New Bayesian Regularized for quantile regression

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Abstract

The statistical analysis provided by quantile regression of the links between random variables is more in-depth. The economic field has made considerable use of quantile regression techniques to study the factors that affect wages, the results of discrimination, and the development of income inequality. Using the Bayesian lasso penalty technique, this article estimates and selects variables in quantile regression models. The Laplace prior distribution of the vector of parameters will be represented by a scale mixture of normal distributions mixing Rayleigh density, and a Bayesian hierarchical model will be constructed to estimate its parameters. Simulation examples and real-world data are taken into account to assess the suggested method's effectiveness and to compare it to other current approaches.

Keywords: *Bayesian Inference, prior distribution, quantile regression, Lasso, MCMC.*

1. INTRODUCTION

Quantile regression is a statistical method for estimating and inferring conditional quantile functions. Conditional mean function models may be estimated in the same way as ordinary linear regression methods can, by minimizing sums of squared residuals. Using quantile regression methods, we can estimate models for the conditional median function and the whole range of conditional quantile functions. The estimate of conditional mean functions is supplemented by methods for estimating a whole family of conditional quantile functions. The statistical analysis provided by quantile regression of the links between random variables is more in-depth.

Quantile regression has found use in many different fields of study. Upper and lower quantile reference curves as a function of age, sex, and other covariates can be estimated using quantile regression methods, which have a long history of use in pediatric medicine and allow for the estimation of reference growth curves without imposing strict parametric assumptions. The economic field has made considerable use of quantile regression techniques to study the factors that affect wages, the results of discrimination, and the development of income inequality.

To estimate conditional quantile functions, Koenker and Basset (1978) presented quantile regression as a generalization of conventional least squares estimation of conditional mean models. Due to its ability to estimate the whole conditional distribution of a response variable, quantile regression is a powerful statistical study that can find more effects than traditional approaches. One of the most important parts of the process is determining how accurate the model is by estimating its parameters. The two most important techniques are the momentum (M) approach and the method of least squares (OLS). estimate based on the principle of maximum likelihood.

There has been a lot of focus on regularization-based subset selection recently. For example, for the lasso (least absolute shrinkage and selection operator), see Tibshirani (1996) and Tibshirani et al. (2005). Bayesian variable selection (Fridley, 2009) is a flexible method for picking variables in light of prior information. Different variable-selection strategies are used within a Bayesian framework. In addition, this research contributed to the Bayesian lasso penalty method for estimating and selecting variables in the quantile regression model. To estimate the parameters, we will construct a Bayesian hierarchical model based on a representation of the scale mixture of normal distribution mixing Rayleigh density for the Laplace prior distribution of the parameters vector.

2. Bayesian variable selection quantile Regression

One of the most important parts of building a model is deciding which variables to use. It is normal in practice to have a high number of prospective predictor variables, and these variables are often included into the first modeling stage to eliminate the possibility of modeling bias (Fan and Li, 2001). However, including extraneous predictors in the final model might lower the model's prediction effectiveness and make it harder to comprehend. In order to achieve variable selection, the regularization framework makes use of a wide variety of penalties. Tibshirani (1996) introduced the LASSO variable selection method, which makes use of the La penalty. Nonconcave penalized least squares regression, as suggested by Fan and Li (2001), combines the processes of variable selection and coefficient estimation into a single step. This technique creates sparse solutions, guarantees the stability of model selection, and produces unbiased estimates for high coefficients by picking an appropriate nonconcave penalty function, thereby retaining many of the advantages of the best subset selection and ridge regression. According to Fan and Li (2001), these are the three most important characteristics of a deterrent.

Based on the asymmetric Laplace likelihood (ALL), which has a unique relationship with the frequentist quantile regression solution (Koenker, 2005), the Bayesian quantile regression method has become increasingly popular since Yu and Moyoed (2001) first introduced it. When using priors other than non-informative or exponential Laplace, ALL-based methods generate incorrect posteriors; the latter leads to the well-known Bayesian Lasso quantile regression (Li et al., 2010; Alhamzawi and Yu, 2013; Alhamzawi et al., 2012; Chen et al., 2013).

$$y_i = x_i' \beta_\tau + u_i, \quad i = 1,2,\dots,n \dots\dots..(1)$$

u_i is random variables from skewed Laplace distribution,

$$f(\theta) = \tau(1-\tau)\theta exp\{-\theta\rho_\tau(u_i)\} \dots\dots.(2)$$

Then the likelihood function of y =(y_1,…, y_n) given $x = (x_1,\dots,x_n)$’ is

$$f(x, \beta_\tau, \theta) = \tau^n (1 - \tau)^n \theta^n exp\left\{-\theta \sum_{i=1}^{n} \rho_\tau (y_i - x_i' \beta_\tau)\right\} \dots . (3)$$

