

Prediction of the number of failures of repairable elements aimed at homogenizers in the dairy sector with the Monte Carlo method

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Abstract

An attempt is made to determine a reliability indicator for repairable assets, performing a probabilistic analysis on repairable elements, homogenizers in the dairy sector. Various failure times were analyzed in recent years. A goodness-of-fit test was performed to determine the distribution that best fits the behavior of the data, an important step to obtain greater certainty in predictive reliability. In the case study, the Kolmogorov-Smirnov test was used. Then, the operating time histories and the number of failures that the asset had in that period were collected, the distribution that best aligned with the data was the Weibull distribution.

For the prediction of the calculation of the number of failures, the Monte Carlo method was used, random values were assigned to the function of the accumulated number of failures $F(t)$. The values of the times obtained were compared with the time t_m (mission time), the values that exceeded the mission time occur when the equipment does not fail. Therefore, as many predictions as history running times must be made, the times that did not exceed the mission time were added, thus determining the number of failures; Performing several data runs, the average was calculated, to obtain a better statistical result.

Keywords: *predictions, reliability, repairable equipment, Probabilistic distributions.*

INTRODUCTION

Systems reliability analyzes are of great interest in the industry, especially in the chemical, electrical, and electronic areas, which can be found in all manufacturing and food sectors hence, the use of new statistical mathematical models that help productivity based on the operational knowledge of the element, and that through engineering and computer, formulas and techniques are developed for the prediction of the performance of the cessation of a function (Antonio & Creus, 2009), these investigations of new statistical methods are carried out in order to minimize failures in engineering designs, maximizing the system performance and use of resources efficiently, it is also important to know the interval time in which different changes failures can occur. This involves predicting the occurrence of failures in a study mission time (Mohammad, Mark P., & Vasiliy, 2016).

In the food industry, various elements can be identified; one of them are consumables, whose main characteristic is that they can be replaced periodically, known as repairable non-active elements, and the others are repairable elements with great maintainability to return them to their proper functioning state (UNE-13306 European Committee for Standardization, 2018). For a correct analysis of these elements, it is necessary to know the circumstances in which they work, that is, their operational context, including the recording of operational data, and failure times (Gomez Romero, 2019).

The measurement of asset reliability is determined by the frequency in which failures occur, that is, if failures do not appear in a mission time, or study time regarding the equipment is reliable, or if one or several failures appear. During the study time, the equipment is unreliable, which is why reliability is determined as a probability that an element works fulfilling a certain function

in a specific mission time (Garcia Palencia, 2012). Random variables are involved in the study of reliability, and the best known are the operating times until a failure event occurs, which are very useful in predicting failures (Mora Gutierrez, 2009), see table 1, and figure 1.

Repairable items are repaired when they fail, allowing the system to continue functioning. An item has regained its ability to perform all required functions after being damaged by means of other than replacement. Modeling the time between item failures is important because a repaired item can fail multiple times (Paschal, 2008).

Figure1 Times for reliability analysis

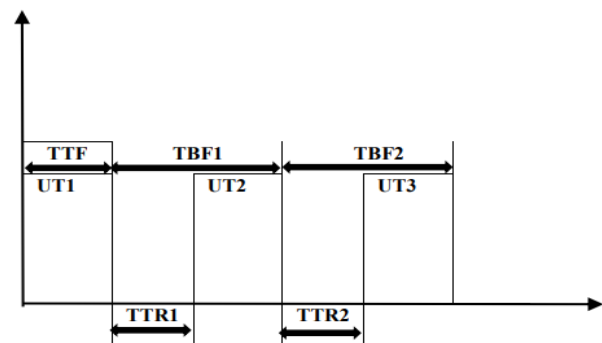


Table 1. Times for reliability analysis

Time	Average times	Name in English	Meaning in Spanish
TTF	$MTTF = \sum TTF / \text{number of failures}$	Time to Failure	Time to failure (non-repairable)
TBF	$MTBF = \sum TBF / \text{number of failures}$	Time Between Failure	time between failures
TTR	$MTTR = \sum TTR / \text{Number of failures}$	Time to repair	Time it takes to repair

