# Realistic mathematical education in the study of ordinary differential equations

**Wilson Marcelo Román Vargas**
*Universidad de las Fuerzas Armadas - ESPE Sangolqui, Ecuador, wmroman@espe.edu.ec*

**Martha Ximena Dávalos Villegas**
*Facultad de Ciencias, Escuela Superior Politécnica de Chimborazo (ESPOCH), Riobamba, 060155, Ecuador, rinsuasti@espoch.edu.ec*

**Alex Eduardo Pozo Valdiviezo**
*Facultad de Ciencias, Escuela Superior Politécnica de Chimborazo (ESPOCH), Riobamba, 060155, Ecuador, eduardo.pozo@espoch.edu.ec*

**Rómel Manolo Insuasti Castelo**
*Facultad de Mecánica, Escuela Superior Politécnica de Chimborazo (ESPOCH), Riobamba, 060155, Ecuador, rinsuasti@espoch.edu.ec*

**Abstract**

The didactic scenario generated by the covid 19 pandemic has affected the learning process of ordinary differential equations, affecting the levels of understanding and applicability of mathematical concepts, in this sense we ask ourselves what mathematical understanding of second-order linear ODEs Is it fostered with a teaching design based on real situations?

In the research, the perspective of Realistic Mathematics Education is considered, where the elements of mathematical understanding are described and didactic activities framed in the solution of a real context problem represented by a dynamic system are designed, the results achieved show achievements in understanding. conceptual and in the fluidity of the procedures, but it is necessary to redesign didactic strategies to strengthen the productive predisposition and the strategic competence in the students of the mechatronics engineering career.

**Keywords:** *Realistic Mathematics Education, Mathematical Comprehension, Ordinary Differential Equations, Mathematical Models.*

## 1. INTRODUCTION

One of the biggest problems that originate in the study of differential equations is to prioritize procedural learning, causing in the student the development of long and complicated analytical and rote processes of a repetitive nature that affect the level of understanding of the mathematical concepts studied analysis carried out by Dullius ( 2011). Arslan (2010a and 2010b) emphasizes that procedural learning in the study of differential equations involves only memorizing operations without understanding the underlying meanings. Conceptual learning involves understanding and interpreting concepts and the relationships between them.

For Artigue (2021), he must consider that the study of ordinary differential equations (EDO) as part of the academic training of an engineering student, has transcendental importance due to the incidence of the application of theoretical and practical knowledge in the solution of problems linked to the student's graduation profile. . In this context, inadequate planning and teaching practice can lead to situations of stress and demotivation that directly affect the learning of EDO.

There are currently countless investigations focused on the teaching-learning process of EDOs. Most of the papers analyze first-order DODs and use problem solving by focusing on the analysis of numerical and graphical solutions; that is, they address the solution of a problem of application of ODS (Dullius, 2011; Perdomo, 2011; Farrás et al., 2011), highlighting the identification of students' abilities to convert symbolic information into graphics and vice versa (Rasmussen and King 2000; Rasmussen and Keene, 2019; Hernández et al., 2017; Hernández et al., 2016).

This work assumes the perspective of Realistic Mathematics Education (RMS) in order to investigate the mathematical understanding of the student. Therefore, the research aims to answer what mathematical understanding of second-order linear ODS is fostered with a teaching design based on real situations?

Considering the perspective of the EMR, it seeks to contribute to the conceptual learning process of the ODE being a complement to the procedural learning that students have acquired when solving proposed exercises and that have no real context of application.

In the research we worked with second-level students of the Mechatronics Engineering career of the University of the Armed Forces-ESPE (Latacunga-Ecuador). It should be noted that the relevance of the work lies in the scarce publications on the teaching of ODE from the perspective of the EMR for second-order differential equations.

## 2. THEORETICAL FRAMEWORK

In the academic training of an engineer, one of the activities consists of representing physical, mechanical, geometric, economic and other quantities through mathematical expressions that link a function with its derivatives giving rise to differential equations; Therefore, various phenomena of nature can be analyzed through these mathematical structures.

Realistic Mathematics Education conceives mathematics as a human activity where the student can carry out his learning process through the mathematization of problems of real context. The didactic process of guided reinvention allows the development of mathematical understanding through different contexts and models, with didactic phenomenology being the methodology for the search for contexts and situations.

### 2.1 Realistic Mathematics Education

Schoenfeld (2016) argues that thinking mathematically is characterized by the capacity for abstraction and generalization, as well as competence in the use of tools typical of mathematics. In this context, the process of teaching mathematics should consist of finding solutions, exploring patterns and formulating conjectures, and not just memorizing procedures and formulas that involve performing exercises without understanding what is being executed.

The EMR refers to Hans Freudenthal (1905-1990), as the pioneer in promoting a change in mathematics education. This author promotes that the teaching of mathematics must be connected to reality and must be relevant to society (Gravemeijer and Terwel, 2000). That is, the EMR promulgates that new knowledge, skills or attitudes in students are achieved from confronting their concrete experience, reflective observation, abstract conceptualization and active experimentation in the solution of a real problem (De Lange, 1996; Bergsteiner et al., 2010).

In addition, the EMR emphasizes the active participation of the student in mathematical work by putting at his disposal a number of organized activities of mathematization. In this way, a didactic proposal is proposed aimed at changing traditional mathematical education focused on mechanical and rote aspects, so that it can be contextualized as a mathematical education related to the real environment of the student, thus generating meaning in their learning (Freudenthal, 2012).

Reviewing Freudenthal (2006), a process of mathematization can be established that allows students to orient the EMR. This process has generally been identified as a conceptual toolkit for the practice of RMS. A brief description of them is provided below.

1. Pose realistic problem situations that can be represented, and that motivate students to imagine different fields of application in their context. In our case, a mass-spring-shock absorber system.

2. Encourage the use of mathematical models through symbolic expressions, diagrams, schemes or others; making them the ideal tools to represent and organize the mathematical activity of the student.

3. Generate interactive teaching; that is, to make students re-invent mathematical ideas and tools from organizing or structuring problematic situations.

