Model Order Reduction Using Eigen Spectrum and Cauer 2nd Form

Deepak Kumar Mahto[1] and Jasvir Singh Rana[2]

[1]Department of Electronics and Communication Engineering, Research Scholar, Shobhit University, Gangoh, Saharanpur, India

[2]Department of Electronics and Electrical Engineering, Prof. & Head, Shobhit University, Gangoh, Saharanpur, India

E-Mail: deepak.deepak2510@gmail.com

jasvirsingh.rana@shobhituniversity.ac.in

Abstract

The model order reduction technique used in the research depends on the eigen spectrum as well as the Cauer second form.. The eigen spectrum is obtained by performing eigen value analysis of the original system matrix and the Cauer second form is used to reduce the order of the system. The Cauer form is derived from the eigen spectrum by performing an inverse z-transform. This technique is used to reduce the order of a system and design a controller with the help of MATLAB. The presented method's outcomes are contrasted to other existing methods to demonstrate the effectiveness of the suggested approach. The findings conclude that the suggested model order reduction approach using eigen spectrum and Cauer second form is effective in reducing the order of the system and designing a controller. The integral square error value calculated for the proposed approach is less than the other existing approaches which validates the proposed method.

Keywords: Reduction Technique, Controller Design, MATLAB, Integral Square Error

INTRODUCTION

In the sphere of science and technology, there is a huge potential for big systems to be simplified. This paper offers an innovative and significant breakthrough in technique with the use of eigen spectrum analysis as well as second Cauer form to implement a mixed reduction strategy to decreasing the order of multivariate feed systems on a large-scale power system. Models of two-area power systems have been used to evaluate the utility of the predicted method. It is also possible to create an efficient reduction controller for massive power systems using the presented methods.[9] The Cauer second form of Continued Fractions is used to lower the linear transformer's higher-order transfer function. This approach has its own advantages and limitations in the field of model order reduction. Indicators of performance were also generated and compared using ISE. In comparison to previous techniques from the literature, the suggested reduced order is stable, settles quickly, and retains all the essential elements of the original system [2]

In both the time domain and the frequency domain, several approaches have been put forward for order-reduction of linear continuous systems. The heuristic criteria used in Eigen/Zero spectrum analysis are as follows: Methods of a higher order The average arithmetic value of the actual segments of the HOS poles (Centroid) could be the same as like LOS. It is standard practice to create lower order simulations by using Cauer's second type of continuing fraction expansion to expand the provided transfer function.

The most common is modal order reduction often utilized idea in engineering because of how simple it is to implement controls, how little it costs, and how quickly it can be calculated. Ghosh and Senroy have offered a method in order to simplify a mechanism using the balanced truncation strategy. To build a system's stable lower order model, Parmar et al. proposed a novel method. The authors suggest Cauer second variant (ESA & CAUER-II), ESA, Pade approximation (ESA & PA) and as a method for system simplification. In maintaining the stability, they placed emphasis. Vishwakarma and Prasad have created an innovative method for the simplification of linear dynamical systems according to ESA and Cuckoo searches. ESA determines the coefficients of the lower order system's denominator polynomial. This method preserves the system's stability while being very straightforward computationally.[1]

ESA calculates the coefficients of the denominator polynomial of the lower order system and uses the cuer second form to compute the coefficients of the numerator polynomial of the higher order system.

A hybrid methodology for model order reduction is suggested in this research. A performance metric called as integral square error is used to assess the effectiveness and potency of the approach (ISE). If the initial system is stable, then this approach ensures stability. In this study, a comparison using the ISE computation is offered.[6]

This article describes the process by which a high-level real mechanism could have been transformed into low-order one while retaining its essential properties. Through contrasting anticipated methodology with the present procedure, it is shown the fact that expected technique ensures the reduced's immutability. The suggested approach could be appropriate for high order system analysis and controller design.[7] The various sections provide a full explanation of the suggested strategy, solve the required numerical computations, and conduct simulated comparisons on the MATLAB platform.

