Results on Intuitionistic Fuzzy Labeling Graphs

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Abstract
In this paper, the Properties of an Intuitionistic Fuzzy Labeling Graph (IFLG) is studied. A graph is said to be an IFLG if it has Intuitionistic fuzzy labeling. Here, Intuitionistic Fuzzy (IF) sub graphs, union, IF-bridges, IF- end vertices, IF- cut vertices and weakest arc of IFL graphs have been discussed. Number of weakest arc, IF-bridge, IF-cut vertex and end vertex of an Intuitionistic fuzzy labeling cycle has been found. Degrees of IF-cut vertex and IF-end vertex have been identified. Also it is proved that If G is a connected IFLG then there exists a strong path between every pair of vertices of G.

Keywords: IF-cut vertex, IF- end vertex, IF-bridge, IFL graph.

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1. Introduction

Based on Zadeh's [29] fuzzy relations, Kaufmann (1973) acquainted us with the primary categorization of fuzzy graphs (1971). Azriel Rosenfeld [27], who developed thoughtful fuzzy relations on fuzzy sets and popularised the notion of fuzzy graph in 1975, is responsible for a more grandiose categorization. Yeh and Banh [28] have familiarised themselves with a number of connectivity perceptions in fuzzy graphs at the same time. Fuzzy graphs have seen amazing progress and a wide range of applications up until this point in engineering and technology. Numerous notions, including cycles, paths, trees, and bridges, have been acquired by Rosenfeld, and some of their characteristics have been developed. K.R. Bhutani et al. [13] examined fuzzy end nodes and cut nodes. In his study "Certain thoughts on fuzzy graphs," Bhattacharya [12] identified some connection principles relating to fuzzy cut nodes and fuzzy bridges.”.

Mathew and Sunitha [21] have studied “types of arcs in fuzzy graphs”. Nagoorgani [23] et al. make known to the concept of fuzzy labeling graphs.Atanassov [7,8,9,10,11] familiarized the concept of IF relations and IFGs. Parvathi et al. [18,19,24,25,26] have studied about “Arcs and operations in IF Graphs”. M.Akram et al. [1,2,3] have particularized “IF hypergraphs with applications, strong IF graphs and IF cycle and tree”. H. Rashmanlou et al.[14] have discussed about " New concepts of Interval-Valued Intuitionistic (S; T)-Fuzzy”. K.AmeenalBibi and M.Devi [4] studied about the concepts of “Bi-Magic labeling on Interval-valued fuzzy graphs”. A comprehensive development of mathematical and enormous applications is found on the theory of research in IF sets. In this work, we addressed a few of the IFLGs' features. The fundamental concepts and notations used in [18, 19, 22, 25, 26] are respected in this article.

2.Preliminaries and Observations

An *IFG* is of the form G: (V,E) where,
(i) $\mathcal{V} \neq \{\emptyset\}$such that $\mu_\alpha: \mathcal{V} \to [0,1]$and $\gamma_\alpha: \mathcal{V} \to [0,1]$ signifies the degree of membership and degree of non-membership respectively of the elements u in $\mathcal{V}$, $0 \leq \mu_\alpha(u) + \gamma_\alpha(u) \leq 1$ for every u in $\mathcal{V}$ (G).
(ii) $\mathcal{V}$x $\mathcal{V}$ is a finite set of edges such that $\mu_\beta: \mathcal{V} \times \mathcal{V} \to [0,1]$ and $\gamma_\beta: \mathcal{V} \times \mathcal{V} \to [0,1]$such that$\mu_\beta(uv) \leq min\{\mu_\alpha(u), \mu_\alpha(v)\}$and $\gamma_\beta(uv) \leq max\{\gamma_\alpha(u), \gamma_\alpha(v)\}and0 \leq \mu_\beta(uv) + \gamma_\beta(uv) \leq 1$ for every (u,v) ∈ E(G).The Length of the Path P ={ $v_1, v_2, \dots, v_{n+1}$} (n>0) is n. A Path P ={ $v_1, v_2, \dots, v_{n+1}$} is called a Cycle if $v_1 = v_{n+1}$ and n ≥ 3. A Path connecting two Vertices is referred to as a connection. The *μ-Strength* of a Path P ={ $v_1, v_2, \dots, v_n$} is defined as $\min_{i,j}\{\mu_\beta(v_i, v_j)\}$------ (1) and is represented by S_{μ_β}. The *γ-Strength*of a Path P = $v_1, v_2, \dots, v_n$ is defined as $\max_{i,j}\{\gamma_\beta(v_i, v_j)\}$ ------ (2) and is represented by S_{γ_β}. If an edge contains both (1) and (2), it is regarded as the Path P's strongest edge and is represented by SP (2).

