Modified New Estimators Depending on Unbiased Ridge Estimator For Linear Regression Model

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Abstract

In this paper, we develop new three types of biased estimators depending on the superiority of unbiased ridge regression estimator comparing to ordinary least square estimator when there is a multicollinearity among the explanatory variables. The bias, variance, mean square error matrix (MSE) and scalar mean square error (mse) of the proposed estimators are derived. The performance of these estimators are evaluated in comparison to that of other estimators by utilizing the MSE criterion. Finally, a numerical example is analyses in order to learn more about the performance of the new proposed estimators.

Keywords: *unbiased ridge estimator; multicollinearity; ordinary ridge estimator; Liu estimator.*

1. INTRODUCTION

Let us consider the multiple linear regression model

$$Y = X\beta + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2 I_n) \qquad (1)$$

where vector Y is a dependent variable of order an $n \times 1$ of observation , X is an $n \times p$ non-stochastic known matrix of explanatory variables of rank p, β is a $p \times 1$ unknown parameters vector , ε is an $n \times 1$ a vector of errors an expectation $E(\varepsilon) = 0$, covariance equal to $\sigma^2 I_n$. The ordinary least squares estimator (OLSE) for model (1) we can the written as follows :

$$\hat{\beta}_{OLSE} = Z^{-1}X'Y, \qquad (2)$$

where $Z = X'X$. For a long time, the OLSE has been considered the best estimator since it has the minimum variance in the class of unbiased estimators known as the best linear unbiased estimator. However, when there is a problem of multicollinearity or an ill-conditioned of design matrix in a linear regression model, many results have shown that the OLSE is no longer a good estimator, leading to the development of biased estimators such as the Stein estimator [1], the ordinary ridge estimator (ORR) was proposed Hoerl and Kennard in [2]. The ordinary ridge estimator (ORR) was proposed as follows:

$$\hat{\beta}_{ORR}(k) = (X'X + kI)^{-1}X'y, \qquad (3)$$

$$= [I - k(X'X + kI)^{-1}]\hat{\beta}_{OLSE} = W\hat{\beta}_{OLSE}$$

where $Z_k = Z + kI, W = [I - k(Z + kI)^{-1}]$, $k > 0$.

The Liu Estimator (LE) was proposed by Liu [3], Almost Unbiased Ridge Estimator (AURE) was in idea indicated Singh and

Chaubey [4] are some of the biased estimators proposed to solve the multicollinearity problem which are based only on sample information. The estimators are given by:

$$\hat{\beta}_{\mathrm{Lu}}(\mathrm{d}) = (\mathrm{S}+\mathrm{I})^{-1}(\mathrm{Z}+\mathrm{dI})\hat{\beta}_{\mathrm{ORR}} = \mathrm{F}_d\hat{\beta}_{\mathrm{OLSE}}, \quad (4)$$

where $\mathrm{F}_d = (\mathrm{Z}+\mathrm{I})^{-1}(\mathrm{Z}+\mathrm{dI})$,

$$\hat{\beta}_{\mathrm{AURE}}(\mathrm{k}) = [\mathrm{I} - k^2(\mathrm{Z}+\mathrm{k})^{-2}]\hat{\beta}_{\mathrm{OLSE}} = \mathrm{A}_k\hat{\beta}_{\mathrm{OLSE}}, \quad (5)$$

where $\mathrm{A}_k = [\mathrm{I} - k^2(\mathrm{Z}+\mathrm{k})^{-2}]$.

This paper is divided into four sections: Section 2 discusses some of the characteristics of the proposed estimator . Section 3 compares the performance of the two estimators theoretically. Finally section 4 gives, a numerical example and some conclusions have been given in Section 5.

2. The new estimators and its properties.

Crouse et al. [5] presented the unbiased ridge estimator (URR) based on the ridge estimator and prior information *J*, which is defined as follows:

$$\hat{\beta}_{URR} = (\mathrm{Z}+\mathrm{kI})^{-1}(\mathrm{X}'\mathrm{y} + \mathrm{k}J), \quad (6)$$

with J being uncorrelated with $\hat{\beta}_{\mathrm{OLSE}}$ and $J \sim \mathrm{N}(\beta, V)$, and in (9) $V = \left(\frac{\sigma^2}{k}\right)I$. They showed that URR estimator is unbiased estimator and its always better than OLS estimator.