The remainder of the article's parts are summarized below. Part 2 has a description of the issue. We get into the specifics of the strategy we suggest in section 3. We provide the findings and a numerical analysis in Section 4. Finally, in Section 5, we provide some thoughts.

METHODOLOGY DESCRIPTION

The HOS transfer function of degree 'n' be:

$$G_n(s) = \frac{A_{21}+A_{22}s+A_{23}s^2+------+A_{2,n}s^{n-1}}{A_{21}+A_{22}s+A_{13}s^2+------+A_{1,n}s^{n-1}+A_{1,n}s^n}$$

(1)

or

$$G_n(s) = \frac{A_{21}+A_{22}s+A_{23}s^2+------+A_{2n}s^{n-1}}{(s+\lambda_1)(s+\lambda_2)-----(s+\lambda_n)}$$

(2)

Where $-\lambda_1 < -\lambda_2 < \ldots\ldots\ldots\ldots\ldots < -\lambda_n$ are poles of HOS

The LOS transfer function of order 'r' be:

$$G_r(s) = \frac{B_{21}+B_{22}s+B_{23}s^2+------+B_{2,n}s^{n-1}}{(s+\lambda_1')(s+\lambda_2')-----(s+\lambda_r')}$$

(3)

Where $-\lambda_1' < -\lambda_2' < \ldots\ldots\ldots\ldots < -\lambda_r'$ are poles of LOS,

As a result, The steps are as below:

Step 1

The eigen spectrum zone (ESZ) of HOS is static as presented in figure 1 If poles −λi (i = 1,n) are positioned at −(Reλi ± Imλi) (i = 1,p) in the ESZ, then the ESZ is formed by two patterns transient over the nearby (Reλ1) and the furthest (Reλp) real poles when expurgated by two lines transitory over the furthest fictitious pole couples (±Im(max)).

Step 2

The mean of actual sections of poles, which defines the HOS pole centroid, is expressed as

$$\lambda_m \triangleq \frac{\sum_{i=1}^{p} R_e\lambda_i}{p}. \qquad (4)$$

The ratio between a system's closest and furthest poles, expressed only in expressions of real amounts, is known as the HOS system stiffness:

$$\lambda_s \triangleq \frac{R_e\lambda_1}{R_e\lambda_p}. \qquad (5)$$

Step 3

The eigen spectral spots of the LOS are as below. If λ′m and λ_s' are the pole centroid and system rigidity, individually, of LOS, and λ′m = λm and $\lambda_s' = \lambda_s$, the subsequent clench:

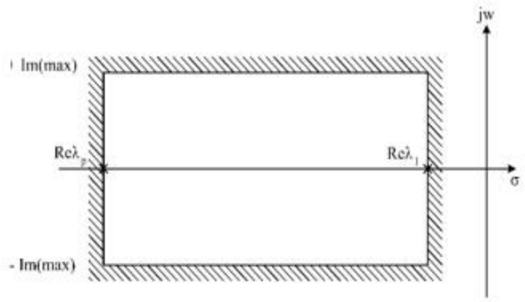
$$\lambda_s' = \frac{Re\lambda_1'}{Re\lambda_{p'}'} = \lambda_s$$

$$\lambda_m' = \frac{Re\lambda_1' + Re\lambda_1' + ----- + Re\lambda_{p'}'}{p'} = \lambda_m \qquad (6)$$

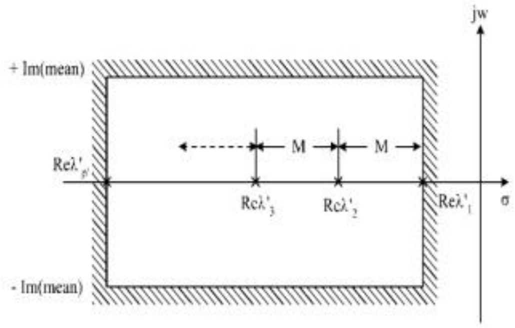
Whereas, λ′ i(i = 1,r)are poles of LOS situated at −(Reλ′ i ± Imλ′ i)i=1,p′ .