As it is known in reliability engineering, for the study of random variables such as time to failure, statistical distributions are used including gamma, normal range, exponential and log-normal, and one of the most inclined towards equipment and industrial machinery is Weibull, in table 2, the equations for the calculation of the functions in the study of

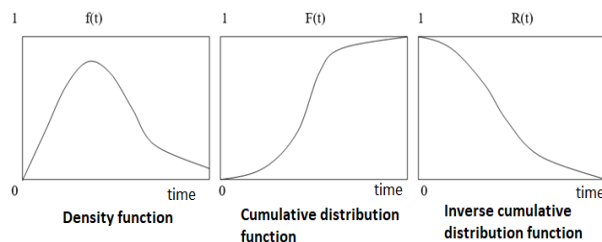
Reliability are shown; Probability density function $f(t)$, Cumulative failure function $F(t)$, Survival function or reliability $R(t)$ and the risk function or instantaneous failure rate $\lambda(t)$, as seen in figure 2. (Reyes Aguilar, 2008).

The difference between probability and occurrence is misleading, probability is a

Table 2. Statistical distributions with their main parameters.

Description	Exponential	Weibull	Normal	gamma
Failure probability density function $f(t)$	$f(t) = \lambda e^{-\lambda t}$	$f(t) = \frac{\beta(t)^{\beta-1}}{\alpha^\beta} e^{-\left(\frac{t}{\alpha}\right)^\beta}$	$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}$	$f(t) = \frac{t^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} e^{-\frac{t}{\beta}}$
Cumulative distribution function of probability of failure. $F(t)$	$F(t) = e^{-\lambda t}$	$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^\beta}$	$F(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left(e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \right) dt$	$F(t) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^t t^{\alpha-1} e^{-\frac{t}{\beta}} dt$
Reliability function $R(t)$	$R(t) = 1 - F(t)$	$R(t) = e^{-\left(\frac{t}{\alpha}\right)^\beta}$	$R(t) = 1 - F(t)$	$R(t) = 1 - F(t)$
The risk $\lambda(t)$	$\lambda(t) = \frac{f(t)}{R(t)}$	$\lambda(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1}$	$\lambda(t) = \frac{f(t)}{R(t)}$	$\lambda(t) = \frac{f(t)}{R(t)}$
Main parameters of the distributions	$\lambda = \frac{n}{\sum_{i=1}^n t_i}$ $n = \text{number of sample teams.}$ $t = \text{time}$	$\alpha = \left(\frac{\sum_{i=1}^n t_i^\beta}{n} \right)^{1/\beta}$ $\frac{\sum_{i=1}^n [t_i^\beta \ln(t_i)]}{\sum_{i=1}^n t_i^\beta}$ $\frac{1}{\beta} = \frac{1}{n} \sum_{i=1}^n \ln(t_i)$	$\mu = \frac{\sum_{i=1}^n t_i}{n}$ $\sigma^2 = \frac{1}{(n-1)} \sum_{i=1}^n (t_i - \mu)^2$	$\alpha = \frac{(n-1)(\sum_{i=1}^n t_i)^2}{n^2 \sum_{i=1}^n (t_i - \mu_x)^2}$ $\beta = \frac{n \cdot \sum_{i=1}^n (t_i - \mu_x)^2}{(n-1) \sum_{i=1}^n t_i}$

Figure2. Probability distributions to failure.



Failure rate $\lambda(t)$ oh(t) is obtained from the ratio between the density function $f(t)$ and the cumulative distribution function $R(t)$, and it represents the probability of survival condition up to time t or the propensity to fail at this time (Arata Andreani, 2005)(Antonio &

possibility, it is a potential event, if there is a 90% probability of failure in 1000 h, does not mean that at that exactly that time the asset will fail which means that there is a 90% probability. Therefore, there is a 10% chance that the equipment will work without failure (Sexto, 2008).

Creus, 2009)(Crespo, Moreu, & Sánchez, 2004).

For greater certainty in the analysis of these variables, goodness-of-fit tests are used, in order to determine that these operational data is used to conform to a certain statistical distribution, and that in the most accurate data fit, the Kolmogorov Smirnov test (Fala, 2022) (Mora, 2012). In this way, a probabilistic model fitted to the data is used to provide an objective statistical basis for further studies, from which reliable estimates and expectations are derived (Moreno & Cruz, 2019).