According to Kozumi and Kobayashi (2009), the Skew Laplace distribution may be seen as a combination of an exponential and scaled normal distribution. Then, we can rewrite the error term as $u = \delta_1 v + \delta_2 \sqrt{v}\ z$, whereas v is standard exponential random variable and z is a standard and

$$\delta_1 = \frac{1-2\tau}{\tau(1-\tau)} \text{ , and } \quad \delta_2 = \sqrt{\frac{2}{\tau(1-\tau)}}$$

Therefore the model can be written:

$$y_i = x_i' \beta_\tau + \theta^{-1} \delta_1 v_i + \theta^{-1} \delta_2 \sqrt{v_i}\ z_i \ \dots \dots \dots (4)$$

follow Li et al. (2010), Let $\tilde{v}_i = \theta^{-1} v_i$, then $\tilde{v}_i$ distributed as exponential $exp\ (\theta^{-1})$.

Lasso quantile regression model can be shown as :

$$\sum_{i=1}^{n} \rho_\tau (y_i - x_i' \beta_\tau) + \lambda \sum_{k=1}^{p} |\beta_{\tau k}| \quad \dots \dots \dots . (5)$$

Laplace distribution set as a prior distribution for the parameters β_τ.

$$\pi(\theta, \lambda) = (\frac{\theta\lambda}{2})^p exp\left\{-\theta\lambda \sum_{k=1}^{p} |\beta_{\tau k}|\right\} \dots \dots \dots . . (6)$$

new hierarchical model representation for Lasso quantile based on (Flaih et al. (2020)),

Scale mixture of normal distribution and Rayleigh distribution

$$\frac{1}{2a} exp\left\{-\frac{|z|}{a}\right\} = \int_0^\infty \frac{1}{\sqrt{2\pi s^2}} exp\left\{-\frac{z^2}{2s^2}\right\} . \ \frac{s}{a}\ exp\left\{-\frac{s^2}{2a}\right\}\ ds$$

Let $\eta = \theta\lambda, a = \frac{1}{\eta}$, then we can rewrite the Laplace distribution:

$$\frac{\eta}{2} exp\{-\eta |\beta_\tau|\} = \int_0^\infty \frac{1}{\sqrt{2\pi s_k^2}} exp\left\{-\frac{\beta_\tau^2}{2s_k^2}\right\} . \ \eta s\ exp\left\{-\frac{\eta s^2}{2}\right\}\ ds$$

3. Hierarchical model and MCMC sampler

Based on what is mentioned above in section 2, the hierarchical model for the new Bayesian variable selection for quantile regression can be shown as follows:

$$y_i = x'_i \beta + \delta_1 \tilde{v}_i + \theta^{-1/2} \delta_2 \sqrt{\tilde{v}_i} z_i$$

$$\tilde{v}_i / \theta \sim \prod_{i=1}^{n} \theta exp\ \{-\theta \tilde{v}_i\}$$

$$z_i \sim \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} exp\ \left\{-\frac{1}{2} z_i^2\right\}$$

$$\beta, s/\eta \sim \prod_{k=1}^{p} \frac{1}{\sqrt{2\pi s_k^2}} e^{-\beta^2/2s^2} \cdot \eta s e^{-\eta s^2/2} ds$$

$$\theta \sim \theta^{a-1} exp\ \{-b\theta)$$

$$\eta \sim (\eta)^{c-1} exp\ \{-d\eta\}$$

For all parameters the fall condition posterior will be given as:

$$\pi(x,v,\tilde{}\beta_\tau,\theta,s,\eta|y) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta^{-1}\delta_2^2 \tilde{v}_i}} exp\left\{-\frac{(y_i - x_i'\beta_\tau - \delta_1 \tilde{v} i)^2}{2\theta^{-1}\delta_2^2 \tilde{v}_i}\right\} \times \prod_{i=1}^{n} \theta ex\{-\theta \tilde{v}_i\} \times \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} exp\ exp\left\{-\frac{1}{2}z_i^2\right\} \times \prod_{k=1}^{p} \frac{1}{\sqrt{2\pi s_k^2}} e^{-\frac{\beta_\tau^2}{2s^2}} \cdot \eta s e^{-\frac{\eta s^2}{2}} ds \times \theta^{a-1} exp\ exp\{-b\theta) \times (\eta)^{c-1} exp\{-d\eta\}$$

Where a,b,c and d are hyperparameters.