$$Re\lambda_{p'}' = M(p' - 1) + Re\lambda_1'$$

$$\frac{Re\lambda_{p'}' - Re\lambda_1'}{p' - 1} = M \qquad (7)$$



Eigen spectrum Zone of HOS



Eigen spectrum Zone of LOS

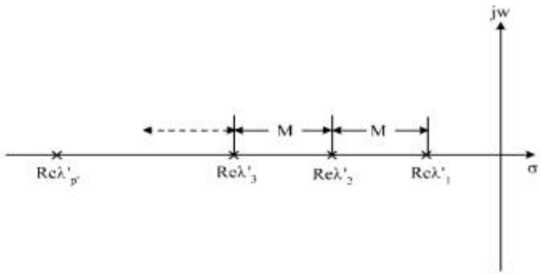


Fig.1 Eigen spectral point of LOS

i.e., $Re\lambda_1' + M = Re\lambda_2', Re\lambda_2' + M = Re\lambda_3'$ and so forth until $Re\lambda_{p'-1}' + M = Re\lambda_{p'}'$,

Equation (3) may be expressed in the following form:

$$\lambda_m = \frac{Re\lambda_1' + Re\lambda_{p'}' + (Re\lambda_1' + M) + (Re\lambda_2' + M) + -- + (Re\lambda_{p'-2}' + M)}{p'}$$

Or

$$\lambda_m p' = Re\lambda_1' + Re\lambda_{p'}' + (Re\lambda_1' + M) + (Re\lambda_1' + 2M) + ----+ (Re\lambda_1' + (p' - 2)M)$$

Or

$$N = Re\lambda_1'(p' - 1) + Re\lambda_{p'}' + QM \qquad (8)$$

Where

$$N = \lambda_m p' \; and \; QM = M + 2M + ---- + (p' - 2)M$$

Placing $\lambda_1' = \lambda_s Re\lambda_{p'}'$, equation 7 & 8 can be written as follows

$$Re\lambda_{p'}'(1 - \lambda_s) + M(1 - p') = 0 \qquad (9)$$

$$Re\lambda_{p'}^{'}[\lambda_s(p^{'}-1)+1]+MQ=N \quad (10)$$

Or

$$\begin{bmatrix} \lambda_s(p^{'}-1)+1 & Q \\ (1-\lambda_s) & (1-p^{'}) \end{bmatrix}\begin{bmatrix} Re\lambda_{p'}^{'} \\ M \end{bmatrix}=\begin{bmatrix} N \\ 0 \end{bmatrix}$$

(11)

Eq (11) can be resolved for $Re\lambda_{p'}^{'}$ *and* M facilitating thus the Figure 1 shows an example of ESP.. Consequently, the denominator polynomial in equation() is recognized that is defined as

$$\Delta_r(s)=B_{11}+B_{12}s+B_{13}s^2+----+B_{1,r+1}s^r$$

Step 4

Evaluate Cauer second form coefficients

$h_p(p=1,2,3,\ldots\ldots\ldots r)$ through constructing the Routh array as follows:

$$h_1=\frac{A_{11}}{A_{21}}< \quad \begin{cases} A_{11} \ A_{12} \ \ A_{13}\ldots \ \ A_{1,n} \ \ A_{1,n+1} \\ A_{21} \ A_{22} \ A_{23}\ldots\ldots\ldots A_{2,n} \ \ A_{21} \end{cases}$$

$$h_2=\frac{A_{21}}{A_{31}} \ \begin{cases} A_{21} \ A_{22} \ A_{23}\ldots\ldots\ldots A_{2,n} \ \ A_{21} \\ A_{31} \ A_{32} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{cases}$$

$$h_3=\frac{A_{31}}{A_{41}} \ \begin{cases} A_{31} \ A_{32} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ A_{41} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{cases}$$

………………………………………….. …..

where the first two rows of this array are copied from the denominator and numerator coefficients of $G_n(S)$ in equation(1), and the rest components are calculated utilizing known Routh's approach..