4. Talk with their peers, in environments specially created so that they can explain, reflect, justify, agree or disagree about the academic activity carried out.

5. Guided reinvention is conceived as the conjunction of roles and responsibilities between teacher and student through interaction.

2.2 Mathematical understanding

According to Rodríguez et al. (2016) the mathematical understanding of a student is given in terms of relating what he knows, how he knows it and what he uses it for; That is, it evidences its ability to integrate the elements that form a concept, to section them for analysis and to relate the concept to other sciences and practical life.

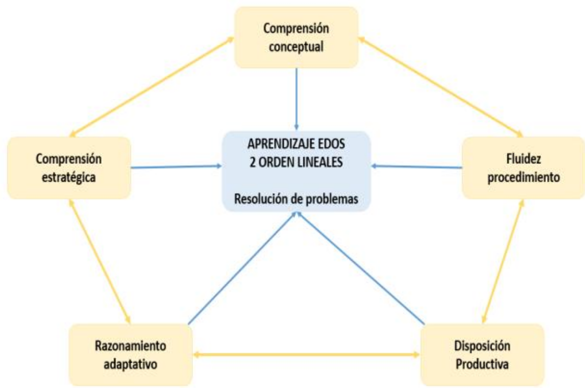
Elements of mathematical understanding

Perdomo (2010) identifies the elements of mathematical understanding, which contribute directly to evidence the cognitive learning of ODE (Zeynivandnezhad and Bates, 2018; Zeynivandnezhad, 2014); Figure 1 shows that the elements need to be interrelated.

-  Conceptual understanding: understands mathematical concepts and their relationships.
-  Fluency in procedures: demonstrates ability in the execution of procedures to solve the problem flexibly and correctly.
-  Strategic competence: shows ability to formulate, represent and solve problems of application of linear WEDs.
-  Adaptive reasoning: reveals your ability to think logically. Analyze, reflect, explain and justify.
-  Productive predisposition: demonstrates interest in learning ODE by linking them to problems in their context or the professional profile of the career.

The above elements are related in figure 1, where the resolution of a real context problem has been located as the motivation center to achieve mathematical understanding framed in the learning of the EDO.

**Figure 1. Elements of mathematical understanding for the study of ODE**




2.3 Second-order linear differential equations

Differential equations are part of functional analysis, constituting one of the branches of mathematics and are present in different fields of science (Khotimah and Masduki, 2016). In particular, in the training of an engineer, that is, in university mathematics education, a second-order linear differential equation is usually represented by the following mathematical expression (Ross, 2021; Zill and Cullen, 2013).

$$a_2(t)\frac{d^2x}{dt^2} + a_1(t)\frac{dx}{dt} + a_0(t)\,x = g(t) \qquad (1)$$

In our research, we focus on the analysis of mass-spring-damper (MRA) systems and electrical circuits (RLC) with a resistance (R), an inductance (L) and a capacitance (C). Table 1 presents the description of its mathematical elements that are part of dynamical systems and that allow the application of the mathematical models of the problem posed.

**Table 1. Mathematical elements of a second-order linear ODS with constant coefficients**

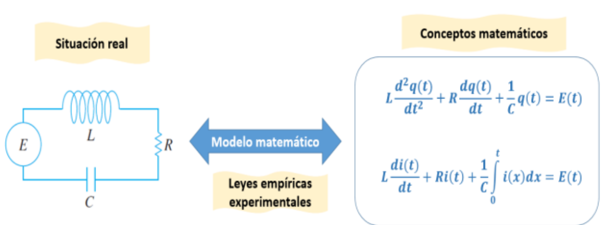
| Dynamical system | Analysis item | Mathematical representation |
|---|---|---|
| **Sistema RLC** $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = g(t)$ | Capacitor (C) | $V_C = \frac{1}{C}q$ |
| | Resistance (R) | $V_R = R\frac{dq}{dt}$ |
| | Inductor (L) | $V_L = L\frac{d^2q}{dt^2}$ |
| **Sistema MRA** $m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = g(t)$ | Displacement | $x$ |
| | Velocity | $v = \frac{dx}{dt}$ |
| | Acceleration | $a = \frac{d^2x}{dt^2}$ |

In the MRA system, the variation of values in the mass, the damper and the (m)(β) constant of elasticity of the spring generate the didactic scenarios of analysis and reflection of the incidence of the mathematical elements and their variations. For example, a small value in the shock absorber means system instability, and precisely this concept, the student associates it with real-life problems when relating it to damping systems of vehicles, bicycles and others.(k)

As mentioned above, one of the principles of EMR is to mathematize a real context problem where the use of mathematical models represents one of the challenges to teach mathematics (Camacho and Guerrero, 2015). According to Shahbari and Daher (2016) to involve the student in mathematical modeling contexts is to provide didactic resources where he can describe, graphically represent and include mathematical language to interpret a real context problem.

The research conducted by Schukajlow et al. (2018) focuses on analyzing the work that has been published regarding mathematical modeling. From the results found, it follows the need to incorporate mathematical models into the curricular plan, especially at the higher level. In this context, the mathematization of a problem is proposed with the perspective that students generate significant mathematical ideas when conceiving the mathematical model as an activity that relates the real situation with the mathematical elements through the empirical laws that intervene in a certain context. Figure 2 presents the elements for symbolizing, analyzing, and modeling a real-world context problem corresponding to a dynamic system of the RLC electrical circuit.

**Figure 2. Mathematical model representing the dynamic system of the RLC electrical circuit**




An indicator of performance of engineering students is to use ODE in a variety of contexts, so that it can be initiated into mathematical modeling that has as input a real-life problem. This contributes to students' meaningful learning (Kwon, 2009; Niss and Blum, 2020; Juárez et al., 2020; Boatman and Jessen, 2020; Czocher, 2017; Greefrath and Vorhölter, 2016).

## 3. METHODOLOGY

According to the problem posed and the research question, the work is qualitative, which seeks to explore the implementation of a didactic methodology and describe the mathematical understanding derived from said implementation.