Sampling $\tilde{v}_i$

The conditional distribution

$$\pi(\tilde{v}_i/x,\beta_\tau,\theta,s,\eta,\tilde{v}_{-i}) \propto f(y/x,\tilde{v}_i,\beta_\tau,\theta,s,\eta) \cdot \pi(\tilde{v}_i,\theta)$$

$$\propto \frac{1}{\sqrt{\tilde{v}_i}} exp\left\{-\frac{(y_i - x'\beta_\tau - \delta_1 \tilde{u}_i)^2}{2\theta^{-1}\delta_2^2 \tilde{v}_i}\right\} \cdot exp\{-\theta \tilde{v}_i\}$$

$$\propto \frac{1}{\sqrt{\tilde{v}_i}} exp\left\{-\frac{1}{2}\left[\left(\frac{\theta\delta_1^2}{\delta_2^2} - 2\theta\right)\tilde{v}_i + \frac{\theta(y_i - x'\beta)^2}{\delta_2^2}\tilde{v}_i^{-1}\right]\right.$$

Generalized inverse Gaussian is the fall conditional distribution of $\tilde{v}_i$

Sampling $\beta_{\tau k}$

The posterior distribution is

$$\pi(\beta_j/x,\theta,\beta_{-j},s,\eta_{,},\tilde{v}_i) \propto f(y/x,\tilde{v},\beta_\tau,s,\theta,\eta) \cdot \pi(\beta,s)$$

$$\propto exp\left\{-\sum \frac{(yi - x'\beta_\tau - \delta_1 \tilde{v}_i)^2}{2\theta^{-1}\delta_2 \tilde{v}_i}\right\} exp\left\{-\frac{\beta_{\tau j}^2}{2s_j}\right\}$$

The full conditional distribution for $\beta_\tau \sim N(\mu,\sigma^2)$ where

$$\sigma_j^2 = \sum \frac{x_i^2}{\theta\delta_2^2 v_i} + \frac{1}{s_j}, \mu_j = \sigma_j^2 \sum_{j=1}^{n} \frac{\tilde{y}_{ij} x_{ij} \tilde{u}_i^{-1}}{\theta\delta_2^2}$$

$$\tilde{y}_{ij} = y_i - \delta_1 \tilde{v}_i - \sum_{j=1}^{p} x_i \beta j$$

- **sampling** η

$$\pi(\eta/x,\beta,\theta,\tilde{v}_i,s) \alpha \pi(s/\eta) \cdot \pi(\eta)$$

$$\propto \prod_{i=1}^{n} \eta s\, e^{-\eta s^2/2} \cdot \eta^{c-1} exp\,(-d\eta)$$

$$\eta^{n+c-1} exp\left\{\left(\sum \frac{s_j^2}{2} - d\right)\eta\right\}$$

Gamma distribution is the Conditioned posterior for η.

- **Sampling** S_k

$$\pi(s_j/x,\beta,\theta,s_{-j},\eta) \alpha \pi(\beta/s_j) \pi(s_j/\eta)$$

$$\propto \frac{1}{\sqrt{s_j}} exp\left\{-\frac{-\beta^2}{2\,s_j^2}\right\} \cdot\ s_j\ exp\left\{-\frac{\eta s_j^2}{2}\right\}$$

$$\propto \sqrt{s_j}\, exp\left\{-\left[\frac{\beta^2}{2s_j^2} + \frac{\eta s_j^2}{2}\right]\right\}$$

The posterior distribution fir S ar GIG $\left(\frac{3}{2},\frac{\beta^2}{2},\frac{n}{2}\right)$

Sampling θ

$$\pi(\theta/x,\beta,\tilde{v},s,\eta) \propto \pi(y/x,\theta,\beta,s,\tilde{v},\eta)\cdot\pi(\tilde{v}/\theta)\cdot\pi(\theta)$$
$$\propto \quad \theta^{n/2} exp\left\{-\frac{1}{2}\sum \quad \frac{(y_i - x_i'\beta - \delta_1 \tilde{v}_i)^2}{\theta^{-1}\delta_2^2\tilde{v}_i}\right\}\theta^n exp\left\{\theta\sum \quad \tilde{v}_i\right\}$$
$$\cdot\,\theta^{a-1}exp\{-b\theta\}$$
$$\propto \quad \theta^{3n/_2+a-1} exp\,\{-\left[\sum \quad \frac{(y_i - x_i'\beta - \delta_1 \tilde{v}_i)^2}{2\delta_2^2\tilde{v}_i} + \sum \quad \tilde{v}_i + b\right]\theta]\}$$

The posterior distribution of θ is gamma distribution.