In the adjustment process, certain steps must be followed, in which the cumulative function $F(t)$ of the used distributions and the empirical function $F^{\wedge}(t)$ intervene directly:

First, the cumulative failure function of each distribution used in the study is calculated (for continuous random variables; Weibull, Normal; exponential, Log-normal, Gamma) then the empirical function of the data, second we plot $F^{\wedge}(t)$ comparing it with each of the functions and the $F(t)$ functions, for greater security to recognize which of the hypotheses are admitted, the KS test (Kolmogorov - Smirnov) is performed and finally, it is compared according to the recommended level of significance (Gallegos, Garcia, & Tenicota, 2022),(Melo, Lara, & Gordillo, 2009).

Using the last analysis, the correct distribution can be chosen, to which the data aligns, which will be useful in predicting a single time that the item will fail, and in this way being ready for a repair, which characterizes repairable items (Park, 2002), but by using these data in consequent times, a new analysis can be carried out, it will allow predicting upcoming events, and using restoration procedures.

These restoration methodologies, which on many occasions are immediate, allow the equipment functions to be restored, leaving them in different states, which are common within the industry, whatever the sector, commonly speaking of five states (Zapata, 2011).

State 1: Repaired as new.

State 2: Same as before.

State 3: Better than before but not like new.

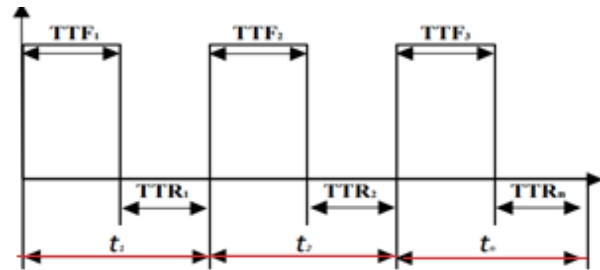
Condition 4: Better than new.

State 5: Worse than before

Most of probability methodologies for estimating failures in a study time are mostly inclined towards two common states:

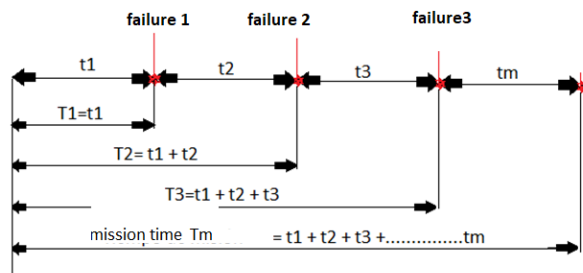
operation or failure, and in this research a probabilistic model is presented to predict failures in a determined time, besides including the repair time, that is, joining these two times is analyzed in a single time t_1 of work for repairable systems, representing it as in figure 3 (Ramires castaño, 2014) as is the case of a homogenizer in the dairy industry, this system is one of the most complex, since it has various components, which are pneumatic, electrical, electronic, and hydraulic, which are subjected to wear out, usually some to a hydro-adhesive wear, considering that after a repair it can take any of the states seen previously, and making a practical application and main use of its operation data be able to convert it into a process that generalizes the homogeneous restoration by mean of the prediction (Muniz Sanchez, 2004).

Figure 3 Operation time and repair time t_n



In these types of elements, variables of interest can be found in the reliability engineering field, reliability, probability of failure and the failure rate names. In the case of repairable elements, the primary factor is time, since different times intervene in which more than one probability of failure can occur in a mission time (t_m).