The *μ-strength of Connectedness* between two vertices v_i *and* v_j is defined as the maximum of μ-strength of all the paths between v_i and v_j excluding the arc joining $v_i$and v_j. It is denoted by CONN$\mu_\beta(v_i , v_j)$.The *γ-strength of Connectedness* between two vertices v_i *and* v_j is defined as the minimum of γ-strength of all the paths between v_i and v_j excluding the arc joining $v_i$and v_j. It is denoted by CONN$\gamma_\beta(v_i , v_j)$.

In an IFG, a path P between any two vertices is called the *strongest path* if $S_{\mu_\beta} =$ CONN$\mu_\beta(v_i , v_j)$and $S_{\gamma_\beta}=$ CONN$\gamma_\beta(v_i , v_j)$ and both the values lie in the same edge. An arc (v_i , v_j) is said to be a *strong arc* if $\mu_\beta(uv) \geq$ CONN$\mu_\beta(uv)$and $\gamma_\beta(uv) \leq$ CONN$\gamma_\beta(uv)$. A (v_i , v_j) path P in an IFG is called a strong path if P comprises only strong arcs. An arc (v_i , v_j) is said to be the weakest arc if $\mu_\beta(uv)$ <CONN$\mu_\beta(uv)$and $\gamma_\beta(uv)$ >CONN$\gamma_\beta(uv)$.

If the removal of an arc (v_i , v_j)decreases the overall connectedness while simultaneously increasing the total connectedness between a particular pair of vertices, the arc is said to be an IF-bridge in the graph G. A vertex is an IF-cut vertex of an IFG if its removal decreases the overall connectivity while simultaneously increasing the total connectedness between another pair of vertices. If a vertex v_i is exactly one of the strongest neighbor it is said to be an IF-end vertex.

d(v_i)= $\left[\sum_{(v_i , v_j) \in E}\left(\mu_\beta(v_i , v_j)\right), \sum_{(v_i , v_j) \in E}\left(\gamma_\beta(v_i , v_j)\right)\right]$ and$\mu_\beta(v_i , v_j)$=$\gamma_\beta(v_i , v_j)$=0 for $(v_i , v_j) \in E$.

3.An Intuitionistic Fuzzy Labeling for Subgraph and Union of Graphs

Definition:3.1

An IF Labeling on a non-empty graph H = (V',E') of G is said to be an intuitionistic fuzzy labeling subgraph (IFLSG) of G if $V' \subseteq V$ and $E' \subseteq E$ .In other words, $\mu'_\alpha(v_i) \leq \mu_\alpha(v_i)$, $\gamma'_\alpha(v_i) \geq \gamma_\alpha(v_i)$ and $\mu'_\beta(v_i, v_j) \leq \mu_\beta(v_i, v_j)$, $\gamma'_\beta(v_i, v_j) \geq \gamma_\beta(v_i, v_j)$ for every i,j=1,2,…,n.

Proposition:3.2

If $\mathcal{H} = (\mathcal{V}',\mathcal{E}')$ is an IFLSG of $G = (\mathcal{V},\mathcal{E})$ then for some $(v_i, v_j) \in E$,$\mu'^{\infty}_\beta(v_i, v_j) \leq \mu^{\infty}_\beta(v_i, v_j)$ and $\gamma'^{\infty}_\beta(v_i, v_j) \geq \gamma^{\infty}_\beta(v_i, v_j) \forall\, i,j = 1,2,\dots,n$.