The participants in the research were students studying the subject of Ordinary Differential Equations (EDO). The subject belongs to the second level of the curriculum of the Mechatronics Engineering career of the University of the Armed Forces – ESPE Latacunga headquarters - Ecuador. Due to problems of the Covid-19 pandemic, classes were held virtually and synchronously. The technological infrastructure consisted of using the Moodle platform, Google Classroom and/or institutional mail; and, for virtual classes, the Meet app.

Students have knowledge of the theoretical contents of differential and integral calculus and linear algebra. At the beginning of the EDO course, shortcomings found in concepts such as continuity, variation rates, the derived function and its link with the EDO were solved (Mkhatshwa, 2020). In addition, students know the classic methods of solving first-order WEDs.

With this background, the research proposal is presented that is based on the theoretical foundations of the EMR and whose purpose is that the student can analyze, reflect, generate ideas and methods to find the solution, explanation and argumentation of the proposed problem (Rasmussen and Ruan, 2008). The teaching approach based on the solution of a real context problem aims to promote in students the motivation to consider mathematics as an active discipline, as well as to establish relationships between mathematical elements. We hope that it will favor the development of skills such as examining, representing, transforming, solving and applying their learned knowledge (Groth, 2017).

The solution of the dynamic system proposed for the teaching and learning process of the EDO, constitutes a challenging task, which reflects a real situation and serves as a starting point for students to begin to investigate and explore (Vajravelu, 2018). Therefore, you must meet the following objectives:

-  Enable the reinvention of mathematical ideas and methods to analyze the ODS solution.

-  Demonstrate a balance between analysis, analytical, numerical and graphical process.

The use of computer systems is used, with the antecedent that students have knowledge about the use of Geogebra, Wxmaxima, Matlab, Simulink and Simscape software (Braun et al., 2018). This implies that the focus is solely on the mathematical understanding of second-order linear OEDs.

The use of technological resources is considered as a didactic resource that allows students to perform different explorations and can facilitate resolution strategies, as well as the verification of their conjectures (Santos, 2016).

Table 2 presents the instrument to analyze student comprehension based on compliance with the indicators established based on the elements of mathematical understanding described by Perdomo (2011).

In the analysis of the information obtained from the student, it can be categorized as SI when it meets the description of the indicator, PARTIAL in the case of having an approximation to the indicator and finally as NO when no feature of the indicator description is identified.

**Table 2. Indicators for elements of mathematical understanding**

| **Element** | **Indicator** | **Description** |
|---|---|---|
| Conceptual Comprehension **(CC)** | CC1. Identify empirical laws and their variables. | Know the context of application of the EDO. |
| | CC2. Determines the ODS that constitutes the mathematical model | Based on the laws of empirical it determines the ODE that represents the problem posed |
| Strategic Competence **(CE)** | CE1. Recognizes stages of<br>Solving a real-world context problem. | Based on the proposed problem, identify the type of EDO, conditions and their possible solutions. |
| | CE2. Demonstrates ability to formulate, represent and solve problems of application of linear WEDs. | Able to select between different methods, the most appropriate for solving the problem. |
| Procedural fluency **(FP)** | FP1. Demonstrates skill in executing procedures to solve the problem | Depending on the EDO that the model represents, apply the solution method. |

| | | |
|---|---|---|
| | flexibly and correctly. | |
| | FP2. Argue about the types of ODS solution. | It distinguishes the general and particular solution of the EDO, and performs an analysis on what has been found. |
| Adaptive reasoning **(AR)** | RA1. It makes known their ability to think logically. Analyze, reflect, explain and justify. | Verifies the analytical solution performed and that obtained through the software, explains what has been developed and interprets the results. |
| | RA2. Demonstrates ability to adapt the model to the varying conditions of the problem. | It performs an analysis of the graphic and numerical representations obtained, generates simulations and establishes relationships with the real context. |
| Productive predisposition **(PP)** | PP1. He answers questions asked by his peers. | It presents its arguments based on the theoretical conceptual foundation of the EDO. |
| | PP2. Demonstrates interest in learning ODE by linking them to problems in their context or the professional profile of the career. | Depending on their professional profile, they formulate a problem that can be expressed through a second-order linear ODOS, assess the restrictions they can make to the different variables involved in the problem. |

For the collection of information, observation is used in the exhibitions of the group work carried out by the students and the information is collected through table 2; Using the same table, the individual productions of the solution of a problem posed in a real context are recorded, where didactic activities are designed aimed at identifying the indicators of mathematical understanding.

The activities carried out are described below.

-  Group work

Pose and realize a real context problem that is framed in a dynamic mass-spring-damper system, in which it is evidenced:

a) The formulation of the problem,

b) The analysis of the mathematical elements involved and their initial conditions of the problem

c) The analytical development executed to solve the problem justifying the processes carried out

d) The use of mathematical software to verify the solution found and make variations to the initial conditions to establish real parameters of analysis.

-  Individual work

Statement of the problem to be solved by the students

Analyze the damping system of a bicycle where the rider is grade one overweight.

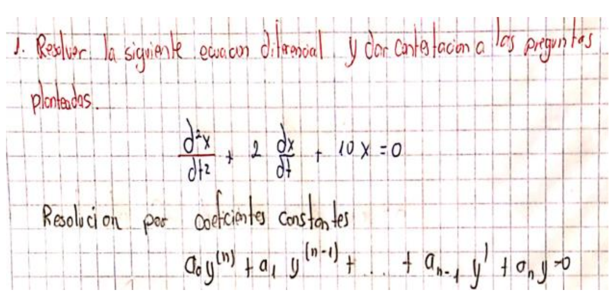
Consider that the person passes through a gap and does not lose the balance to continue transiting. Relate the mathematical elements with the context of the problem and execute the following didactic activities described in table 3. Problem adapted from Fauzan (2021).