Since URR is better than OLS estimator, we propose new three modified unbiased estimators as a generalized form depending on the estimators in (3) to (5) and we call the Modified Unbiased Ordinary Ridge Estimator (MUORE), the Modified Unbiased Ordinary Liu Estimator (MUOLE) and the Modified Unbiased Almost Unbiased Ridge Estimator (MUAURE). We can write them in the following generalized form to be easy to find the statistical properties:

$$\hat{\beta}_{\mathrm{G}} = A_i\hat{\beta}_{\mathrm{URR}}, \quad (7)$$

where (A_i) is a positive definite matrix , $i = 1,2,3$ and $(A_1 = \mathrm{W},\ A_2 = \mathrm{F}_d,\ A_3 = \mathrm{A}_k)$. The bias vector, dispersion matrix and MSE matrix of $\hat{\beta}_{\mathrm{G}}$ are given as :

$$\mathrm{E}(\hat{\beta}_{\mathrm{GMUE}}) = (A_i\hat{\beta}_{\mathrm{URR}}) = A_i\, E(\hat{\beta}_{\mathrm{URR}}) = A_i\beta, \quad (8)$$

$$\mathrm{Bais}(\hat{\beta}_{\mathrm{GMUE}}) = \mathrm{E}(\hat{\beta}_{\mathrm{GMUE}}) - \beta = A_i\beta - \beta = (A_i - I)\beta$$

The covariance matrix for any estimator β^* for β is defined as follows:

$$\mathrm{Cov}(\hat{\beta}) = \mathrm{E}(\beta^* - \beta)(\beta^* - \beta)'.$$

Consequently,

$$\mathrm{Cov}(\hat{\beta}_{\mathrm{G}}) = \sigma^2 A_i\, Z_k^{-1} A_i'. \quad (9)$$

In the context of biased estimation, the best criterion to assess an estimator's performance is the mean squared error (MSE) matrix, since it can simultaneously calculate both the variance-covariance matrix and the biased vector with one formula.

$$\mathrm{MSE}(\hat{\beta}) = \mathrm{Cov}(\hat{\beta}) + \mathrm{Bias}(\hat{\beta})\mathrm{Bias}(\hat{\beta})',$$

The definition of the scalar mean square error (mse) is:

$$\mathrm{mse}(\hat{\beta}) = tr\left(\mathrm{MES}(\hat{\beta})\right),$$

where *tr* is the trace that defined to be the sum of the main diagonal of matrix. So, the MSE of $\hat{\beta}_{\mathrm{G}}$ is given by:

$$\mathrm{MES}(\hat{\beta}_{\mathrm{G}}) = \sigma^2 A_i\, Z_k^{-1} A_i' + (A_i - I)\beta\beta'(A_i - I)' \quad (10)$$

The properties of ORR are defined as:

$b_2 = \mathrm{Bias}(\hat{\beta}_{\mathrm{ORR}}) = -k(Z+kI)^{-1}\beta$ and $\mathrm{Cov}(\hat{\beta}_{\mathrm{ORR}}) = \sigma^2 Z(Z+kI)^{-2}$

MSE($\hat{\beta}_{\mathrm{ORR}}) = \sigma^2 Z(Z+kI)^{-2} + k^2(Z+k)^{-1}\beta\beta'(Z+k)^{-1}$ (11)

The MSE of URR is defined as,

MSE($\hat{\alpha}_{URR}) = \sigma^2 Z_k^{-1}$ (12)

The properties of LU are defined as:

$b_3 = $ Bias $(\hat{\beta}_{\mathrm{LU}}) = (\mathrm{F}_d - I)\beta$ and Cov $(\hat{\beta}_{\mathrm{LU}}) = \sigma^2 \mathrm{F}_d Z^{-1} \mathrm{F}_d{}'$

MSE($\hat{\beta}_{\mathrm{LU}}) = \sigma^2 \mathrm{F}_d Z^{-1} \mathrm{F}_d{}' + (\mathrm{F_d} - \mathrm{I})\beta\beta'(\mathrm{F_d} - \mathrm{I})'$ (13)