Proof:

Let $G = (\mathcal{V},\mathcal{E})$ be an IFLG and $\mathcal{H} = (\mathcal{V}',\mathcal{E}') \subset G = (\mathcal{V},\mathcal{E})$

By definition (3.1), $\mathcal{V}' \subseteq \mathcal{V}$ and $\mathcal{E}' \subseteq \mathcal{E}$ which implies,

$$\mu'_\alpha(v_i) \leq \mu_\alpha(v_i)\,,\ \gamma'_\alpha(v_i) \geq \gamma_\alpha(v_i)$$

for every $v_i \in \mathcal{V}$. (3)

and $\mu'_\beta(v_i, v_j) \leq \mu_\beta(v_i, v_j)$

(4)

$\gamma'_\beta(v_i, v_j) \geq \gamma_\beta(v_i, v_j)$, for every $v_i, v_j \in \mathcal{V}$.

(5)

Let us take a Path P = $v_1, v_2, \dots, v_n$ of $\mathcal{H}$.

$$\mu'^{\infty}_\beta(v_i, v_j) = \min_{k=1,2,\dots n}\left\{\mu'_\beta(v_i, v_j)^k\right\}$$

(6)

$$\gamma'^{\infty}_\beta(v_i, v_j) = \max_{k=1,2,\dots n}\left\{\gamma'_\beta(v_i, v_j)^k\right\}$$

and $\mu^{\infty}_\beta(v_i, v_j) = \min_{k=1,2,\dots n}\left\{\mu_\beta(v_i, v_j)^k\right\}$

$$\gamma^{\infty}_\beta(v_i, v_j) = \max_{k=1,2,\dots n}\left\{\gamma_\beta(v_i, v_j)^k\right\}$$

Therefore, we have

$\mu'^{\infty}_\beta(v_i, v_j) = \min_{k=1,2,\dots n}\left\{\mu'_\beta(v_i, v_j)^k\right\}$
$\leq \min_{k=1,2,\dots n}\left\{\mu_\beta(v_i, v_j)^k\right\}$

by using (4)

$= \mu^{\infty}_\beta(v_i, v_j)$.

Also, $\gamma'^{\infty}_\beta(v_i, v_j) = \max_{k=1,2,\dots n}\left\{\gamma'_\beta(v_i, v_j)^k\right\}$
$\geq \max_{k=1,2,\dots n}\left\{\gamma_\beta(v_i, v_j)^k\right\}$

by using (5)

$= \gamma^{\infty}_\beta(v_i, v_j)$.

Hence proved.

Proposition 3.3[20]

If the membership and non-membership values of the edges between G_1 and G_2 are distinct, the union of any two IFLGs G_1 and G_2 is likewise an IFLG.

Proof:

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two IFL Graphs with $V_1 \cap V_2 \neq \varphi$ and $G = G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$. Then the union of an IFL of graphs G_1 and G_2 is also an IFLG defined by,

$$(\mu_{1\alpha} \cup \mu_{2\alpha})(v) = \begin{cases} \mu_{1\alpha}(v) & if \quad v \in V_1 - V_2 \\ \mu_{2\alpha}(v) & if \quad v \in V_2 - V_1 \end{cases}$$

$$(\gamma_{1\alpha} \cup \gamma_{2\alpha})(v) = \begin{cases} \gamma_{1\alpha}(v) & if \quad v \in V_1 - V_2 \\ \gamma_{2\alpha}(v) & if \quad v \in V_2 - V_1 \end{cases}$$

and $(\mu_{1\beta} \cup \mu_{2\beta})(v_i, v_j) = \begin{cases} \mu_{1\beta}(v_i, v_j) & if \quad (v_i, v_j) \in E_1 - E_2 \\ \mu_{2\beta}(v_i, v_j) & if \quad (v_i, v_j) \in E_2 - E_1 \end{cases}$ (7)