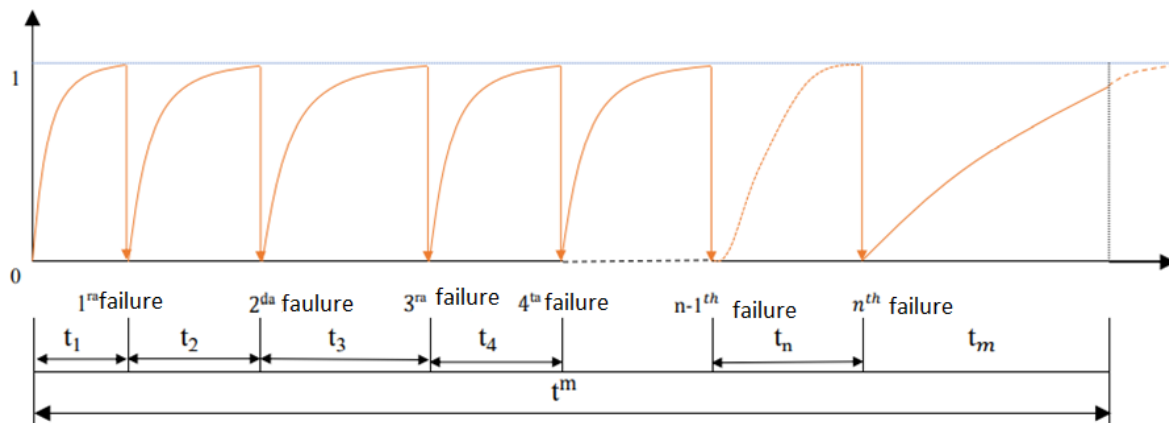
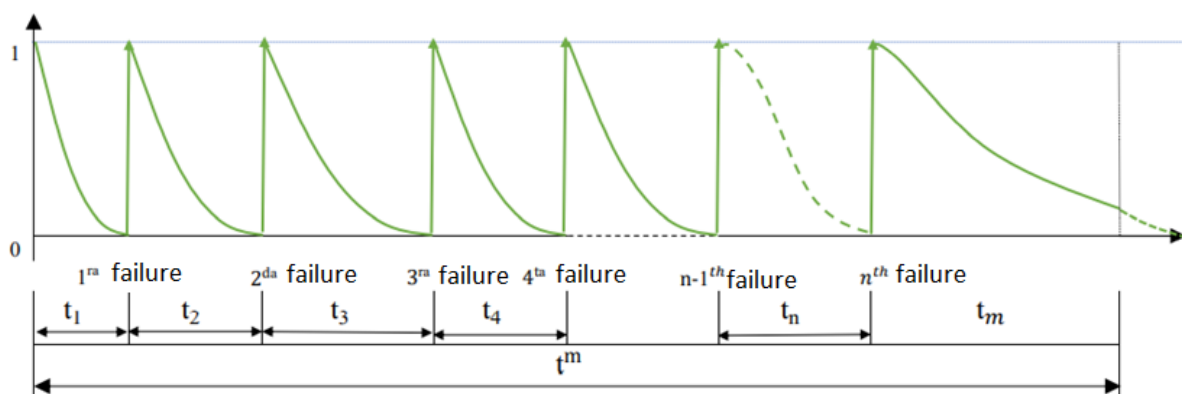
A scheme of an operative productive process of the repairable element is made as can be seen in figure 4, which is clearly explained that after a time t_1 failure occurs and the production come to the end until it is repaired (Rasay, Taghipour, & Sharifi, 2022).

Figure 4 Consequential failure process

Observing the previous figure it is considered that the variable t demonstrates the operated time by the element between two consecutive failures (operating time between the first and second failure) which is t_1 , between the

second and third failure, which is t_2 ; between the third and fourth failure, such as t_3 , and so on until the umpteenth failure t_n , since they are random, they can take any time value and each of these can be represented probabilistically (Navas Alvarez, 2017).

In figure 5 different probabilities of failure can be seen with respect to each studied time, which take ascending values from 0 to 1, and in figure 6 it can be seen that the reliability in these same times decreases from 1 to 0.

Figure 5 Probability of occurrence of failures in repairable equipment**Figure 6 Reliability behavior in repairable equipment**