The properties of AURE are defined as,

$b_4 =$ Bias $(\hat{\beta}_{\mathrm{AURE}}) = \mathrm{I} - k^2(\mathrm{S}+\mathrm{k})^{-2}\beta$ and Cov $(\hat{\beta}_{\mathrm{AURE}}) = \sigma^2 \mathrm{A}_K Z^{-1} \mathrm{A}_K{}'$

MSE($\hat{\beta}_{\mathrm{AURE}}) = \sigma^2 \mathrm{A}_K Z^{-1} \mathrm{A}_K{}' + k^4(S+kI)^{-2}\beta\beta'(S+kI)^{-2}$ (14)

Therefore, The MSE of $\hat{\beta}_{\mathrm{MUORE}}, \hat{\beta}_{\mathrm{MUOLE}}, \hat{\beta}_{\mathrm{MUAURE}}$ are given by

$\mathrm{MES}(\hat{\beta}_{\mathrm{MUORE}}) = \sigma^2 A_1 Z_k^{-1} A_1{}' + (A_1 - I)\beta\,\beta'\,(A_1 - I)'$ (15)

$\mathrm{MES}(\hat{\beta}_{\mathrm{MUOLE}}) = \sigma^2 A_2\, Z_k\, A_2{}' + (A_2 - I)\beta\,\beta'\,(A_2 - I)'$ (16)

$\mathrm{MES}(\hat{\beta}_{\mathrm{MUAURE}}) = \sigma^2 A_3\, Z_k^{-1}\, A_3{}' + (A_3 - I)\beta\,\beta'\,(A_3 - I)'$ (17)

3. Comparison of Estimators

We need to provide some lemmas that will help us do and prove our theoretical results. We also take into account the new estimators.

Lemma 3.1 Let $n \times n$ matrices A > 0 and B > 0 (or B > 0) the A is a positive definite (p.d) , then as B is a (p.d) the A > B if and only if $\lambda_{max}(BA^{-1}) < 1$,where $\lambda_{max}(BA^{-1}) < 1$ is the maximum eigenvalue for the matrix BA^{-1}.[6]

Lemma 3.2 Let $\hat{\beta}_i = A_i\, y$ i=1,2 be the given two linear estimators of β. Suppose that $D_1 = $ Cov($\hat{\beta}_1) -$ Cov($\hat{\beta}_2)$ is p.d.,where Cov($\hat{\beta}_i$) i=1,2 is the covariance matrix of $\hat{\beta}_i$ and $b_i = $ Bias($\hat{\beta}_i) = (A_i X - I)\alpha$, i=1,2 consequently

$\Delta =$ MSE($\hat{\beta}_1) -$ MSE($\hat{\beta}_2) = \sigma^2 D_1 + b_1' b_1 - b_2' b_2$

is p.d. iff $b_2'\,(\sigma^2 D_1 + b_1' b_1) b_2 < 1$, where $\mathrm{MSE}(\hat{\beta}_i) = \mathrm{Cov}(\hat{\beta}_i) + b_i' b_i$. [7]

Lemma 3.3 Let E a (p.d) , matrix and F a (n.n.d) matrix F-E > 0 if only if $F^{-1} - E^{-1} > 0$. [8]

3.1 The Comparison with URR Estimator

Let $\Delta_1 = \mathrm{MSE}(\hat{\beta}_{\mathrm{URR}}) - \mathrm{MSE}(\hat{\beta}_{\mathrm{G}}) = \sigma^2 D_1 + b_1' b_1$, where $D_1 = \mathrm{Cov}(\hat{\beta}_{\mathrm{URR}}) - \mathrm{Cov}(\hat{\beta}_{\mathrm{G}}) = Z_k^{-1} - A_i Z_k^{-1} A_i{}'$ and, $b_1 = (A_i - I)\beta$. We can observe that $D_1 > 0$ if $\mathrm{diag}\left\{\frac{1}{\lambda_i+k} - \frac{a_i^2}{\lambda_i+k}\right\} > 0$, where a_i is the diagonal elements of A_i. Since $a_i < 1$ for all i , then $\frac{1}{\lambda_i+k} - \frac{a_i^2}{\lambda_i+k} > 0$. Therefore, we can state the following theorem:

Theorem 3.1 The estimator $\hat{\beta}_{\mathrm{G}}$ is always better than the estimator $\hat{\beta}_{\mathrm{URR}}$ in the MSE sense.