**Table 3. Design of didactic activities**

| **Design of didactic activities** | **Elements of mathematical comprehension that you want to activate** | **It is expected to identify in the student the ability to** |
|---|---|---|
| Make a graphic representation that relates the mass of the person, the shock absorber of the bicycle and the constant of elasticity of the spring. | ▪ Conceptual Comprehension (CC) | Establish relationships between different types of representations and connect different mathematical concepts |
| Present the problem of real context through formal language of mathematics, considering the mechanical elements and conditions involved in the problem and induce the graphic representation of the possible solution of the problem posed. | ▪ Strategic Competence (CE) | Represent and analyze the coefficients of the mathematical model for a qualitative description of the differential equation |
| Present the analytical development executed and the verification of the solution through the use of mathematical software. | ▪ Procedural fluency (FP)<br>▪ Adaptive reasoning (AR) | Run procedures to resolve the issue flexibly and correctly |
| Propose new fields of application of second-order linear EDO in the field of engineering. Indicate what could be solved by applying EDO. | ▪ Productive predisposition (PP) | Ability to integrate the elements that make up a mathematical concept and relate it to other sciences and practical life. |

Prior to the research experience, students know and perform exercises to solve second-order linear differential equations they do not present any context of application and only strengthen the algorithmic aspect of development, an example is attached.

**Image 1. Formulation of a homogeneous second-order linear EDO with constant coefficients**



In image 1 you have the proposal to solve an exercise that does not represent any context of application.

**Image 2. Proposal to solve an exercise of application of a linear EDO of second order**

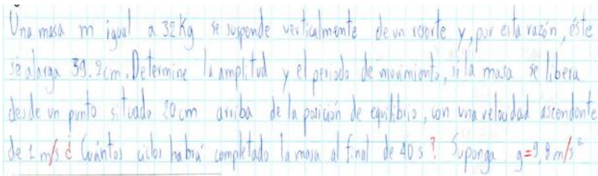

Una masa m igual a 32 Kg se suspende verticalmente de un resorte y, por esta razón, éste se alarga 39.2 cm. Determine la amplitud y el periodo de movimiento, si la masa se libera desde un punto situado 20 cm arriba de la posición de equilibrio, con una velocidad ascendente de 1 m/s ¿Cuántos ciclos habrá completado la masa al final de 40 s? Suponga g=9.8 m/s²

Image 2 presents the solution of an application exercise where the statement defines the data in a particular context, limiting the capacity for analysis and integration of mathematical foundations with other sciences and with the real context of the problem.

A mass equal to 32 kg is suspended vertically from a spring and, for this reason, it is lengthened by 39.2 cm. Determine the amplitude and period of movement, if the mass is released from a point 20 cm above the equilibrium position with an ascending velocity of 1 m/s. How many cycles will the mass have completed at the end of 40 s? Suppose g = 9.8 m/sm2.

## 4. RESULTS

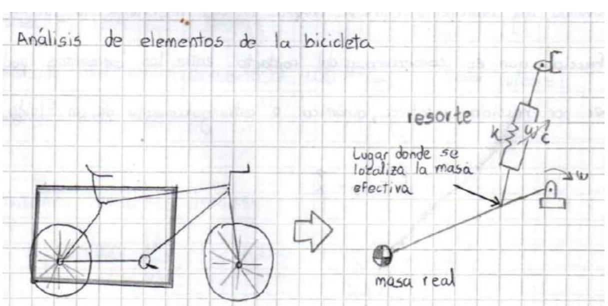
In the first instance, we proceed to analyze the activity carried out by three students according to the activities requested in table 3 and then identify descriptors that guide towards the indicators of mathematical comprehension described in table 2.

Image 2 transcript

| Didactic activity 1 | Principles of realistic mathematics education |
|---|---|
| Make a graphic representation that relates the mass of the person, the shock absorber of the bicycle and the constant of elasticity of the spring. | It uses intuition, common sense and experience to contextualize the problem, identifying elements that could be analyzed in the solution of the problem. |

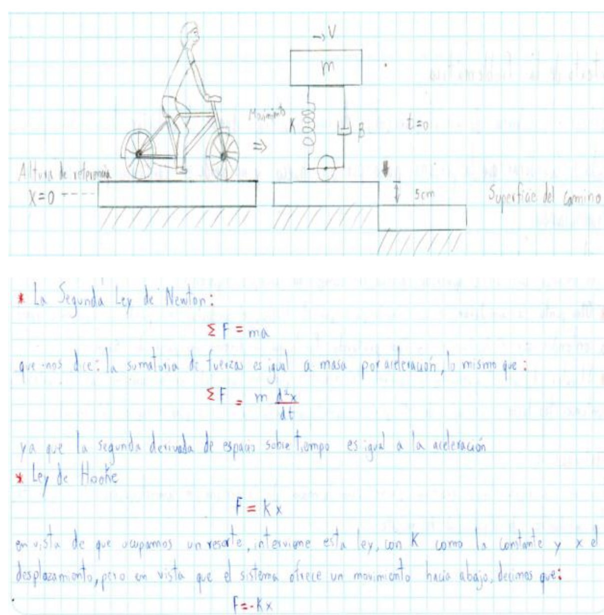
Activity carried out by students

**Image 3. Activity carried out by Solange**



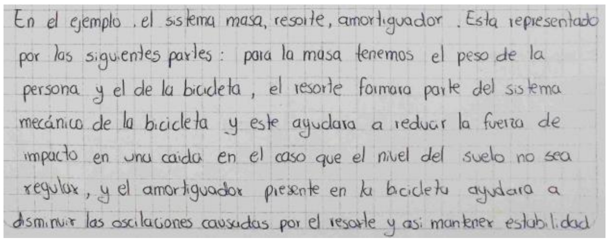

It identifies the variables mass, spring and damper (CC1), but does not determine the relationship between the elements of the dynamic system (CC2).

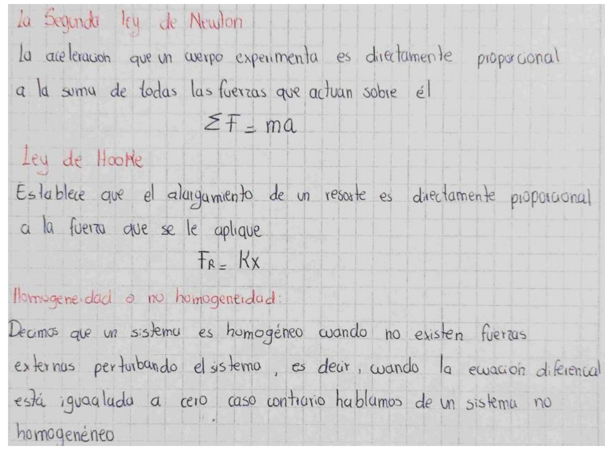
**Image 4. Activity executed by Jonathan**



In the graphical representation it is observed that it identifies the variables, establishes a reference system, establishes initial values of the problem (CC1). In addition, it establishes relationships between the mechanical elements of the dynamic system (CC2).