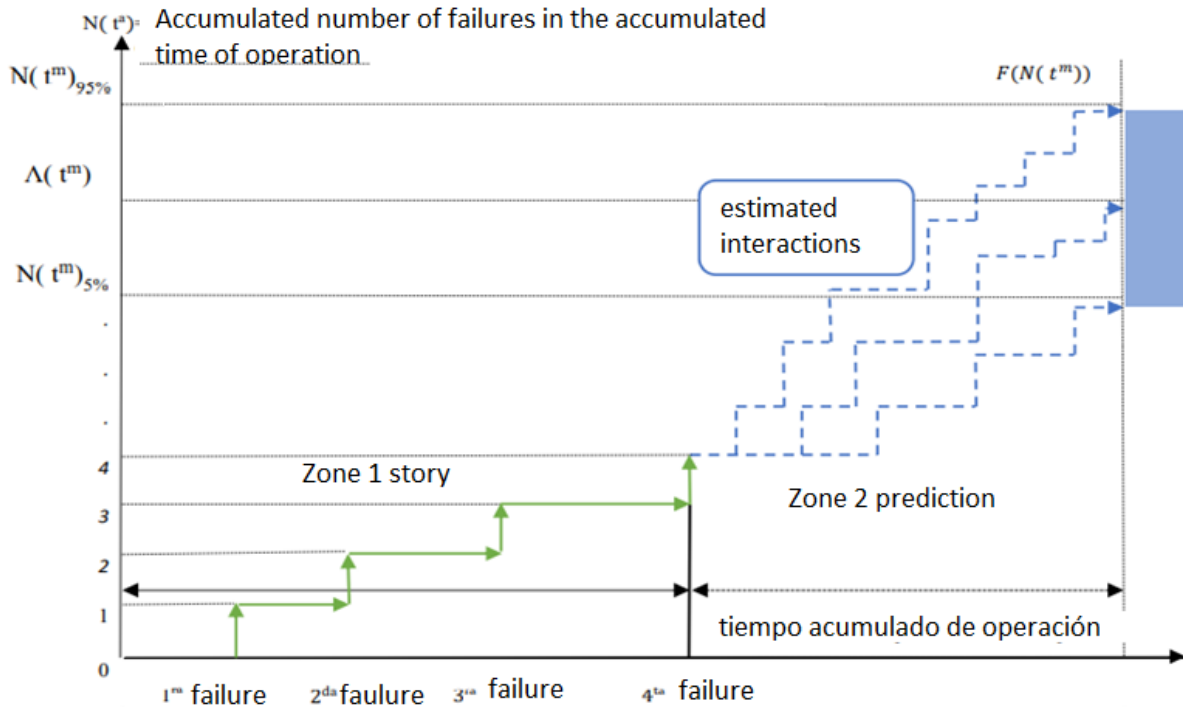
Then the correct form of prediction can be determined by the number of failures that have occurred cumulatively $N(t)$, in an accumulated operating time as shown in figure 7, where we appreciate 2 zones, a history that has occurred

consistently, with useful times to predict zone two of the estimated times. In this second prediction zone are the failures of the element that can be repaired in the future, also set as a random zone, whose objective is to determine

the number of failures from the last time the last failure occurred to an estimated mission time modeling it mathematically using a probability distribution and predict with great

certainty the expected number of failures, all focused on using the Monte Carlo simulation (Zhang, Chen, Zeng, Liu, & Beer, 2022).

Figure 7 Cumulative number of failures in a mission time



Methodology.

Keep data durations until the failures (t_i) of the repairable elements of the dairy homogenizing system occur.

With the data collected, analyze and graph the cumulative failure probability functions $F(t)$ for each distribution, using the formulas in Table 2.

For a better estimation, the K-S, Kolmogorov – Smirnov goodness fit test is performed to select the distribution that best aligns with the data from the history of the element studied.

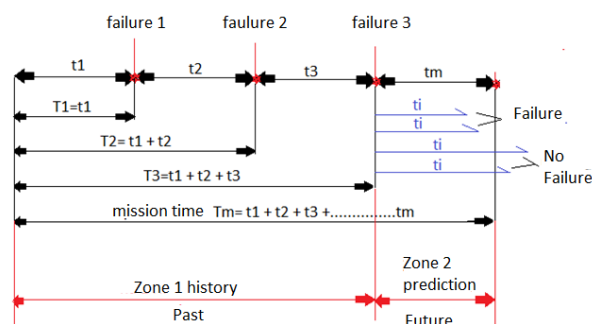
From the selected probability distribution (Weibull), the time t_i , Time of successive failures, is cleared:

$$t_i = \alpha^\beta (-\ln(1-F(t_i)))^{1/\beta} \quad (1)$$

In a programmable sheet perform the estimated numerical calculation, where m is the number of iterations and n number of failures, Figure 8. The Monte Carlo method is used to create random values of $F(t)$ and calculate the time t_i , perform as many interactions as number of failures. $\Lambda(t^m) = 1/m(\sum_{i=1}^m n_i)$

Define a mission time (t_m) time at which we want to estimate the number of failures. The t_i calculated above is compared with the mission time, if the times are higher, it means that there are no failures and if the time is less than (t_m) it means that the equipment failed, the failure events are added finally, the number of estimated failures is obtained.