The above theorem demonstrates that all proposed estimators are better than URR and according to that, the proposed estimators will obviously better than OLS estimator.

3.2 The Comparison Between MUORE and ORR Estimators

Let $\Delta_2 = \mathrm{MSE}(\hat{\beta}_{\mathrm{ORR}}) - \mathrm{MSE}(\hat{\beta}_{\mathrm{MUORE}}) = \sigma^2 D_2 + b_2' b_2 - b_5' b_5$, where

$D_2 = $ Cov($\hat{\beta}_{\mathrm{ORR}}) -$ Cov($\hat{\beta}_{\mathrm{MUORE}}) = WZ^{-1}W - WZ_k^{-1}W = W(Z^{-1} - Z_k^{-1})W$,

and $b_5 = (A_1 - I)\beta$. So, $D_2 > 0$ if $\mathrm{diag}\left\{\frac{1}{\lambda_i} - \frac{1}{\lambda_i+k}\right\} > 0$ and that is satisfied because $(\lambda_i + k) > \lambda_i$ for all i. As a result, we can now state following theorem after applying Lemma 3.2.

Theorem 3.2.The MUORE estimator is better than the ORR estimator in the MSE sense, if and only if $b_5' \, (\sigma^2 D_2 + b_2' b_2) b_5 < 1$.

3.3 The Comparison Between MUOLE and LU Estimators

Since the MUOLE and LU estimators

$$\Delta_3 = \mathrm{MSE}\left(\hat{\beta}_{\mathrm{LU}}\right) - \mathrm{MSE}\left(\hat{\beta}_{\mathrm{MUOLE}}\right)$$

$$= \sigma^2 Z^{-1}(Z+I)^{-2}(Z+dI)^2 - [\sigma^2(A_2 Z_k^{-1} A_2{}' + b_3' b_3 - b_6' b_6]$$

$$= \sigma^2[\mathrm{G}_2 - \mathrm{G}_1] + b_3' b_3 - b_6' b_6.$$

where $G_2 = \mathrm{Cov}\left(\hat{\beta}_{\mathrm{LU}}\right)$, and $G_1 = \mathrm{Cov}\left(\hat{\beta}_{\mathrm{MUOLE}}\right)$, $b_6 = (A_2 - I)\,\beta$. Thus, the following theorem can be stated

Theorem 3.3 Let $\lambda_i^{\mathrm{G}_2}(\mathrm{G}_1) \le 1$. Therefore, the MUOLE estimator is superior to the LU in the sense of MSE $\Leftrightarrow$ $b_6'(\sigma^2[\mathrm{G}_2 - \mathrm{G}_1] + b_3 b_3')^{-1} b_6 \le 1$.

Proof : Since $\mathrm{G}_1 > 0$ and $\mathrm{G}_2 > 0$, we can get $\mathrm{G}_2 - \mathrm{G}_1 \ge 0 \Leftrightarrow \lambda_i^{\mathrm{G}_2}(\mathrm{G}_1) \le 1$ using Lemma 3.1. Since $b_3' b_3 \ge 0$, deciding whether to take action $\Delta_3 > 0$ is reduced to that of deciding $\sigma^2(\mathrm{G}_2 - \mathrm{G}_1) + b_3' b_3 - b_6' b_6 > 0$, then $\Delta_3 > 0$, the proof is completed after using Lemma 3.2.

3.4 Comparison between MUAURE and AURE Estimators

The MSE difference the MUAURE and AURE estimators is as follows

$\Delta_4 = \mathrm{MSE}(\hat{\beta}_{\mathrm{AURE}}) - \mathrm{MSE}(\hat{\beta}_{\mathrm{MUAURE}}) = \sigma^2 D_4 + b_4 b_4' - b_7 b_7'$,where $D_4 = Cov(\hat{\beta}_{\mathrm{AURE}}) - \mathrm{Cov}(\hat{\beta}_{\mathrm{MUAURE}}) = M - N$, where $\mathrm{M} = A_k \; Z^{-1} A_k{}'$, $\mathrm{N} = A_3 Z_k^{-1} A_3{}'$, $\lambda_{max}(\mathrm{M}N^{-1}) < 1$ is the largest eigenvalue of the matrix $\mathrm{M}N^{-1}$ and $b_7 = (A_k - I)\,\beta$. Now , the following theorem can be stated .

