On subclass of Bazilevic function With Chebyshev polynomial

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Abstract

In the current work, we determine specific coefficients of the subclass S (β, r) of univalent functions using chebyshev polynomials and estimate the pertinent relationship of the renowned traditional functions in this class are subject to the Fekete-Szegö inequality.Introduction and definitions:

If the unit disk is U since

U= {w : w $\in\not\subset$, |w| <1}.

let A represent the class of analytical operationsinU, fulfilling the requirements £(0)=0 and £^˴ (0)=1.

$£(w) = w + \sum_{n=2}^{\propto} a_n w^n$ Let S make reference to the category of all univalent functions that are members of the normalized analytic function A.

INTRODUCTION

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$£(w) = w + \sum_{n=2}^{\propto} a_n w^n$ Let S make reference to the category of all univalent functions that are members of the normalized analytic function A.
If the functions £(w) and g(w) are analytic on U. We refer to that as £(w) is subordinate to g(w) in U, written £(w) < g(w) if a Schwarz function is present ϕ(w), This is analytical in U, and fulfills ϕ(0)=0 , |ϕ(w)|<1. (w∈U)
Such that £(w)= g(ϕ(w)) (w∈U)

In addition, g(w) is univalent in U, we obtain the equivalent (see[10])
£(w) < g(w) (w∈U) ⇔ £(0)= g(0) and £(U)⊂g(U)

The Fekete-Szegö functional $|a_3 - r a_1^2|$ for the normalized Taylor series.
£(w) =w+$a_2 w^2$ +a_3 w^3+….

in the theory of geometrical functions is well known. The disproof of the 1933 study by Fekete-Szegö served as its foundation.Gvess Paley and Littlewoods' contention that the maximum number of odd univalent function coefficients is unity (see[12]), has since attracted a lot of interest, especially in several classes of univalent functions. Because of this, further classes of univalent functions were discovered using the Fekete-Szegö function that was introduced by numerous authors. (see[4, 11, 15, 16, 17]).

The key findings of research publications using Chebyshev orthogonal polynomials typically come from the first kind. $T_k(r)$ and second kind $U_k(r)$ of chebyshev polynomials and their numerous applications in other fields.

Moreover, one can see the papers in ([2, 3, 5, 6, 9, 14, 7]. The chebyshev polynomials' first and second forms are well-known in the involving a verified variable.

-1<t<1 They are described as follows:

$T_k(t) = \cos k\theta$

$U_k(t) = \frac{\sin (k+1)\theta}{sin\theta}$

where K is the subyript's degree in the polynomial and $t=\cos\theta$.

Definition: 1. (1) For £∈A a Al-oboudi operator is defined by D_n^m : A → A where $n\geq 0$, $m\in N_0$ and

D_n^0 £(w) = £(w),

D_n^1£ (w) = (1-n)£(w) + nw£`(w),

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D_n^m £(w) = D_n^1 (D_n^{m-1} £ (w)) m≥2

Moreover, the operator D_n^m can be stated as follows using a power series.:

D_n^m £(w) = w +$\sum_{k=2}^{a} [1+n(k-1)]^m a_k w^k$

Definition 2. (9) A $n\ operater\ D_n^m f$ ∈A is reportedly the class S(β, r) , β≥0 and r∈($\frac{1}{2}$, 1), if the subordination follows hold.

$$(1-\beta)\frac{w(D_n^m f)(21\acute{)}}{(D_n^m s(w))} + \beta(1+\frac{w(D_n^m f)(21)"}{(D_n^m s(w))\acute{}} <$$

$$H(w,t) := \frac{1}{1-2tw+w^2} \quad (z\in U)\ldots\ldots(2)$$

Noting that if v=cos∝ with ∝∈(-π/3, π/3), then

$$Ή(w,v)= \frac{1}{1-2vw+w^2}$$

$$=1+\sum_{n=1}^{\infty}\frac{\sin (n=1)\propto}{sind\propto} w^n \quad (z\in U)$$

Thus

Ή(w,v) = 1+2cos∝w+(3cos²∝-sin²∝) $w^2 +$ ... ... (w∈U)

From [18], we write

Ή(w,v)= 1+U_1(v)w+U_2(v) +….. (w∈U, v∈(-1, 1))

Where

$U_{k-1}=\frac{\sin (k\ are\ cos v)}{\sqrt{1-v^2}}$, (k ∈ N = {1, 2, 3, …})

pertaining to the second class of Chebyshev polynomials. Moreover, it is acknowledged that $U_k(v)$ = zt $U_{v-1}(v)$ –$U_{v-2}(v)$

And $U_1(v) = 2v$

$U_2(v) = 4v^2-1$

…..(3)

$U_3(v) = 8v^3-4v$.

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The first variety of the chebyshev polynomials' common generating function $T_k(v)$, v∈[-1, 1], has the following structure:

$\sum_{k=0}^{\infty} T(v) = \frac{1-vw}{1-2vw-w^2}$ (wϵU)

(The first kind of chebyshev polynomials $T_k(v)$ and the second kind $U_k(v)$ have the relationships have the going to follow connections::

$$\frac{dT_k(v)}{dv} = KU_{k-1}(v),$$
$$K_k(v) = U_k(v) - vU_{k-1}(v),$$
$$2K_k(v) = U_k(v) - U_{k-2}(v),$$

Let Ω be the class of functions of the form

ϕ(w) = C_1w+$C_2 w^2$+c3w^3+….

Satistying │ϕ(w)│<1 for w∈U

The following lemma is necessary for us to prove our findings.