Our proposed posterior distributions offer efficient algorithms that can provide us with precise estimates. In simulation and real-world dataset scenarios, the proposed method algorithm was executed 13,000 times, with the initial 3000 iterations omitted as burn-in.

4. Simulation examples:

The effectiveness of the offered approaches is examined here using simulated situations. This model was used to evaluate the suggested strategy against both Bayesian and non-Bayesian alternatives. The alternative to Bayesian analysis is a conventional quantile regression model implemented in the R programming environment through the rq function in the package quantreg. Bayesian quantile regression is the R package MCMCpack's MCMCquantreg function that implements the Bayesian approach. In the present investigation, we evaluate the effects of using quantiles of =0.15, =0.35, =0.55, =0.75, and =0.95. Using root mean square error (MSE) and mean absolute deviations (MAD), we compare four different distributions for the random error terms i used in our simulation examples: the normal distribution with mean 0 and variance 1 (iN(0,1)), the normal distribution with mean 3 and variance 4 (iN(3,4)), a mixture of normal distributions (i0.5N1,1+0.5N2,1), and the t4 distribution with 4 degrees of freedom it(4).

4.1 Simulation First Example

In this simulation example, we focus on very sparse model, that take the following formula:

$$y_i = 5x_{i1} + \epsilon_i$$

Where $i = 1,2,3 \ldots\ldots n$ $\quad ,n = 100$, with true parameters $\beta = (5,0,0,0,0,0,0,0)^t$.

We generate eight explanatory variables from multivariate normal distribution with mean ($\underline{0}$) and variance- covariance $(\Sigma_x)_{ij} = \left(\frac{1}{2}\right)^{|i+j|}$.

Table 1: Mean square error (MSE) for the first simulation example. mean absolute deviation (MAD) displayed in parentheses. The results are averaged over 100 independent simulations.

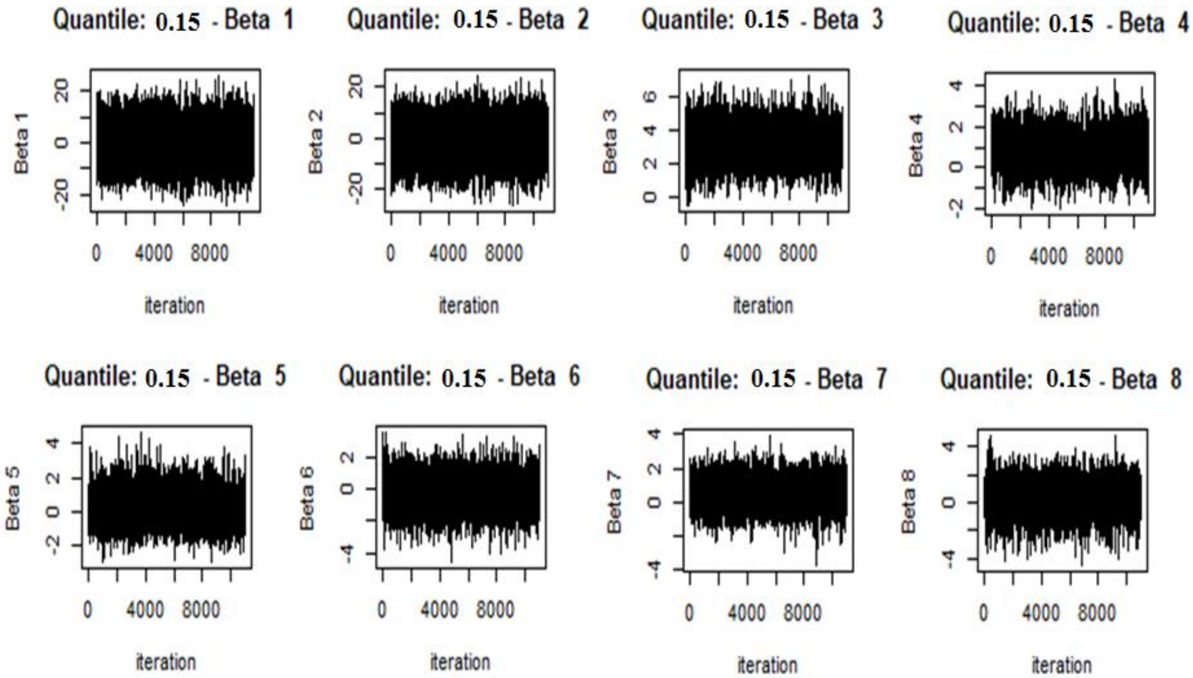
Methods	$\varepsilon_i \sim N(0,1)$	$\epsilon_i \sim N(3,4)$	$\epsilon_i \sim 0.5N(1,1)$ $+0.5N(2,1)$	$\varepsilon_i \sim t_{(5)}$
$rq_{0.15}$	1.137 (0.879)	1.262 (0.953)	1.122 (0.845)	1.013 (0.828)