For best results, several data runs should be carried out and the average of these as the final result.

Figure 8 Estimated Iterations**Results**

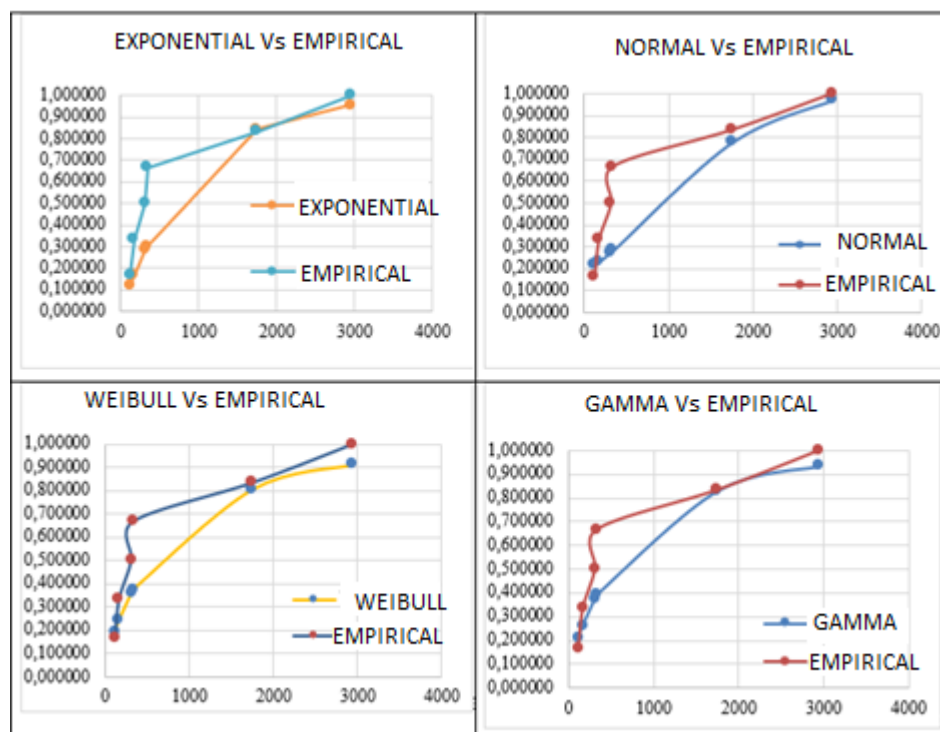
For the study, 7 failures were recorded in the pumping piston, of the pumping group of a homogenizer of a dairy company, which is a repairable component, the study is emphasized in the pumping piston head, these data were collected in a period of three years, taken from the maintenance record of the manual, in which each active maintenance action carried out on the system was recorded. Table 3 shows the data.

Table 3 Fault Data Sample

Equipment	stop date	hour meter	Operating time (hours)
pumping group	01/09/2018	8303	
pumping group	03/02/2018	8642	339
pumping group	05/01/2019	11592	1753
pumping group	07/05/2020	13345	2950
pumping group	09/08/2020	13660	315
Group of. Pumping	10/10/2020	13930	270
Pumping Group	11/13/2020	14250	320

The K_S goodness-of-fit test was performed, calculating the parameters for each

distribution and comparing the theoretical vs. empirical function, graph 10.

Figure 9 Graphs of Theoretical Vs Empirical Functions

A calculation was made for the previous table 4, as well as its parameters shown in graphs, obtaining as a result the data shown in table 5.

Table 4. Result of probability of failure of the distributions.

