^4}$
2	t	$\frac{1}{v^6}$
3	e^{at}	$\frac{1}{v^2(v^2-a)}$
4	$\sin(at)$	$\frac{a}{v^2(v^4+a^2)}$
5	$\cos(at)$	$\frac{1}{v^4+a^2}$

2. Main Results:

Example 2.1:

Consider the homogeneous wave equation in the form

$$U_x = c^2 U_{xx}$$

$$U(x,0) = sinx\,, U_t\,(x,0) = 2$$

$$U(0,t) = 2t\,, U_x\,(0,t) = \cos ct$$

By taking the double Iman transform

$$v^4 l(u,v) - l(u,0) - \frac{1}{v^2}\frac{\partial l(u,0)}{\partial t} = c^2\left[u^4 l(u,v) - l(0,v) - \frac{1}{u^2}\frac{\partial l(0,v)}{\partial x}\right]$$

The single Imantransform of initial conditions gives

$$l(u,0) = \frac{1}{u^2(u^4+1)}, \frac{\partial l(u,0)}{\partial t} = \frac{2}{u^4}$$

$$l(0,v) = \frac{2}{v^6}, \frac{\partial l(0,v)}{\partial x} = \frac{1}{v^4+c^2}$$

Then

$$v^4 l(u,v) - c^2 u^4 l(u,v) = \frac{1}{u^2(u^4+1)} + \frac{2}{u^4 v^2} - \frac{2c^2}{v^6} - \frac{c^2}{u^2(v^4+c^2)}$$

$$(v^4 - c^2u^4) l(u,v) = \frac{1}{u^2(u^4+1)} + \frac{2}{v^2}\left(\frac{1}{u^4} - \frac{c^2}{v^4}\right) - \frac{c^2}{u^2(v^4+c^2)}$$

$$(v^4 - c^2u^4) l(u,v) = \frac{1}{u^2(u^4+1)} - \frac{c^2}{u^2(v^4+c^2)} + \frac{2}{v^2}\left(\frac{v^4 - c^2u^4}{v^4u^4}\right)$$

$$(v^4 - c^2u^4)l(u,v) = \frac{v^4 - c^2u^4}{u^2(u^4+1)(v^4+c^2)} + \frac{2}{u^4v^6}(v^4 - c^2u^4)$$

Then

$$l(u,v) = \frac{1}{u^2(u^4+1)(v^4+c^2)} + \frac{2}{u^4v^6}$$

Applying inverse double Iman transform [1]

$$U(x,t) = sinx\cos(ct) + 2t$$

Example 2.2:

Solving the heat equation

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, t > 0$$

$$U(x,0) = sinx\,,$$

$$U(0,t) = 0\,, U_x\,(0,t) = e^{-t}$$

By taking the double Iman transform we get

$$v^2 l(u,v) - u^4 l(u,v) = \frac{1}{v^2} l(u,0) - l(0,v) - \frac{1}{u^2}\frac{\partial l(0,v)}{\partial x}$$

The single Iman transform of initial conditions gives

$$l(u,0) = \frac{1}{u^2(u^4+1)}, \qquad \frac{\partial l(0,v)}{\partial x} = \frac{1}{v^2(v^2+1)}\,, l(0,v) = 0$$

$$(v^2 - u^4)l(u,v) = \frac{1}{v^2}\left[\frac{1}{u^2(u^4+1)}\right] - \frac{1}{u^2}\left[\frac{1}{v^2(v^2+1)}\right]$$

$$(v^2 - u^4)l(u,v) = \frac{(v^2 - u^4)}{u^2v^2(u^4+1)(v^2+1)}$$

$$l(u,v) = \frac{1}{u^2v^2(u^4+1)(v^2+1)}$$

Applying inverse double Iman transform

$$U(x,t) = e^{-t} sinx$$

Conclusion:

In this paper, we use double Iman Transform and the standard properties of Iman Transform are discussed for solving one-dimensional boundary value problems.

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