**Image 5. Activity carried out by Yordan**


En el ejemplo , el sistema masa, resorte, amortiguador . Esta representado por las siguientes partes : para la masa tenemos el peso de la persona y el de la bicicleta , el resorte formara parte del sistema mecánico de la bicicleta y este ayudara a reducir la fuerza de impacto en una caida en el caso que el nivel del suelo no sea regular , y el amortiguador presente en la bicicleta ayudara a disminuir las oscilaciones causadas por el resorte y asi mantener estabilidad


La Segunda ley de Newton

la aceleracion que un cuerpo experimenta es directamente proporcional a la suma de todas las fuerzas que actuan sobre él

$$\Sigma F = ma$$

Ley de Hooke

Establece que el alargamiento de un resorte es directamente proporcional a la fuerza que se le aplique

$$F_R = Kx$$

Homogeneidad o no homogeneidad:

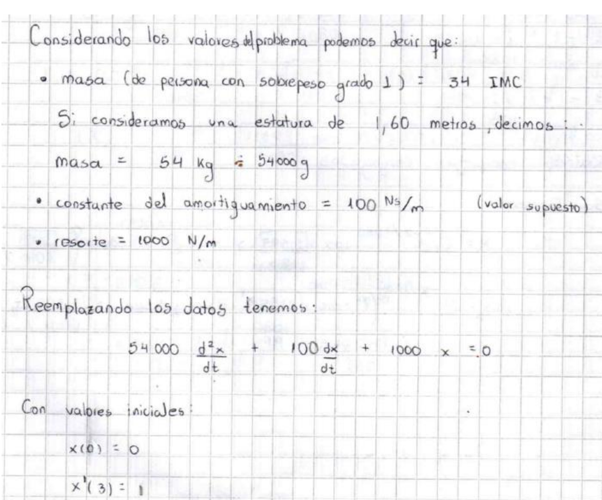
Decimos que un sistema es homogéneo cuando no existen fuerzas externas perturbando el sistema , es decir , cuando la ecuación diferencial está igualada a cero caso contrario hablamos de un sistema no homogenéneo

It concentrates on the role played by the mechanical elements involved in the dynamic system (CC1) and formulates the laws that allow it to determine the EDO of the system (CC2).

| Didactic activity 2 | Principles of realistic mathematics education |
|---|---|
| Present the problem of real context through formal language of mathematics, considering the mechanical elements and conditions involved in the problem and induce the graphic representation of the possible solution of the problem posed. | Mathematization of the problem through mathematical models that allow formalizing, structuring and solving. It shows its ability to think logically, analyzes, reflects and justifies. |

Activity carried out by students

**Image 6. Activity carried out by Solange**


Considerando los valores del problema podemos decir que:

- masa (de persona con sobrepeso grado 1) = 34 IMC

Si consideramos una estatura de 1,60 metros, decimos :

masa = 54 Kg = 54000 g

- constante del amortiguamiento = 100 Ns/m (valor supuesto)
- resorte = 1000 N/m

Reemplazando los datos tenemos :

$$54\,000 \frac{d^2x}{dt} + 100\frac{dx}{dt} + 1000\,x = 0$$

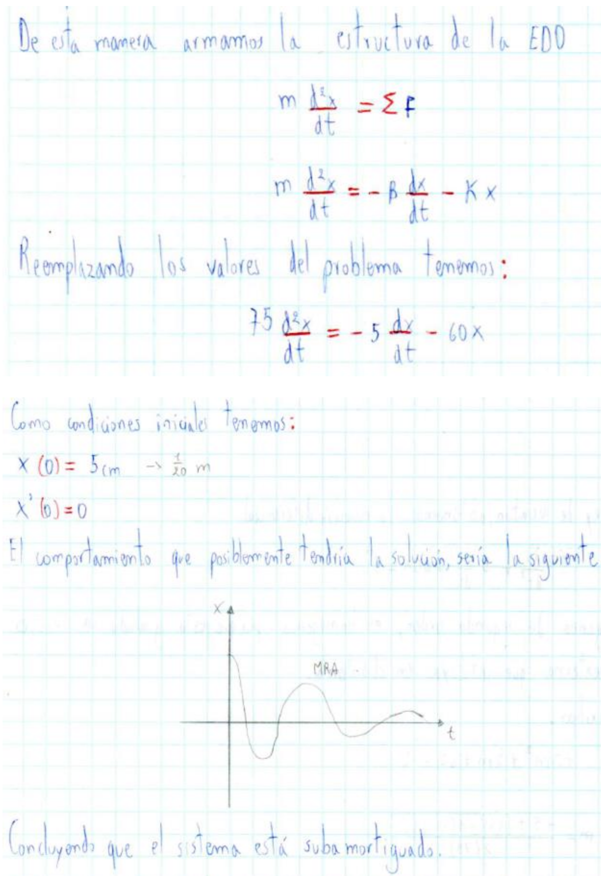
Con valores iniciales :

$x(0) = 0$

$x'(3) = 1$

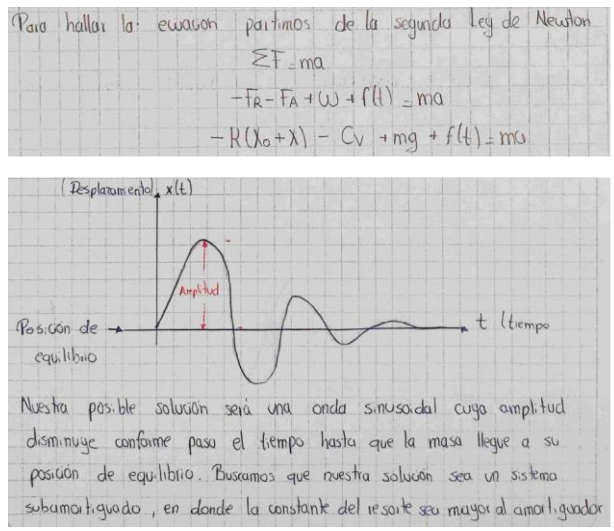
It assigns values to the problem and describes the ODE (EC1), but does not explain the formulation of the differential equation; that is, it does not resort to the empirical laws that are required to use the mathematical model. It does not relate the mathematical concepts that would allow you to visualize a behavior of the solution (CE2).