$MCMCpack_{0.15}$	0.942 (0.507)	0.844 (0.264)	0.782 (0.386)	0.921 (0.562)
$NBRQReg_{0.15}$	**0.727 (0.256)**	**0.572 (0.183)**	**0.581 (0.117)**	**0.513 (0.105)**
$rq_{0.35}$	1.231 (0.925)	1.102 (0.781)	1.022 (0.791)	0.943 (0.905)
$MCMCpack_{0.35}$	0.826 (0.461)	0.782 (0.351)	0.891 (0.461)	0.651 (0.361)
$NBRQReg_{0.35}$	**0.681 (0.176)**	**0.573 (0.102)**	**0.517 (0.281)**	**0.414 (0.132)**
$rq_{0.55}$	1.453 (0.952)	1.264 (0.934)	1.344 (0.862)	0.709 (0.361)
$MCMCpack_{0.55}$	0.936 (0.572)	0.838 (0.573)	0.854 (0.475)	0.816 (0.672)
$NBRQReg_{0.55}$	**0.682 (0.254)**	**0.482 (0.184)**	**0.491 (0.262)**	**0.411 (0.156)**
rq_{075}	1.274 (0.892)	1.107 (0.738)	1.074 (0.693)	1.341 (0.851)
$MCMCpack_{0.75}$	0.681 (0.186)	0.764 (0.176)	0.727 (0.375)	0.917 (0.534)
$NBRQReg_{0.75}$	**0.439 (0.101)**	**0.381 (0.071)**	**0.363 (0.228)**	**0.331 (0.191)**
$rq_{0.95}$	1.064 (0.761)	1.096 (0.734)	1.124 (0.892)	1.122 (0.845)
$MCMCpack_{0.95}$	0.739 (0.268)	0.679 (0.361)	0.617 (0.316)	0.782 (0.486)
$NBRQReg_{0.95}$	**0.563 (0.112)**	**0.428 (0.203)**	**0.406 (0.174)**	**0.381 (0.217)**

Note: In the parentheses are MAD.

Table (1) shows that the suggested technique (NBRQReg) outperforms Bayesian and non-Bayesian alternatives across all error distributions and quantile levels studied. In contrast, both the mean square error (MSE) and the mean absolute deviation (MAD) obtained by the suggested technique (NBRQReg) are much lower than those generated by the other two methods, rq and MCMCpack. We will build a trace map for the model parameters at a low quantile level (=0.15) to learn more about the consistency of our technique.

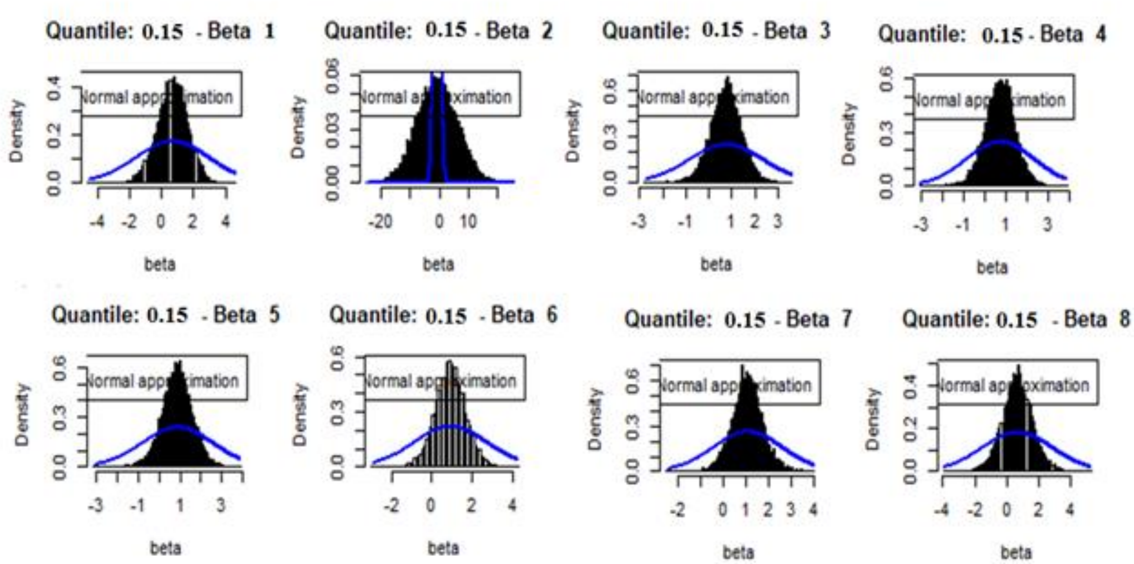
Figure 1- show trace plot of first simulation example at quantile level $\theta = 0.15$.