$$(\gamma_{1\beta} \cup \gamma_{2\beta})(v_i, v_j) = \begin{cases} \gamma_{1\beta}(v_i, v_j) & if \quad (v_i, v_j) \in E_1 - E_2 \\ \gamma_{2\beta}(v_i, v_j) & if \quad (v_i, v_j) \in E_2 - E_1 \end{cases} \quad (8)$$

where$(\mu_{1\alpha},\gamma_{1\alpha})$ and $(\mu_{2\alpha},\gamma_{2\alpha})$ are the membership and non- membership values of vertices of G_1 and G_2 respectively.$(\mu_{1\beta},\gamma_{1\beta})and(\mu_{2\beta},\gamma_{2\beta})$ are the membership and non- membership values of edges of $G_1$and G_2 respectively.

Here, $\mu_\alpha(v) = \max\{\mu_{1\alpha},\mu_{2\alpha}\}$ if $v \in V_1 \cap V_2$ and $\gamma_\alpha(v) = \min\{\gamma_{1\alpha},\gamma_{2\alpha}\}$ if $v \in V_1 \cap V_2$
Here, $\mu_\beta(v_i , v_j) = \max\{\mu_{1\beta},\mu_{2\beta}\}$ if $(v_i , v_j) \in E_1 \cap E_2$ and

$\gamma_\beta(v) = \min\{\gamma_{1\beta},\gamma_{2\beta}\}if(v_i , v_j) \in E_1 \cap E_2$.

Hence, μ_α=$(\mu_{1\alpha} \cup \mu_{2\alpha})$, γ_α=$(\gamma_{1\alpha} \cup \gamma_{2\alpha})$, μ_β=$(\mu_{1\beta} \cup \mu_{2\beta})$ and γ_β=$(\gamma_{1\beta} \cup \gamma_{2\beta})$.

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4. Properties of an Intuitionistic Fuzzy labeling cycle

Proposition: 4.1
The IFL cycle G has precisely one weakest arc if G* is a cycle.
Proof:
Assuming that $G = (V, E)$ is an IFL cycle, let's assume.

$\mu_\beta(v_x , v_y) = \min\limits_{i,j=1,2,\ldots n} \mu_\beta(v_i , v_j)and\gamma_\beta(v_x , v_y) = \max\limits_{i,j=1,2,\ldots n} \gamma_\beta(v_i , v_j)$ for all $(v_i , v_j) \in V$.

Since $G = (V, E)$ has an IFL, it will have only one arc with $\{\mu_\beta(v_x , v_y)$,$\gamma_\beta(v_x , v_y)\}$. If we remove an edge$(v_x , v_y)$, then it will not decrease the strength of connectedness which denotes that the arc (v_x , v_y) is the weakest arc. Therefore, there exists only one weakest arc in any IFL cycle graph.

Proposition: 4.2

If G* is a cycle, then the IFL cycle G has (n-1) bridges.
Proof:

Let G be an IFL cycle graph. By 4.2, it follows that there exists only one weakest arc. According to a known finding, the weakest arc is not a fuzzy bridge, which means that removing any other arcs outside the weakest arc will weaken G's degree of connectedness. Thus, there are exactly (n-1) bridges in each IFL cycle.

Proposition 4.3

Let $G = (V, E)$ be an IFL cycle graph such that $G *$is a cycle. Then a vertex is an IF cut vertex of $G *$ if and only if it is a common vertex of two IF bridges.

Proposition 4.4

If $G = (V, E)$ is an IFL graph, then it has (n-2) cut vertices.
Proof:
By Proposition 4.3, every IFL cycle graph has (n-1) bridges. This implies that G will have only one weakest arc say $\{\mu_\beta(v_x , v_y)$,$\gamma_\beta(v_x , v_y)\}$.Therefore, excluding$\mu_\alpha(v_x)$,$\mu_\alpha(v_y)$, $\gamma_\alpha(v_x)$,$\gamma_\alpha(v_y)$, all the remaining (n-2) vertices are common vertices of two IF bridge. Hence by Proposition 4.4, an IFL cycle with (n-2) cut vertices.