**Image 7. Activity executed by Jonathan**




It shows knowledge about the empirical laws involved in the problem, relates the assigned values to the mathematical model (CE1). It is evident that there is no analysis between the values representing the coefficients of the mass, the damping constant and the spring elasticity constant to induce the type of movement that would be generated in the solution of the problem (CE2).

**Image 8. Activity carried out by Yordan**

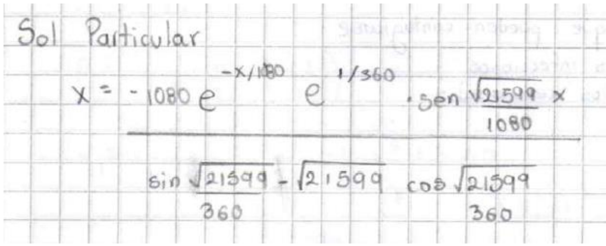



The student assigns values to the problem, clearly explains the empirical laws involved in the formulation of the mathematical model, describes the elements of the model and induces the possible solution of the problem depending on the assigned conditions (CE1); however, the design of the graphical representation is not supported according to the differential equation formulated or the appropriate method of solution (EC2).

| **Didactic activity 3** | **Principles of realistic mathematics education** |
|---|---|
| Present the analytical development executed and the verification of the solution through the use of mathematical software. | Students analyze their own mathematical activity through the process of mathematization that promotes reflection and understanding. |

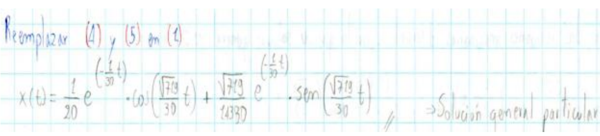
Activity carried out by students

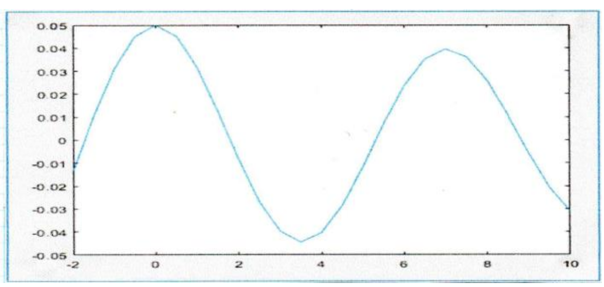
**Image 9. Activity carried out by Solange**



It presents the analytical development of the problem (FP1), does not justify the processes executed or performs the analysis of the solution found, does not establish graphic representations that allow analyzing the problem in a real context.

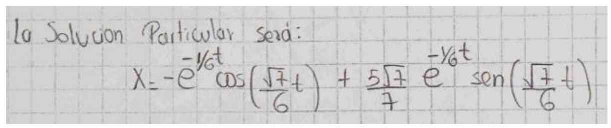
**Image 10. Activity executed by Jonathan**

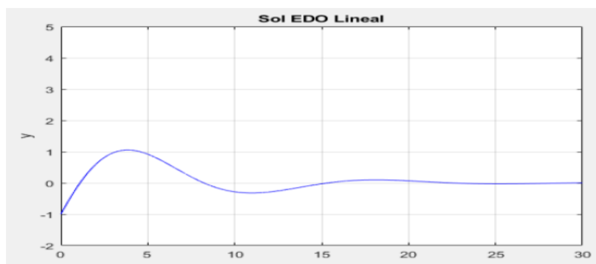




Correct processes are evidenced in the solution of the mathematical model (FP1) and verifies what is executed analytically when comparing with results obtained through Matlab software, adaptive reasoning (AR) partially fulfills it because it lacks the ability to argue about the analysis and interpretation of the results depending on the real context.

**Image 11. Activity carried out by Yordan**



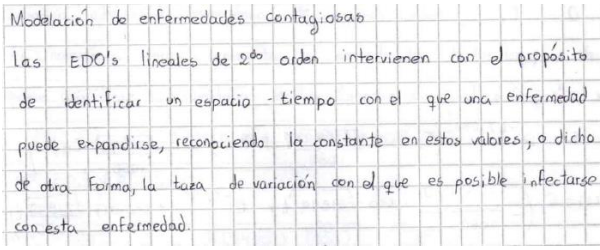



It correctly executes the processes to solve the problem (FP1), identifies the general and particular solution, but does not reflect on the executed processes (FP2). In addition, it does not analyze or interpret the results found (RA), it uses mathematical software to solve the problem posed.

| **Didactic activity 4** | **Principles of realistic mathematics education** |
|---|---|
| Propose new fields of application of second-order linear EDO in the field of engineering. Indicate what could be solved by applying mathematical models. | Educational mathematics opens the space for guided reinvention, where the student proposes mathematical models to be analyzed based on the theoretical foundations studied. |

Activity carried out by students

**Image 12. Activity carried out by Solange**




The contagion of sick women is related as a variation rate, but it is necessary to identify the variables that could intervene or how the contagion could be caused (PP2).