$A_{i,j}=A_{i-2,\ j+1}-h_{i-2}A_{i-1,j+1}$ and

$i=3,4,\ldots\ldots\ldots\ldots\ldots\ldots$

$j=1,2,\ldots\ldots\ldots\ldots\ldots\ldots$ (12)

$h_i=\frac{A_{i,1}}{A_{i+1,1}}; i=1,2,3,\ldots\ldots\ldots\ldots$ (13)

Equal the co-efficients $B_{1,j}(j=1,2,\ldots\ldots\ldots,(r+1))$ of Step 3 and Cauer quotients

$h_p(p=1,2,\ldots\ldots\ldots\ldots,r)$ of Step 4 to determine the reduced order model's numerator expressions $G_r(s)$. Create an inverse Routh algorithm as shown below.

$$B_{i+1,1}=\frac{B_{i,1}}{h_i};\ i=1,2,\ \ldots\ldots\ldots\ldots,r \ and \ r\le n$$

$$B_{i+1,j+1}=\frac{(B_{i,j+1}-B_{i+2,j})}{h_i};$$

$i=1,2,\ldots\ldots,(r-j)$

$j=1,2,\ldots\ldots,(r-1)$ (14)

NUMERICAL EXAMPLE AND RESULTS

For correlating LOS with the original HOS, three numerical samples taken from the literature were employed. The findings from the different reduction techniques are shown in a table format. The second-order model shown below serves to compare:

An error index ISE [15], The quality of the LOS, measured by the integral square error (ISE) (i.e., the lower the ISE, the closer Gr (s) to Gn(s)), is governed by the interaction of the transitory components of original and reduced order systems. ISE is supplied by:

$$ISE = \int_0^{\infty} [y(t) - y_r(t)]^2 dt \qquad (15)$$

where y(t) and yr (t) are unit impulse reactions of the original and low-order systems, individually.

The IRE [10, 19] is estimated and evaluated for the original model and additional reduced-order models. IRE is provided by

$$IRE = \int_0^{\infty} g^2(t) dt \qquad (16)$$

where g(t) is program's impulse response

Example 1.

Considering 10th order system that Mukherjee [17], Edgar [20], Therapos and Diamessis [21], and others have previously studied. The system's gain is very large, its numerator dynamics are nonexistent, and all of its poles are actual.

The HOS transfer function G10(s) is given by

<u>Original T/F</u>:

$$\frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}$$

Denominator $s^4 + 10s^3 + 35s^2 + 50s + 24$

HOS pole centroid and stiffness estimation: -

$$\lambda_m = \frac{\sum_{i=1}^{4} x_i}{4} = \frac{4+3+2+1}{4} = 2.5$$

$$\lambda_s = \frac{R_e \lambda_1}{R_e \lambda_0} = 1/4 = 0.25$$

We know that from Eigen spectrum method

$$\begin{pmatrix} \lambda_s(p^1 - 1) + 1 & Q \\ (1 - \lambda_s) & (1 - p^1) \end{pmatrix}$$

$$\begin{bmatrix} R_e \lambda^1 p^1 \\ m \end{bmatrix} = \begin{bmatrix} N \\ o \end{bmatrix}$$