Fault Not "i"	you" (hours)	F(t) Exponential	F(t) Weibull	F(t) Normal	F(t) gamma	F(t) empirical
1	270	0.238455	0.268047	0.240268	0.293463	0.166667
2	315	0.272257	0.298480	0.254171	0.326231	0.333333
3	320	0.275919	0.301736	0.255741	0.329736	0.500000
4	339	0.289667	0.313893	0.261755	0.342826	0.666667
5	1753	0.829432	0.769582	0.771931	0.821748	0.833333
6	2950	0.949018	0.895476	0.972325	0.936155	1.000000

The level of significance was taken according to 0.05, which is chosen a priori, it corresponds to the amount of data for a critical value of 0.486. Table 5.

Table 5 Parameter values and critical values by K_S test for each distribution

distributions	parameters	Maximum critical values
Exponential	$\lambda=$ 0.00100891	0.37700
gamma	$\alpha=$ 1102.57938	0.352774
	$\beta=$ 0.82774543	
Normal	$m=$ 1465.63261	0.40491
	$\sigma=$ 1417.68355	
Weibull	$\alpha=$ 0.78336131	0.323840
	$\beta=$ 1265,27395	

From the maximum critical values of the distributions, we compare them with the study significance value and we can realize that the four hypotheses meet the requested significance value, choosing the lowest value for a better appreciation, which is the Weibull distribution.

With the selected distribution we can specify the construction of curves of its reliability function $R(t)$, Unreliability $F(t)$, figure 10; Risk $\lambda(t)$, figure 11 and its probability density $f(t)$, figure 12. Which were needed to determine the history zone of the repairable element and with the data already obtained from it, it was used to predict the estimated number of failures in a mission time that can be any of the study times.

Figure 10 Reliability and Unreliability in Weibull

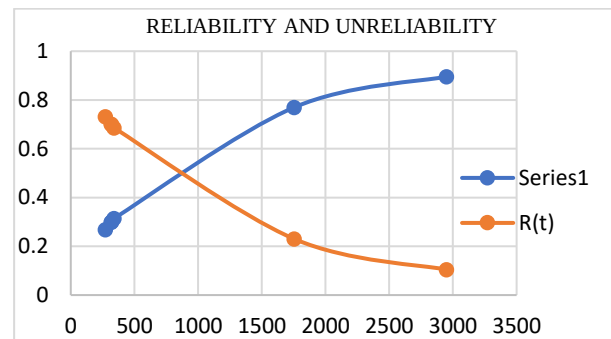
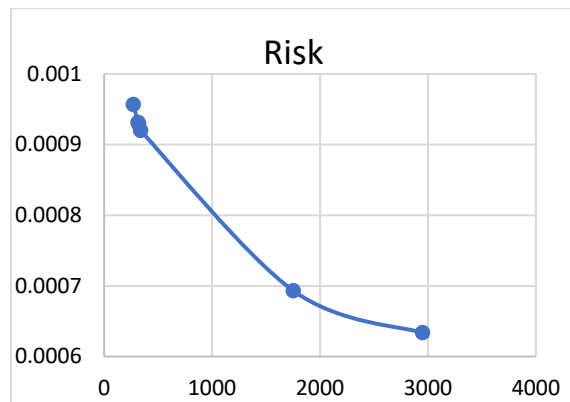
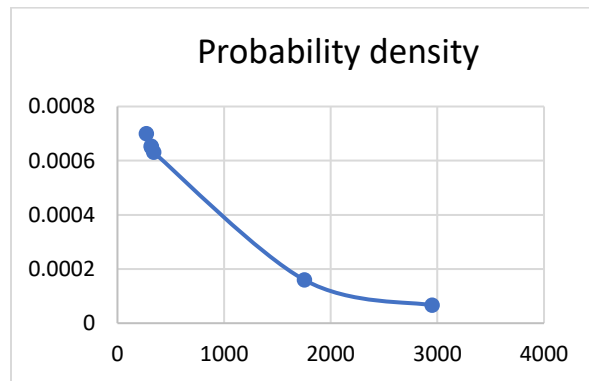


Figure 11 Risk in Weibull**Figure 11 Weibull probability density**

Weibull distribution parameters are calculated, Table 6

Shape parameter:

$$\frac{\sum_{i=1}^n [t_i^\beta \ln(t_i)]}{\sum_{i=1}^n t_i^\beta} - \frac{1}{\beta} = \frac{1}{n} \sum_{i=1}^n \ln(t_i) \quad (2)$$