Theorem 3.4. When $\lambda_{max}(\mathrm{M}N^{-1}) < 1$, the estimator $\hat{\beta}_{\mathrm{MUAURE}}$ is superior to the estimator $\hat{\beta}_{\mathrm{AURE}}$ in the mean squared error matrix sense if and only if

$$b_7'(\sigma^2 D_4 + b_4 b_4')^{-1} b_7 \le 1$$

proof : The M and N are (p . d) and based on (Lemma 3.3) one can say that $\mathrm{M} - N > 0$ if and only if $\lambda_{max}(\mathrm{M}N^{-1}) < 1$. Now , according to (Lemma 3.2), $\mathrm{MSE}(\hat{\beta}_{\mathrm{AULE}}) - \mathrm{MSE}(\hat{\beta}_{\mathrm{MUAURE}}) \ge 0$ if and only if,

$$b_7'(\sigma^2 D_4 + b_5 b_5')^{-1} b_7 \le 1$$

So, the proof is completed .

4.Numerical Example

In order to go for further illustrating for the behavior of the proposed estimators, we consider the data set on Portland cement originally due to [9]. This data set has been used by many researchers such as [10]-[13]. The cement data set came from an experimental investigation of the heat evolved during the setting and hardening of Portland cements of varied composition and the dependence of this heat on the percentages of four compounds in the clinkers from which the cement was produced. The data set compounds considered by [9]. Since β and σ^2 are unknown, we estimated β by OLS estimator for estimators in Eq's (11,13,14) as well as we estimated it by URR estimator in Eq's (15-17). For σ^2 we take several values to have a clear perception of the performance of the estimators and the extent of the impact of that value on the performance of these estimators, small, medium and high values were taken as (0.05,0.1,0.9,5). Also different values of k and d are taken as showing the following tables. We want to mention that all estimated mse values in following tables are multiplied by 10^{-2} in order to be clear for comparing.

Table 1: The mse for different estimators with proposed estimators when σ^2=0.05

σ^2	d=0.01						
	k	**0.01**	**0.1**	**0.5**	**0.9**	**1**	**1.5**
0.05	**OLS**	0.33708	0.33708	0.33708	0.33708	0.33708	0.33708
	URR	0.046809	0.007315	**0.002192**	**0.001402**	**0.001293**	**0.000942**
	ORR	0.009981	0.00335	0.002709	0.002869	0.002922	0.003201
	LU	**0.002923**	**0.002923**	0.002923	0.002923	0.002923	0.002923
	AURE	0.024145	0.004477	0.00289	0.002695	0.002682	0.002694
	MUORE	0.005333	0.00342	0.002863	0.002869	0.002874	0.002889
	MUOLE	0.003403	0.003286	0.002982	0.002847	0.002822	0.002731
	MUAURE	0.006902	0.003638	0.002552	0.002483	0.002481	0.002497
	d=0.5						
	OLS	0.33708	0.33708	0.33708	0.33708	0.33708	0.33708
	URR	0.046809	0.007315	0.002192	0.001402	0.001293	**0.000942**
	ORR	0.009981	0.00335	0.002709	0.002869	0.002922	0.003201
	LU	0.085647	0.085647	0.085647	0.085647	0.085647	0.085647
	AURE	0.024145	0.004477	0.00289	0.002695	0.002682	0.002694
	MUORE	**0.005333**	0.00342	0.002863	0.002869	0.002874	0.002889
	MUOLE	0.012981	**0.002975**	**0.001503**	**0.001229**	**0.001189**	0.00105
	MUAURE	0.006902	0.003638	0.002552	0.002483	0.002481	0.002497
	d=0.9						
	OLS	0.33708	0.33708	0.33708	0.33708	0.33708	0.33708
	URR	0.046809	0.046809	0.002192	0.001402	0.001293	0.000942
	ORR	0.009981	0.009981	0.002709	0.002869	0.002922	0.003201
	LU	0.273296	0.273296	0.273296	0.273296	0.273296	0.273296
	AURE	0.024145	0.024145	0.00289	0.002695	0.002682	0.002694
	MUORE	**0.005333**	**0.005333**	0.002863	0.002869	0.002874	0.002889
	MUOLE	0.038099	0.038099	**0.001876**	**0.00122**	**0.001129**	**0.000834**
	MUAURE	0.006902	0.006902	0.002552	0.002483	0.002481	0.002497