**Image 13. Activity executed by Jonathan**

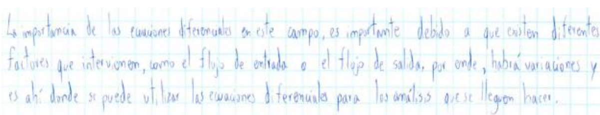
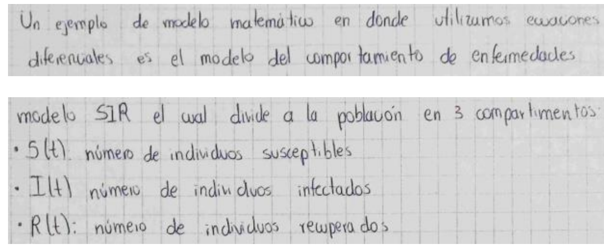



Image transcript 13

The importance of differential equations in this field is important because there are different factors involved, such as the input flow or the output flow therefore, there will be variations and that is where differentials can be used for the analyses that are made.

Implicitly, the structure of a function and its variation process are related, but it is not specified what its variables could be for the formulation of the mathematical model (PP2).

**Image 14. Activity carried out by Yordan**




The elements involved in the mathematical model are pointed out, conceptualizing that a differential equation has variables in its mathematical formulation, does not identify constraints to the problem posed (PP2).

Therefore, through the implementation of the proposed activity, the sequence of instructions that helps students to achieve the learning objective is emphasized, allowing an adequate presentation of the work and the search for future applications in the field of real life (Habre, 2020; Lozada and Fuentes, 2018). In this sense, the student who solves a real context problem, designed under the characteristics and principles of the EMR, is able to build and reconstruct the formal mathematical knowledge of the EDO.

Table 4 presents the summary of the results identified about the mathematical comprehension described in Table 2 that are related to the design of the didactic activities described in Table 3.

**Table 4. Results of indicators for elements of mathematical understanding**

| Comprehension | Description | While | Jonathan | Yordan |
|---|---|---|---|---|
| Conceptual Comprehension **(CC)** | CC1 | YES | YES | YES |
| | CC2 | NO | YES | YES |
| Strategic Competence **(CE)** | CE1 | YES | YES | YES |
| | CE2 | NO | NO | NO |
| Procedural fluency | FP1 | YES | YES | YES |

| | | | | |
|---|---|---|---|---|
| **(FP)** | FP2 | NO | YES | YES |
| Adaptive reasoning **(AR)** | RA1 | NO | PARTIAL | PARTIAL |
| | RA2 | NO | PARTIAL | PARTIAL |
| Productive predisposition **(PP)** | PP1 | YES | YES | YES |
| | PP2 | NO | NO | NO |

Through table 4 it can be evidenced how the solution of a real context problem oriented to the learning of the ODE carried out under the principles of the EMR allows to achieve achievements in conceptual understanding, fluency in procedures, but it is necessary to design didactic strategies that are aimed at strengthening the productive predisposition and strategic competence framed in formulating a problem that can be expressed through a second-order linear ODE, assessing the constraints to the different variables involved in the problem and their corresponding mathematical model.

## 5. CONCLUSIONS

The results of the qualitative research described in Table 4 show the elements of mathematical understanding that have been fostered with a teaching design based on real situations for the study of second-order linear ODES. Table 2 determined the indicators that allow assessing the activity carried out by the student according to the fulfillment of the didactic activities designed under the notions of the EMR and described in table 3 oriented in the solution of a problem of real context. The results obtained are related to what Barquero and Jessen (2020) described on the theoretical perspectives in the design of the mathematical modeling task in real problems.

In general terms, it is concluded that conceptual understanding (CC) has been achieved by students, and it is necessary to redesign activities to achieve strategic competence, fluency in procedures (FP), adaptive reasoning (AR) and productive predisposition (PP) which have been achieved in a partial positive way, compared to the work of Camacho and Rodríguez (2015) it has not been fully possible to identify and explore the relationships between situations contextual and ordinary differential equations posed and solved by students.

As a consequence of the results of the research and considering the academic activities that were executed before it, the initiative of the students to improve their capacity for analysis, reflection and creativity by activating the knowledge of mathematical concepts and their relationship with the mechanical and electrical elements that were used for the approach of problems of real context is highlighted. In addition, good performance was evidenced in the calculation procedures and the use of mathematical software.

## Reference

Arslan, S. (2010a). ¿Do students really understand what an ordinary differential equation is? International Journal of Mathematical Education in Science and Technology, 41(7), 873-888.

https://doi.org/10.1080/0020739X.2010.486448

Arslan, S. (2010b). Traditional instruction of differential equations and conceptual learning. Teaching mathematics and its applications: An International Journal of the IMA, 29(2), 94-107.

https://doi.org/10.1093/teamat/hrq001

Artigue, M. (2021). Mathematics education research at university level: Achievements and challenges. In Research and development in University Mathematics Education, 2-21. https://doi.org/10.4324/9780429346859

Barquero, F. y Jessen, B. (2020). Impact of theoretical perspectives on the design of mathematical modelling task. Avances de investigación en educación matemática. https://hdl.handle.net/11162/198309

Bergsteiner, H., Avery, G. y Neumann, R. (2010). Kolb's experiential learning model: critique from a modelling perspective. Studies in Continuing Education, 32(1), 29-46. https://doi.org/10.1080/01580370903534355

Braun, B., Bremser, P., Duval, A., Lockwood, E. y White, D. (2018). ¿What does active learning mean for mathematicians? The Best Writing on Mathematics 2018, edited by Mircea Pitici, Princeton: Princeton University Press, 2018, 169-178. https://doi.org/10.1515/9780691188720-015

Camacho, M. y Guerrero, O. (2015). Identifying and exploring relationships between contextual situations and ordinary differential equations. International Journal of Mathematical Education in Science and Technology, 46(8), 1077-1095. https://doi.org/10.1080/0020739X.2015.1025877

Czocher, J. (2016). Introducing modeling transition diagrams as a tool to connect mathematical modeling to mathematical thinking. Mathematical Thinking and Learning, 18(2), 77-106. https://doi.org/10.1080/10986065.2016.1148530

De Lange, J. (1996). Using and applying mathematics in education. In International handbook of mathematics education, Springer, Dordrecht, 49-97.

https://doi.org/10.1007/978-94-009-1465-0_3

Dullius, M. (2011). Teaching and learning in differential equations with graphic, numerical and analytical approach. University of Burgos. https://core.ac.uk/download/pdf/61545447.pdf

Farrás, B., Bosch, M. y Gascón, J. (2011). Study and research paths and mathematical modelling in university teaching of experimental sciences. Science Education: Journal of Research and Didactic Experiences, 29(3), 339-352.