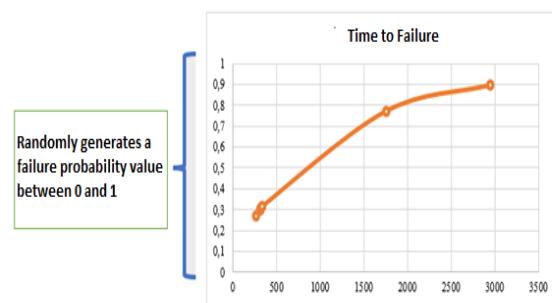
Scale parameter:

$$\alpha = \left(\frac{1}{n} \right)^{1/\beta} \quad (3)$$

Table 6 Weibull parameters

Weibull parameters		Worth
shape parameter	β	0.8277
intercept	C.	-5.7987
scale parameter	α	1102.5794

To continue with the analysis, the probability of failures $F(t)$ was calculated, in order to generate a probability of random failures between 0 and 1, Figure 12.

Figure 12 F(t) random

With equation (1) the estimated times for failures were calculated, randomly estimating the values of $F(t)$.

In table 7 the results obtained in the prediction zone can be seen, based on the history of the asset, an estimated mission time was determined, and that through the random value of $F(t)$, iterations of possible failures could be established. estimated in the study time, which can be changed depending on what range of hours are wanted to predict a failure.

Table 7 Result of the prediction in an estimated mission time of 800 hours

F(t)	F(t) Random	ti	cumulative t	tm	t1	t2	nf
0.26804728	0.06629542	43.3	5947	800	6747	5990.3	1
0.29848049	0.3554595	408.1	5947	800	6747	6355.1	0
0.301736	0.33750878	377.4	5947	800	6747	6324.4	0
0.31389285	0.86675003	2571.2	5947	800	6747	8518.2	1

Prediction of the number of failures of repairable elements aimed at homogenizers in the dairy sector with the Monte Carlo method

0.76958207	0.376617	445.8	5947	800	6747	6392.8	0
0.89547568	0.01064393	4.6	5947	800	6747	5951.6	0
expected number of failures $\Lambda(t^m)$							2

As the mission time t_m varies, the expected number of failures can also change. An example can be seen in table 8 where the mission time is calculated for the next 1000 hours.

Table 8 Result of the prediction in an estimated mission time of 1000 hours

F(t)	F(t) Random	ti	cumulative t	tm	t1	t2	nf
0.268047283	0.511513958	737.0	5947	1000	6947	6684.0	1
0.298480486	0.316082818	342.5	5947	1000	6947	6289.5	0
0.301736004	0.82657719	2170.9	5947	1000	6947	8117.9	1
0.313892852	0.961940403	4611.2	5947	1000	6947	10558.2	1
0.769582071	0.45681706	607.2	5947	1000	6947	6554.2	0
0.895475676	0.315904178	342.2	5947	1000	6947	6289.2	0
expected number of failures $\Lambda(t^m)$							3

Tables 7 and 8 show an example of how an iteration should be carried out for different study times, and in this article, it is recommended to carry out several iterations for each study time and then take an average of said values.

Conclusions

The reliability study in repairable equipment, can be used in two types of assets, the first are those that constitute elements that are replaced after a while, and the second is the equipment that has components that after a restoration procedure can be reused, as in our case study, the homogenizers.

A minimum of five data is required for the analysis, and the use of goodness-of-fit tests is required for a correct choice of the distribution that best fits the data studied.

The object of study of the reliability of repairable equipment is the main variable of the accumulated number of failures, which is analyzed in a certain accumulated mission time, which allows us to graph an important

indicator that is the expected number of failures.

These mathematical procedures can also be linked to the analysis of other repairable elements, which have components that must be replaced, to return to the operating state, with a criticality analysis of spare parts, to determine a number of real spare parts stock, that is, in other words, according to the prediction of failures, also have the logistics ready for future maintenance.

The number of iterations carried out will give us a greater certainty of the estimated number of failures in the prediction time.

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alimentos con la mayor capacidad del mundo

con el costo total de propiedad más bajo.

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