Table 2: The mse for different estimators with proposed estimators when σ^2=0.1

σ^2	d=0.01						
	k	**0.01**	**0.1**	**0.5**	**0.9**	**1**	**1.5**
0.1	**OLS**	0.674159	0.674159	0.674159	0.674159	0.674159	0.674159
	URR	0.093617	0.014631	0.004383	**0.002804**	**0.002585**	**0.001883**
	ORR	0.018821	0.005106	0.00325	0.003194	0.003216	0.003396
	LU	**0.003271**	**0.003271**	0.003271	0.003271	0.003271	0.003271
	AURE	0.047429	0.007518	0.004006	0.003402	0.003333	0.00316
	MUORE	0.009098	0.004648	0.00318	0.003044	0.003028	0.00298
	MUOLE	0.003707	0.003551	0.00318	0.003011	0.00298	0.002863

	MUAURE	0.012793	0.005566	**0.003104**	0.002818	0.002784	0.002697
	d=0.5						
	OLS	0.674159	0.674159	0.674159	0.674159	0.674159	0.674159
	URR	0.093617	0.014631	0.004383	0.002804	0.002585	0.001883
	ORR	0.018821	0.005106	0.00325	0.003194	0.003216	0.003396
	LU	0.170637	0.170637	0.170637	0.170637	0.170637	0.170637
	AURE	0.047429	0.007518	0.004006	0.003402	0.003333	0.00316
	MUORE	**0.009098**	**0.004648**	0.00318	0.003044	0.003028	0.00298
	MUOLE	0.025172	0.00518	**0.002297**	**0.001775**	**0.001698**	**0.001438**
	MUAURE	0.012793	0.005566	0.003104	0.002818	0.002784	0.002697
	d=0.9						
	OLS	0.674159	0.674159	0.674159	0.674159	0.674159	0.674159
	URR	0.093617	0.014631	0.004383	0.002804	0.002585	0.001883
	ORR	0.018821	0.005106	0.00325	0.003194	0.003216	0.003396
	LU	0.546566	0.546566	0.546566	0.546566	0.546566	0.546566
	AURE	0.047429	0.007518	0.004006	0.003402	0.003333	0.00316
	MUORE	**0.009098**	**0.004648**	0.00318	0.003044	0.003028	0.00298
	MUOLE	0.076167	0.012111	0.003724	**0.002413**	**0.00223**	**0.00164**
	MUAURE	0.012793	0.005566	**0.003104**	0.002818	0.002784	0.002697

Table 3: The mse for different estimators with proposed estimators when σ^2=0.9

σ^2	**d=0.01**						
	k	**0.01**	**0.1**	**0.5**	**0.9**	**1**	**1.5**
0.9	**OLS**	6.067431	6.067431	6.067431	6.067431	6.067431	6.067431
	URR	0.842554	0.131676	0.039449	0.025236	0.023268	0.016947
	ORR	0.160257	0.033203	0.011904	0.008385	0.007921	0.006528
	LU	0.008831	0.008831	0.008831	0.008831	0.008831	0.008831
	AURE	0.419968	0.056175	0.021858	0.01472	0.013744	0.010609
	MUORE	0.069336	0.024296	0.00825	0.005842	0.005505	**0.00444**
	MUOLE	**0.008572**	**0.007798**	**0.006357**	**0.005639**	**0.005503**	0.004972
	MUAURE	0.107063	0.036408	0.011943	0.008171	0.007642	0.005891
	d=0.5						
	OLS	6.067431	6.067431	6.067431	6.067431	6.067431	6.067431
	URR	0.842554	0.131676	0.039449	0.025236	0.023268	0.016947
	ORR	0.160257	0.033203	0.011904	0.008385	0.007921	0.006528
	LU	1.530472	1.530472	1.530472	1.530472	1.530472	1.530472
	AURE	0.419968	0.056175	0.021858	0.01472	0.013744	0.010609
	MUORE	**0.069336**	**0.024296**	**0.00825**	0.005842	**0.005505**	**0.00444**
	MUOLE	0.220224	0.040459	0.01499	0.010498	0.00984	0.007636
	MUAURE	0.107063	0.036408	0.011943	**0.008171**	0.007642	0.005891
	d=0.9						
	OLS	6.067431	6.067431	6.067431	6.067431	6.067431	6.067431