Fauzan, A. (2021). The Development of Student Worksheet Based on Realistic Mathematics Education in Ordinary Differential Equations of Order-1. In Journal of Physics: Conference Series.

https://doi.org/10.1088/1742-6596/1742/1/012018

Freudenthal, H. (2006). Revisiting mathematics education: China lectures (Vol. 9). Springer Science y Business Media.

Freudenthal, H. (2012). Mathematics as an educational task. Springer Science y Business Media.

Gravemeijer, K. y Terwel, J. (2000). Hans Freudenthal: a mathematician on didactics and curriculum theory. Journal of curriculum studies, 32(6), 777-796. https://doi.org/10.1080/00220270050167170

Greefrath, G. y Vorhölter, K. (2016). Teaching and learning mathematical modelling: Approaches and developments from German speaking countries. Springer

Nature. https://library.oapen.org/bitstream/handle/20.500.12657/27708/1002298.pdf?sequence=1

Groth, R. (2017). Classroom data analysis with the five strands of mathematical proficiency. The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 90(3), 103-109. https://doi.org/10.1080/00098655.2017.1301155

Habre, S. (2000). Exploring students' strategies to solve ordinary differential equations in a reformed setting. The Journal of Mathematical Behavior, 18(4), 455-472. https://doi.org/10.1016/S0732-3123(00)00024-9

Hernandez, R., MariñoL. y Penagos, M. (2017). Second-order linear differential equations as mathematical models. Mathematical Echo, 54-60. https://doi.org/10.22463/17948231.1383

Hernandez, S., Jaimes, C. y Chaves, E. (2016). Models of application of first-order differential equations with Geogebra: activities to solve mixing problems. FESC World, 6(11), 7-15. Retrieved from https://www.fesc.edu.co/Revistas/OJS/index.php/mundofesc/article/view/77

Juarez, R., Chamoso, S. and González, A. (2020). Interaction in virtual forums by integrating mathematical modeling to train engineers. Science Education: Journal of Research and Didactic Experiences, 38(3), 161-178. https://doi.org/10.5565/rev/ensciencias.3041

Khotimah, R. y Masduki, M. (2016). Improving teaching quality and problem solving ability through contextual teaching and learning in differential equations: A lesson study approach. JRAMathEdu (Journal of Research and Advances in Mathematics Education), 1(1), 1-13.

Kwon, O. (2009). Conceptualizing the realistic mathematics education aproach in the teaching and learning of ordinary differential equations. Colección Digital Eudoxus, (11).

Lozada, J. y Sources, R. (2018). Problem-solving methods and the development of mathematical thinking. Bolema: Bulletin of Education Mathematics, 32, 57-74. https://doi.org/10.1590/1980-4415v32n60a03

Mkhatshwa, T. (2020). Calculus students' quantitative reasoning in the context of solving related rates of change problems. Mathematical Thinking and Learning, 22(2), 139-161. https://doi.org/10.1080/10986065.2019.1658055

Niss, M. y Blum, W. (2020). The learning and teaching of mathematical modelling. Routledge. https://doi.org/10.4324/9781315189314

Perdomo, D. (2011). Teaching module for the introduction of ordinary differential equations in a problem-solving environment with technology. NUMBERS. Journal of Didactics of Mathematics, 78, 113-134.

Rasmussen, C. y King, K. (2000). Locating starting points in differential equations: A realistic mathematics education approach. International Journal of Mathematical Education in Science and Technology, 31(2), 161-172.

https://doi.org/10.1080/002073900287219

Rasmussen, C., y Keene, K. (2019). Knowing solutions to differential equations with rate of change as a function: Waypoints in the journey. The Journal of Mathematical Behavior, 56, 100695.

https://doi.org/10.1016/j.jmathb.2019.03.002

Rasmussen, C. y Ruan, W. (2008). Teaching for understanding: A case students learning to use the uniqueness theorem as a tool in differential equations. Making the connection: Research and teaching in undergraduate mathematics: MAA notes, (73), 153-164.

Rodriguez, L., Ponce, And. and Pérez, A. (2016). Mathematical understanding of functions in interdisciplinarity with physics through practical life problems. UNION. Revista Iberoamericana de Educación Matemática, 47, 176-191.

Ross, S. (2021). Differential equations. I reversed.

Santos, L. (2016). The resolution of Mathematical Problems and the coordinated use of digital technologies. Cuadernos de Investigación y Formación en Educación Matemática, 2016, 333-346.

Schoenfeld, A. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). Journal of Education, 196(2), 1-38.

Schukajlow, S., Kaiser, G. y Stillman, G. (2018). Empirical research on teaching and learning of mathematical modelling: a survey on the current state-of-the-art. ZDM: The International Journal on Mathematics Education. 50(43132), 5-18.

https://doi.org/10.1007/s11858-018-0933-5

Shahbari, J. y Daher, W. (2016). Pre-service teachers' mathematical models' features. European Journal of Science and Mathematics Education, 4(4), 523-533. https://doi.org/10.30935/scimath/9491

Vajravelu, K. (2018). Innovative Strategies for Learning and Teaching of Large Differential Equations Classes. International Electronic Journal of Mathematics Education, 13(2), 91-95.

https://doi.org/10.12973/iejme/2699

Zeynivandnezhad, F. y Bates, R. (2018). Explicating mathematical thinking in differential equations using a computer algebra system. International Journal of mathematical education in science and technology, 49(5), 680-704. https://doi.org/10.1080/0020739X.2017.1409368

Zeynivannezhad, F. (2014). Mathematical thinking in Differential Equations through a computer algebra system (Doctoral dissertation, Universiti Teknologi Malaysia). https://core.ac.uk/download/pdf/199241733.pdf

Zill, D. y Cullen, M. (2013). Differential equations. McGraw-Hill Interamericana.