From Figure (1), we readily observed that the algorithm belonging to our proposed method is very stationary through 10000 iterations.

In figure 2- we see closed the posterior parameters estimate from the normal distribution:

Figure 2. Histograms based on posterior distribution of the parameters for Simulation 1 at quantile level $\theta = 0.15$



4.2 Simulation second Example

In this simulation example, we focus on dense model, that take the following formula:

$$y_i = 0.85x_{i1} + 0.85x_{i2} + 0.85x_{i3} + 0.85x_{i4} + 0.85x_{i5} + 0.85x_{i6} + 0.85x_{i7} + 0.85x_{i8} + \epsilon_i$$

Where $i = 1,2,3 \ldots\ldots n$, $n = 100$, with true parameters $\beta = (5,0,0,0,0,0,0,0)^t$

We generate eight explanatory variables from multivariate normal distribution with mean ($\underline{0}$) and variance- covariance $(\Sigma_x)_{ij} = \left(\frac{1}{2}\right)^{|i+j|}$

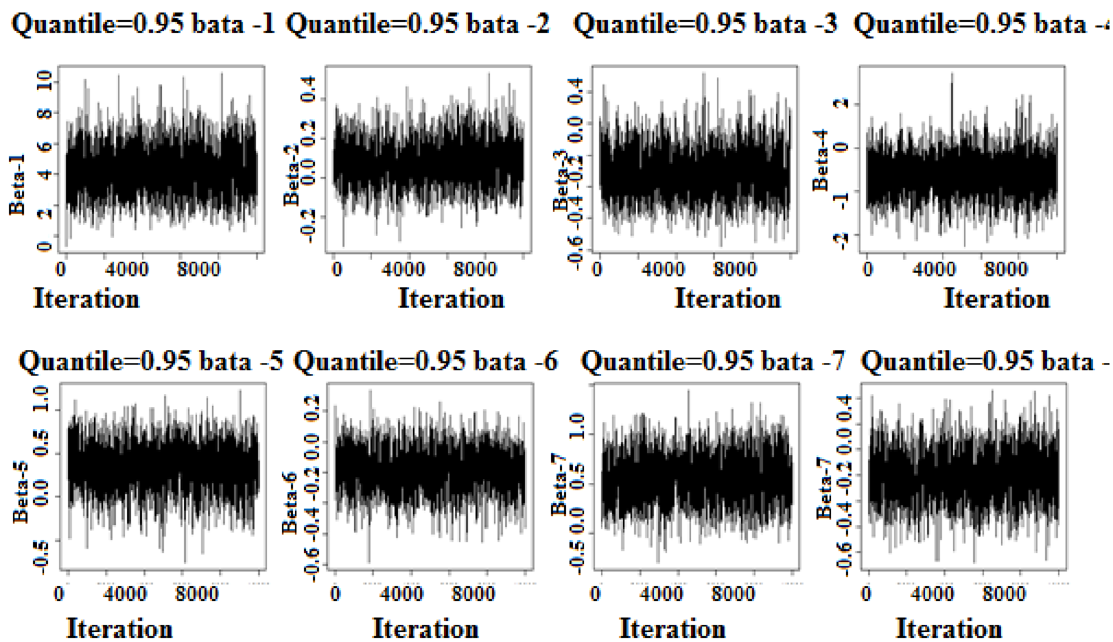
Table 2: Mean square error (MSE) for first simulation example. mean absolute deviation (MAD) displayed in parentheses. The results are averaged over 100 independent simulations.