Lemma.1. [16] let $\phi\in\Omega$, thus for any complex number r.
$|C_2- \varepsilon C_1^2| \leq \max\{1;|\varepsilon|\}$ …………(2)
For the services provided by, the outcome is precise by $\phi(w)=w$ or $\phi(w)=w^2$

We attempt to define a subclass in this work. S(β, r) (see[9]) and operator Al-Aboudi (see[1]) using the chebyshev polynomial and by offering initial coefficient estimations. The Fekete-Szegö dilemma in this class is also explained in addition to that.
Theorem (1): Let D_n^m £(w) presented by (1) belong to the class S(β, r). Then
$|a_2|\leq\frac{2r}{(1+n)^m(1+\beta)}$

$|a_3|\leq \frac{(1+3\beta)r^2}{(1+2\beta)(1+\beta)^2(1+2n)^m}+\frac{r}{(1+2\beta)(1+2n)^m}+\frac{1}{2(1+2\beta)(1+2n)^m}$

proof: Let £∈ S(β, r). From ( ), we have
$(1+\beta)\frac{wf`(w)}{f(w)}+\beta(1+\frac{wf"(w)}{f`(w)}=$
$1+U_1(r)\phi(w)+U_2(r)\phi^2(w)+$….(3)

If w is an analytical function, that way $\phi(0)=0$ and
$|\phi(w)|<1$ for all w∈U. based on the equational ( ) and ( ), That's what we get:

$(1+\beta)\frac{wf`(w)}{f(w)}+\beta(1+\frac{wf"(w)}{f`(w)}=$
$1+U_1(r)C_1w+[U_1(r)C_2+U_2(r)C_1^2]w^2$ ……(4)

It is generally acknowledged that if $|\phi(w)| = |C_1w+C_2w^2+C_3w^3+\ldots|<1$ z∈U

Then $|C_j| \leq 1$, for all j∈N. …(5)

And $|C_2- \varepsilon C_1^2| \leq \max\{1, |\varepsilon|\}$, for all $\varepsilon\in R$ ……(6)

From (2) it follows that
$(1+n)^m(1+\beta)\, a_2 = U_1(r)C_1$ …..(7)

$[2(1+2\beta)(1+2n)^m]a_3- (1+n)^2(1+3\beta)\, a_2^2= U_1(r)C_2+U_2(r)C_1^2$…..(8)

From (4), (6) we obtain
$a_2=\frac{U_1(r)C_1}{(1+n)^m(1+\beta)}$
$|a_2|\leq\frac{2r}{(1+n)^m(1+\beta)}$ ….(9)

And by using (7), (8), to find the bound on $|a_3|$, we obtain

$2(1+2\beta)(1+2n)^m\, a_3= \{U_2(r)+\frac{(1+n)^{2m}(1+3\beta)}{(1+n)^{2m}(1+\beta)^2}U_1^2(r)\}C_1^2$ …….(10)

then, in view of (5) and (6), we have form (10)

$|a_2|\leq \frac{(2\beta^2+10\beta+4)r^2}{(1+2\beta)(1+\beta)^2(1+2n)^m}+\frac{r}{(1+2\beta)(1+2n)^m}+\frac{1}{(1+2\beta)(1+2n)^m}$ …….(11)

Theorem2. Let D_n^m £(w) belong to the class S(β, r). Then

$|a_3- \varepsilon a_2^2|\leq\frac{r}{2(1+2n)^m(1+2\beta)}\max\{1, \frac{4r^2-1}{2r}+\frac{2(1+3\beta)r}{(1+\beta)^2}-4\varepsilon\frac{(1+2\beta)(1+2n)^m r}{(1+n)^{2m}(1+\beta)^2}\}$

Proof: from (9) and (11), we have

$|a_3- \varepsilon a_2^2|\leq\frac{U_1(r)}{2(1+2n)^m(1+2\beta)}| C_2\ \{1, \frac{U_2(r)}{U_1(r)}+\frac{(1+3\beta)r}{(1+\beta)^2}U_1(t)-2\varepsilon\frac{(1+2\beta)(1+2n)^m U_1(r)}{(1+n)^{2m}(1+\beta)^2}\}C_1^2|$.

Then, in view of (2) we obtain that

$|a_3- \varepsilon a_2^2|\leq\frac{U_1(r)}{2(1+2n)^m(1+2\beta)}\max\{1, |\frac{U_2(r)}{U_1(r)}+\frac{(1+3\beta)r}{(1+\beta)^2}U_1(r)-2\varepsilon\frac{(1+2\beta)(1+2n)^m}{(1+n)^{2m}(1+\beta)^2}U_1(r)|\}$.

Finally, by substituting (5) and (6) into (2) we get

$|a_3 - \varepsilon\ a_2^2| \leq \frac{r}{2(1+2n)^m(1+2\beta)}$ max $\{1, \frac{4r^2-1}{2r} + \frac{2(1+3\beta)r}{(1+\beta)^2} - 4\varepsilon \frac{(1+2\beta)(1+2n)^m r}{(1+n\ )^{2m}(1+\beta)^2}\}$.

Taking β=1 Theorem above yields the following consequence..

Corollary: If D_n^m £(w) belong to the class S(r). Then

$$|a_2| \leq \frac{r}{(1+2)^m}$$

$$|a_3| \leq, \frac{4r^2}{3(1+n\ )^{2m}} + \frac{r}{3(1+2n\ )^m} - \frac{1}{6(1+2\ )^m}$$

$$|a_3 - \varepsilon\ a_2^2| \leq \left| \frac{8r^2-6\varepsilon r^2-1}{6(1+2n)^m} \right|$$

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