Here $\lambda_s = 0.25, Q = 0, P^1 = 2, N = 5$

Then we get $\text{Re}\, \lambda^1 p^1 = 6.6667, M = 5$

We know that

$$\frac{\text{R}_e \lambda^1 p^1 - R_e \lambda_1^1}{p^1 - 1} = M$$

Hence $R_e \lambda_1^1 = 1.6667$

The resulted Denominator is

$$(s + 1.6667)(s + 6.6667)\lambda$$

$$= s^2 + 8.3334s + 11.1114$$

Numerator N(s) $= s^3 + 7s^2 + 24s + 24$

Routh array

$s^3 \mid 1 \ \ 24$

$s^2 \mid 7 \ \ 24$

$s \mid 20.5714$

$s^0 \mid 24$

1^{st} order numerator

$$D(s) = 20.57145s + 24$$

Finally $\dfrac{N(s)}{D(s)} = \dfrac{(20.57145s + 24)}{s^2 + 8.334s + 11.1114}$

Table 1 Parameters

Models	**Rise time**	**Settling time**	**Settling minimum**	**Settling maximum**	**Overshoot**	**Undershoot**	**Peak**	**Peak time**
Original	2.2603	3.9308	0.9019	0.9990	0	0	0.9990	6.88447
Proposed	2.2251	6.2764	0.9123	1.0389	3.8909	0	1.0389	4.5548
G_paramar	0.0409	4.3942	0.9140	2.4214	142.14	0	2.4214	0.3454
Mukherjee	0.0817	3.3477	1.0052	2.3795	136.82	0	2.3795	0.5685
Mittal	0.1415	5.4717	0.9953	2.0621	107.19	0	2.0621	0.8750

RESULTS AND DISCUSSION

Figure 1 shows the step responses of the proposed and current methods.and its parameters are tabulated in below table 1.

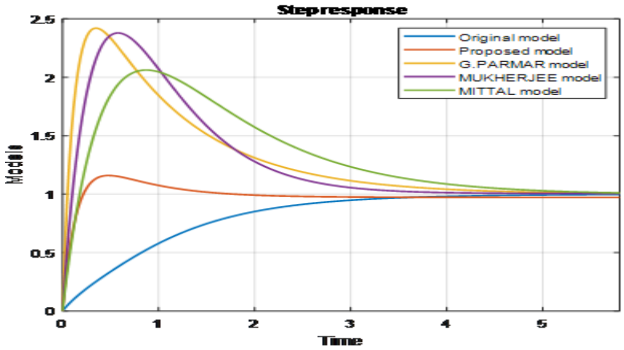


Figure 1 Step response of the five models

The aforementioned table displays the peak and peak time of the proposed and current models as well as their rise and settling times, settling minimum and maximum, overshoot, and undershoot values. The aforementioned statistic makes it obvious that the proposed model performs better than alternative models.

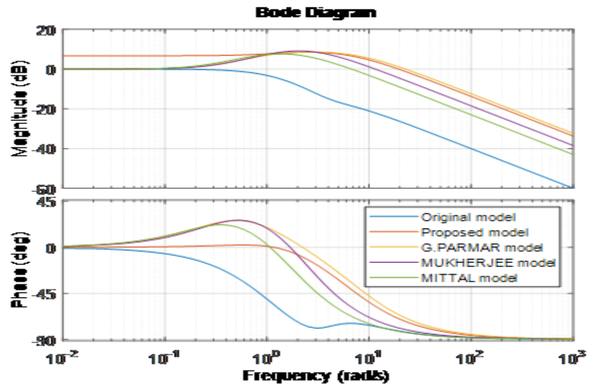


Figure 2 Bode plot

The bode diagram of all five models is shown in the previous picture. It can be seen from the diagram that the suggested model has a higher stability margin than the other models.

Table 2 Integral square error values

Models	**ISE**
Proposed	0.8086
G_paramar	170.4108
Mukherjee	165.8482
Mittal	142.6026