Proposition 4.5

If the graph $G = (V, E)$ is a cycle with an IFL, it contains exactly two end vertices.
Proof:

By Proposition 4.2, G* has precisely only one weakest arc, say (v_x , v_y) which implies $\mu_\alpha(v_x)$,$\mu_\alpha(v_y)$, $\gamma_\alpha(v_x)$,$\gamma_\alpha(v_y)$ are the end vertices of $\{\mu_\beta(v_x , v_y)$,$\gamma_\beta(v_x , v_y)\}$.

Hence every IFL graph has precisely two end vertices.

Proposition 4.6

If and only if each and every IF bridge is strong, G will be a cycle graph with an IFL.

Proof:

Necessary Part:

Let G = (V, E) be an n-vertex IFL cycle graph. According to propositions 4.2 and 4.3, G has exactly one weakest arc and (n-1) IF bridges.We now suppose that each of these (n-1) bridges is robust. Let's choose one edge from the (n-1) edges, (v_x, v_y). G is a cycle, hence there are two ways to get from vertex v_x to vertex v_y. Namely one path with $\mu_\beta(v_x, v_y) > 0$ and $\gamma_\beta(v_x, v_y) < 1$ and the other paths $\mu_\beta(v_x, v_{x+1}, \dots, v_y) > 0$and $\gamma_\beta(v_x, v_{x+1}, \dots, v_y) < 1$.

Hence $\mu_\beta^\infty(v_x, v_y) = \mu_\beta(v_x, v_y)\gamma_\beta^\infty(v_x, v_y) = \gamma_\beta(v_x, v_y)$ which indicates $(v_x, v_y)$is strong arc.By reiterating the procedure for the left over edges, we will acquire (n-1) strong arcs.

Sufficient Part:

Conversely, suppose that every IF bridge is strong. Then by definition, it follows that

$$\mu_\beta^\infty(v_x, v_y) = \mu_\beta(v_x, v_y) \text{and } \gamma_\beta^\infty(v_x, v_y) = \gamma_\beta(v_x, v_y).$$

That is $\mu_\beta(v_x, v_y) > 0$and $\gamma_\beta(v_x, v_y) < 1$ and the other paths $\mu_\beta(v_x, v_{x+1}, \dots, v_y) > 0$and $\gamma_\beta(v_x, v_{x+1}, \dots, v_y) < 1$.

Since G is a cycle, then only there exists two paths between the vertices $v_x \; and \; v_y$.Hence, G is a cycle graph with an IFL. Thus proved.

5. IF labeling with bridge and Strong edge

Proposition 5.1

If G is an IFLG, then at least one IF bridge exists in G.

Proof:

Take an edge $(v_x, v_y) \in E$ of the IFL graph $G = (V, E)$.

$$\mu_\beta(v_x, v_y) = \max\{\mu_\beta(v_i, v_j)\}$$

$$\gamma_\beta(v_x, v_y) = \min\{\gamma_\beta(v_i, v_j)\} \text{ for all } v_i, v_j \in V.$$

Therefore, $\mu_\beta(v_x, v_y) > 0$ and $\gamma_\beta(v_x, v_y) < 1$.

Therefore atleast one edge (v_m, v_n) different from (v_x, v_y) such that $\mu_\beta(v_m, v_n) < \mu_\beta(v_x, v_y)$and $\gamma_\beta(v_m, v_n) > \gamma_\beta(v_x, v_y)$.

Assumed to be a bridge of $G$is (v_x, v_y). The level of connectivity between v_x and v_y in the IF subgraph so acquired is less if we remove the edge (v_x, v_y) from G.

Remark 5.2: The converse of the above is not true. The proof is obvious.

Proposition 5.3

Every pair of G's vertices has a strong route connecting them if G is a linked IFLG.