Methods	$\varepsilon_i \sim N(0,1)$	$\epsilon_i \sim N(3,4)$	$\epsilon_i \sim 0.5N(1,1) + 0.5N(2,1)$	$\varepsilon_i \sim t_{(5)}$
$rq_{0.15}$	1.747 (0.936)	1.591 (0.984)	1.602 (0.957)	1.108 (0.829)
$MCMCpack_{0.15}$	0.984 (0.581)	0.918 (0.603)	0.823 (0.593)	0.956 (0.361)
$NBRQReg_{0.15}$	**0.619 (0.318)**	**0.584 (0.285)**	**0.692 (0.205)**	**0.356 (0.089)**
$rq_{0.35}$	1.471 (0.945)	1.256 (0.879)	1.175 (0.938)	1.022 (0.791)
$MCMCpack_{0.35}$	0.805 (0.491)	0.937 (0.404)	0.857 (0.412)	0.891 (0.461)
$NBRQReg_{0.35}$	**0.657 (0.317)**	**0.725 (0.306)**	**0.527 (0.203)**	**0.517 (0.281)**
$rq_{0.55}$	**0.237 (0.102)**	**0.375 (0.098)**	**0.293 (0.084)**	1.281 (0.795)
$MCMCpack_{0.55}$	1.351 (0.862)	1.241 (0.691)	0.947 (0.481)	0.861 (0.582)
$NBRQReg_{0.55}$	0.648 (0.378)	0.582 (0.380)	0.486 (0.194)	**0.578 (0.273)**
rq_{075}	1.526 (0.822)	1.289 (0.982)	0.942 (0.582)	1.074 (0.693)
$MCMCpack_{0.75}$	0.821 (0.372)	0.519 (0.257)	0.482 (0.178)	0.727 (0.375)
$NBRQReg_{0.75}$	**0.388 (0.142)**	**0.292 (0.167)**	**0.257 (0.074)**	**0.263 (0.028)**
$rq_{0.95}$	1.189 (0.735)	1.106 (0.672)	0.835 (0.285)	0.835 (0.285)
$MCMCpack_{0.95}$	0.573 (0.282)	0.511 (0.204)	0.493 (0.286)	0.493 (0.286)
$NBRQReg_{0.95}$	**0.235 (0.083)**	**0.124 (0.056)**	**0.104 (0.056)**	**0.104 (0.056)**

Note: In the parentheses are MAD.

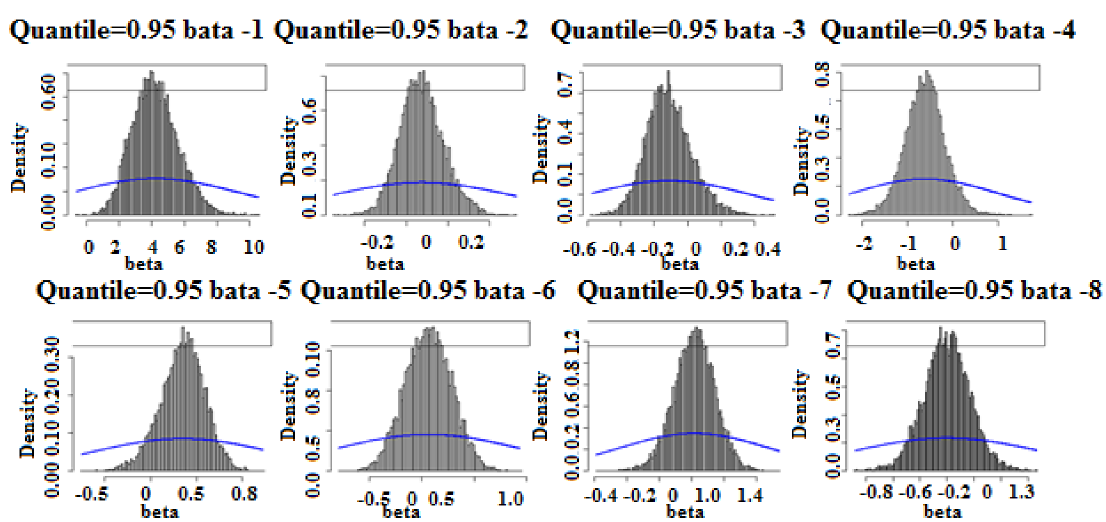
From results listed in Table (2), we see that the proposed method (NBRQReg) has a good performance compared to Bayesian and non-Bayesian methods via all error distributions and the quantile levels under considerations. Because of the mean square error (MSE) and mean absolute deviation (MAD) are computed by our proposed method (NBRQReg) are much smaller than mean square error (MSE) and mean absolute deviation (MAD) are computed by other two methods rq and MCMCpack. To investigate from the stability of our algorithm, we will draw trace plot for the model parameters at high quantile level (θ=0.95)

Figure 3- show trace plot of second simulation example at quantile level θ=0.95.