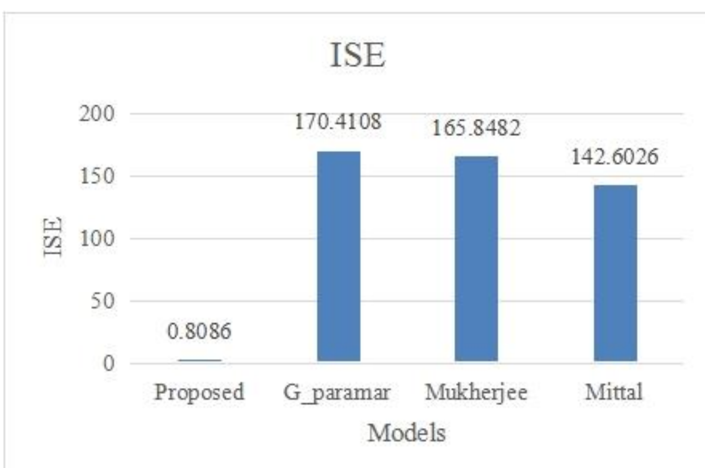


Figure 3 ISE of the models

The suggested model's Integral square error is quite small in comparison to the other models, as can be seen from the above image.

CONCLUSION

A controller employing the model order reduction approach was suggested in the research. In order to evaluate the performance, the original 4th order model is scaled down to the 2nd order and put up against four other models: Original, G Paramar, Mukherjee, and Mittal. Simulated results of the reactions of the proposed and current models are provided in the relevant section. The suggested model's achieved ISE value is very low when compared to other models, and the data indicated that it produced superior outcomes than other models.

REFERENCES

[1] Sikander and R. Prasad, "New technique for system simplification using Cuckoo search and ESA," *Sādhanā*, vol. 42, no. 9, pp. 1453–1458, Sep. 2017, doi: 10.1007/s12046-017-0710-0.

[2] D. K. Sambariya, A. K. Sharma, and T. Gupta, "ORDER REDUCTION OF AIR CORE TRANSFORMER USING CONTINUED FRACTION," vol. 14, p. 12, 2019.

[3] K. Choudhary and S. K. Nagar, "Order Reduction Techniques via Routh Approximation: A Critical Survey," *IETE Journal of Research*, vol. 65, no. 3, pp. 365–379, May 2019, doi: 10.1080/03772063.2017.1419836.

[4] R. V. S. Sengar, K. Chatterjee, and J. Singh, "System reduction using modified inverse distance measure and modified Cauer continued fraction method," *IOP Conf. Ser.: Mater. Sci. Eng.*, vol. 691, no. 1, p. 012006, Nov. 2019, doi: 10.1088/1757-899X/691/1/012006.

[5] R. S. Sengar, K. Chatterjee, and J. Singh, "System Simplification Using Pole Spectrum Analysis (PSA) with the Advantage of Dominant Pole Retention," *Circuits Syst Signal Process*, vol. 41, no. 1, pp. 102–121, Jan. 2022, doi: 10.1007/s00034-021-01792-9.

[6] S. Lavania and D. Nagaria, "Eigen spectrum based moment matching technique for model order reduction," in *2015 39th National Systems Conference (NSC)*, Greater Noida, India, Dec. 2015, pp. 1–5. doi: 10.1109/NATSYS.2015.7489100.

[7] R. Singh, V. M. Mishra, and J. Singh, "Model order reduction via time moment modeling and clustering technique," in *2017 4th IEEE Uttar Pradesh Section International Conference on Electrical, Computer and Electronics (UPCON)*, Mathura, Oct. 2017, pp. 638–641. doi: 10.1109/UPCON.2017.8251124.

[8] Parmar, S. Mukherjee, and R. Prasad, "System reduction using eigen spectrum analysis and Padé approximation technique," *International Journal of Computer Mathematics*, vol. 84, no. 12, pp. 1871–1880, Dec. 2007, doi: 10.1080/00207160701345566.

[9] R. Singh, A. Singh, J. Singh, and J. Kumar, "Model order reduction using logarithmic assembly technique and second cauer form for power system models," *J. Phys.: Conf. Ser.*, vol. 2007, no. 1, p. 012013, Aug. 2021, doi: 10.1088/1742-6596/2007/1/012013.

[10] S. Mukherjee, Satakshi, R.C Mittal, "Model order reduction using response matching", Journal of Franklin Inst., Vol.342, pp.503-519, 2005.