	URR	0.842554	0.131676	0.039449	0.025236	0.023268	0.016947
	ORR	0.160257	0.033203	0.011904	0.008385	0.007921	0.006528
	LU	4.918887	4.918887	4.918887	4.918887	4.918887	4.918887
	AURE	0.419968	0.056175	0.021858	0.01472	0.013744	0.010609
	MUORE	**0.069336**	**0.024296**	**0.00825**	**0.005842**	**0.005505**	**0.00444**
	MUOLE	0.685251	0.108755	0.033292	0.021498	0.019855	0.014552
	MUAURE	0.107063	0.036408	0.011943	0.008171	0.007642	0.005891

Table 4: The mse for different estimators with proposed estimators when σ^2=5

σ^2	**d=0.01**						
	k	**0.01**	**0.1**	**0.5**	**0.9**	**1**	**1.5**
5	**OLS**	33.70795	33.70795	33.70795	33.70795	33.70795	33.70795
	URR	4.680854	0.731536	0.219159	0.140198	0.129266	0.094152
	ORR	0.885114	0.177195	0.056258	0.034993	0.032034	0.022574
	LU	0.037327	0.037327	0.037327	0.037327	0.037327	0.037327
	AURE	2.329232	0.305547	0.113351	0.072726	0.0671	0.048786
	MUORE	0.37806	0.124992	0.034234	0.020182	**0.018199**	**0.011922**
	MUOLE	**0.033504**	**0.029562**	**0.022637**	**0.019107**	0.018432	0.015779
	MUAURE	0.590197	0.194477	0.057239	0.035606	0.032537	0.022262
	d=0.5						
	OLS	33.70795	33.70795	33.70795	33.70795	33.70795	33.70795
	URR	4.680854	0.731536	0.219159	0.140198	0.129266	0.094152
	ORR	0.885114	0.177195	0.056258	0.034993	0.032034	0.022574
	LU	8.499629	8.499629	8.499629	8.499629	8.499629	8.499629
	AURE	2.329232	0.305547	0.113351	0.072726	0.0671	0.048786
	MUORE	**0.37806**	**0.124992**	**0.034234**	**0.020182**	**0.018199**	**0.011922**
	MUOLE	1.219865	0.221265	0.080046	0.055205	0.051569	0.039401
	MUAURE	0.590197	0.194477	0.057239	0.035606	0.032537	0.022262
	d=0.9						
	OLS	33.70795	33.70795	33.70795	33.70795	33.70795	33.70795
	URR	4.680854	0.731536	0.219159	0.140198	0.129266	0.094152
	ORR	0.885114	0.177195	0.056258	0.034993	0.032034	0.022574
	LU	27.32703	27.32703	27.32703	27.32703	27.32703	27.32703
	AURE	2.329232	0.305547	0.113351	0.072726	0.0671	0.048786
	MUORE	**0.37806**	**0.124992**	**0.034234**	**0.020182**	**0.018199**	**0.011922**
	MUOLE	3.806805	0.604055	0.184828	0.119308	0.110181	0.080725
	MUAURE	0.590197	0.194477	0.057239	0.035606	0.032537	0.022262

Through Table 1, when the value of d = 0.01 and for all values of k and σ^2 is 0.05 and 0.1, the performance of the proposed estimators was not good compared with the estimators within the limits of this paper. It can also be noted that the proposed estimators began to improve when the value of d was increased, as the MUOLE estimator was the best in most of the k values. Also, there is an obvious effect of the value of σ^2 on the performance of the proposed estimators, where when σ^2 is increased, the estimated mse of MUORE, MUOLE and MUAURE will be less. If we like to check the performance of the proposed estimators between the, the MUORE is best when d is closed to 1 and σ^2 has an high values and that can be observed for Tables 1-4.

5. Conclusions

In this paper, new biased estimators depending on unbiased ridge regression estimator (URR) in a multiple linear regression when there exists multicollinearity problem are proposed. These estimators are superior to other exists estimators which are based on sample information. Based on Tables 1-4, we can say that the proposed estimators has smallest mse values compared with OLS,URR,ORR,LU and AURE. We can also suggest that MUORE is the best estimator with compared to other proposed estimators.

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