Proof:

Let G be a connected IFLG and let (v_m, v_n) be a pair of vertices of G which indicates that $\mu_\beta^\infty(v_m, v_n) > 0$ and $\gamma_\beta^\infty(v_m, v_n) < 1$. Now select any edge (v_m, v_p) in (v_m, v_n), if $\mu_\beta(v_m, v_p) = \mu_\beta^\infty(v_m, v_p)$ and $\gamma_\beta(v_m, v_p) = \gamma_\beta^\infty(v_m, v_p)$then it is strong otherwise take some other edge, say (v_m, v_q) which satisfies $\mu_\beta(v_m, v_q) = \mu_\beta^\infty(v_m, v_q)$ and $\gamma_\beta(v_m, v_q) = \gamma_\beta^\infty(v_m, v_q)$. By reiterating this process, we can find a path in (v_m, v_n)in which all arcs are strong.

Proposition 5.4

Every IFL graph has atleast one weakest arc.

Proof:

Let G be an IFLG and let (v_m, v_n) be an edge of G such that
$\mu_\beta(v_m, v_n) =$
$\min\{\mu_\beta(v_i, v_j)\}and\gamma_\beta(v_m, v_n) =$
$\max\{\gamma_\beta(v_i, v_j)\}$ for all $v_i, v_j \in V$.
If we delete the edge (v_m, v_n) from G, it does not reduce/increase the strength of any path.In otherwords, after the exclusion of an edge in its subgraph H, we have $\mu_\beta(v_m, v_n) < \mu_\beta'^\infty(v_m, v_n)$ and $\gamma_\beta(v_m, v_n) > \gamma_\beta'^\infty(v_m, v_n)$ which implies $\mu_\beta(v_m, v_n)$ is neither a fuzzy bridge nor a strong arc. Therefore, it must be atleast one weakest arc.

Proposition 5.5

In any IFL graph G, $\delta_\mu(G)$ *and* $\Delta_\gamma(G)$ have an IF end vertex of G such that the number of arcs incident on $\{\delta_\mu(G), \Delta_\gamma(G)\}$ is atleast one.

Proof:

Let G be an IFL graph and there exists atleast one vertex v with degree $\delta_\mu(G)$ *and* $\Delta_\gamma(G)$ which denotes that the arcs which are incident on v may have lower membership value and higher membership value respectively and it is not possible to have all the arcs which are incident on v as the weakest arc and strongest arc respectively. Therefore G must have both a strong neighbour and weak neighbour. Hence, $\delta_\mu(G)$ *and* $\Delta_\gamma(G)$ have an end vertex of G.

Proposition 5.6

Every IFL graph has atleast one end vertices.

Proof:

In any IFL graph, there exists atleast one vertex with degree $\{\delta_\mu(G), \Delta_\gamma(G)\}$. Therefore, by proposition 5.6, $\{\delta_\mu(G), \Delta_\gamma(G)\}$ has an end vertex of G.

Proposition 5.7

Every IFL graph has atleast one cut vertex.

Proof:

For any IFL graph G, We can find atleast one vertex with $\Delta_\mu(G)$and $\delta_\gamma(G)$. Let v be a vertex with degree $\{\Delta_\mu(G), \delta_\gamma(G)\}$ which implies the edges which are incident in v may have higher membership and lower non-membership values respectively. As a result, the loss of such a vertex v will weaken both the connectedness and the connectedness strength. Consequently, v is a cut vertex.

6. Conclusion

This discussion looks into the features of an IFL for a graph. Intuitionistic fuzzy labelling graphs' weakest arc, union of IF graphs, IF bridges, IF end vertices, IF cut vertices, and IF sub graph have all been investigated. There are a certain number of weakest arcs, an IF bridge, an IF cut vertex, and the cycle's end vertex. There have been recognized levels of intuitionistic fuzzy cut vertex and intuitionistic fuzzy end vertex. Future research on additional arc type characteristics in IFL graphs is what we proposed.

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