From figure 3-, we readily observed that the algorithm belong our proposed method is very stationary through 10000 iterations. In figure 4- we see closed the posterior parameters estimate from the normal distribution:

Figure 4. Histograms based on posterior distribution of the parameters for Simulation 2 at quantile level θ=0.95



5. Real dataset

The "bayesQR" R package contains all the approaches being considered, and it is used for data on prostate cancer. There are 97 rows for observations and 8 columns for the different factors being studied. Logarithm of cancer amount (lcavol) is referred to as (x1), logarithm of the weight of the prostate (lweight) is referred to as (x2), age is referred to as (x3), logarithm of the volume of benign enlargement of the prostate (lbph) is referred to as (x4), seminal vesicle invasion (SVI) is referred to as (x5), and logarithm of Cap

Similar to the simulation example , here we compare three methods: Our proposed method (NBRQReg) ,rq and MCMCpack . The our method and other two methods investigated based on standard division (SD) and (MSE) for five quantile levels θ=(θ=0.15,θ=0.35,θ=0.55,θ=0.75 and θ=0.95 .

Table -3- show the mean square error (MSE) and standard division (SD) for the prostate cancer data

	$\theta = 0.15$	$\theta = 0.35$	$\theta = 0.55$	$\theta = 0.75$	$\theta = 0.95$
Methods	MSE (SD)	MSE (SD)	MSE (SD)	MSE (SD)	MSE (SD)
rq	0.893 (0.542)	0.838 (0.682)	0.676 (0.490)	0.689 (0.505)	0.584 (0.351)
MCMCpack	0.763 (0.474)	0.742 (0.527)	0.563 (0.374)	0.539 (0.328)	0.427 (0.281)
NBRQReg	**0.433 (0.272)**	**0.472 (0.362)**	**0.339 (0.203)**	**0.313 (0.193)**	**0.273 (0.151)**

Note: In the parentheses are SD

From results in Table 3, we see the MSE computed by our proposed method (NBRQReg) is much smaller than MSE computed by other two methods rq and MCMCpack. Also the SD computed by our proposed method (NBRQReg) is much smaller than MSE computed by other two methods rq and MCMCpack. Based on these results, we can conclude that the proposed method has a good performance compared with two other methods. The coefficients estimate for our proposed method via all quantile levels under considerations, listed in Table 4

Table 4- Coefficients estimates for our method via five quantile level for the prostate cancer data

Variables	$\theta = 0.15$	$\theta = 0.35$	$\theta = 0.55$	$\theta = 0.75$	$\theta = 0.95$
lcavol	1.671	1.515	1.142	0.547	1.687
lweight	0.851	0.837	0.773	0.067	0.000
age	0.000	0.000	0.000	0.031	0.000
lbph	-1.023	-0.952	-0.741	0.000	-1.741
SVI	0.863	0.819	0.397	-0.132	0.471
lcp	0.918	0.764	0.191	0.655	0.023

gleason	1.034	0.000	0.000	0.000	0.000
pgg45	0.981	0.297	1.281	0.009	0.085

The results listed in Table 4 show coefficients estimated in direct way for the proposed method based on five quantiles as shown in the table above. In quantile level θ=0.15, the variables (age) ineffective on lpsa , but the rest independent variables have positive and negative effects on lpsa . In quantile level θ=0.35 , the variable (age) and (gleason) ineffective on lpsa , but the rest independent variables have positive and negative effective on lpsa. . Also in quantile level θ=0.55 , the variable (age) and (gleason) ineffective on lpsa , but the rest independent variables have positive and negative effective on lpsa. In quantile level θ=0.75 , the variable (lbph) and (gleason) ineffective on lpsa , but the rest independent variables have positive and negative effective on lpsa. In quantile level θ=0.95 , the variable (lweight) (Age) and (gleason) are ineffective on lpsa , but the rest independent variables have positive and negative effective on lpsa. From the results listed in Table 4, we can see that the proposed method have get a good performance for coefficients estimate and variable selection in quantile regression model.

6. Conclusion:

In this paper , we suggested a novel Bayesian estimation strategy that makes use of the lasso penalty for the purpose of estimating and choosing variables for use in the quantile regression model. We create a Bayesian hierarchical model to estimate the parameters based on a new scale of mixture representation of normal distribution combining Rayleigh density to the Laplace prior distribution of the parameters vector. This model is based on a new scale of mixture representation of normal distribution.

Simulation examples and actual data are taken into account in order to evaluate the performance of the suggested approach that took into account the scale mixture of normal distribution mixing Rayleigh density and to evaluate the performance of this method in comparison to other methods that are already in use. The results of the simulation and the actual data that are provided in the tables and figures that are located above have proved that the newly suggested approach is superior to the other ways that are now being used as competitors.

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