New Transmuted Rayleigh-Pareto distribution: Properties and Applications

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Abstract

The Rayleigh- Pareto distribution is the most commonly used statistical distribution in the engineering and medical fields. We propose to generalize the Rayleigh-Pareto distribution referred to as the transformed Rayleigh-Pareto distribution (*T*RP). The new transformed distribution was developed using the square-rank transformation map studied by Shaw et al. (2009).

A comprehensive account of the mathematical properties of the new distribution is provided. Explicit expressions for moments, moment generation function, entropy, mean deviation, Bonferroni and Lorenz curves, and moments are derived for system statistics. Distribution parameters (TRP) are estimated using the maximum likelihood method. Monte Carlo simulations are performed in order to investigate the performance of MLEs. The cirrhosis data demonstrate the usefulness of the proposed model.

Keywords: *The Rayleigh- Pareto distribution, moments, square-rank transformation map ,parameter estimation, maximum likelihood estimation.*

1. Introduction

Many lifetime data used for statistical analysis follows a particular statistical distribution. Recent days technological world nearly everyone depends on the continued functioning a broad collection of complex machinery and equipment for our every day to day life security, safety, ability to move easily and economic welfare. We expect our electronic devices, lights, hospital monitoring control, next generation aircraft, nuclear power plants ect., to function whenever we need them. It will be fail, the results can be disastrous illness or even loss of life. Statistical distributions have long been employed in the assessment of semiconductor device and product reliability. The use of the exponential distribution which more preferred over mathematically very difficult distribution, such as the Weibull and the lognormal among others, suggest that most of the engineers interest the application of simple model to find out failure rates and reliability results quickly. It is therefore proposed that the new Rayleigh Pareto distribution be considered as a simpler alternative which in some situations , may place a good fit for failure data and provide more appropriate information about reliability and hazard rates. The New Rayleigh Pareto distribution is also used to appropriate representation of the lower tail of the distribution of random variable having fixed lower bound. In the concept of „strength-reliability‟, the stress-strength model describes the life of a component, which has a random strength Y and is subjected to a random stress X . The component fails at the instant that the stress applied to it exceeds the strength , and the component will function satisfies whenever Y □□X . Thus, Y □□X is a measure of component reliability. It has number of applications in engineering concepts, deterioration of rocket motors ect., the reliability estimation of a single component stress-strength version has been considered by several authors assuming various lifetime distributions for the stress-strength random variates. Enis and Geisser (1971), Downtown (1973),Awad and Gharraf (1986), McCool (1991), Nandi and Aich (1994),Kundu and Gupta (2005, 2006), Arulmozhi (2003). In this paper, we will discuss probabilistic properties, including probability mass function, survival function, kurtosis coefficient, and skewness coefficient, and maximum likelihood estimates (MLEs) for unknown parameters are discussed. The entropy and mean deviations are derived. A probability density function (pdf) is derived for the order and moment statistics. We evaluate the performance of MLEs using simulations. The distribution was applied to the data of patients with liver cirrhosis, and the results described in the conclusions paragraph were drawn, and in light of it, recommendations were made

Let us consider the New Rayleigh Pareto distribution with probability density function (pdf)

$$f(x,\theta,\alpha) = \frac{\theta}{\alpha^{\theta}} x^{\theta-1} e^{-\left(\frac{x}{\alpha}\right)^{\theta}} \qquad (1)$$

$Where\ 0 < X < \infty, \theta > 0, \alpha > 0$

The cumulative distribution function (cdf) of the RPD is given by

$$F(x,\theta,\alpha) = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\theta}} \qquad (2)$$

$$Where\ 0 < X < \infty, \theta > 0, \alpha > 0$$

The survival function

$$S(x) = e^{-\left(\frac{x}{\alpha}\right)^{\theta}} \qquad (3)$$

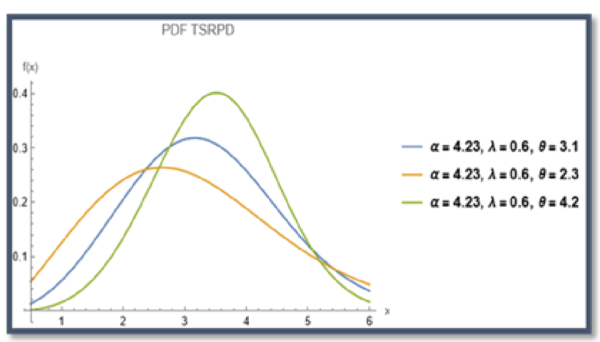
2. Transmuted Rayleigh- Pareto distribution:

In this thesis, we study the quadratic (Rayleigh-Pareto) complex distribution, as we substitute the cumulative distribution function of the (Transmuted Rayleigh-Pareto) distribution (cdf) contained in equation (2) into the quadratic transformation function contained in equation (5), we get the transformed complex distribution The cumulative squared is as follows:

$$f(x,\theta,\alpha,\gamma) = f(x,\theta,\alpha)[1+\gamma - 2\gamma F(x,\theta,\alpha)] \qquad (4)$$

$$f(x,\theta,\alpha,\gamma) = \frac{\theta}{\alpha^{\theta}} x^{\theta-1} e^{-\left(\frac{x}{\alpha}\right)^{\theta}} \left[1+\gamma - 2\gamma\left(1 - e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\right)\right]$$

The reasonable shapes of PDF Transmuted Rayleigh- Pareto distribution



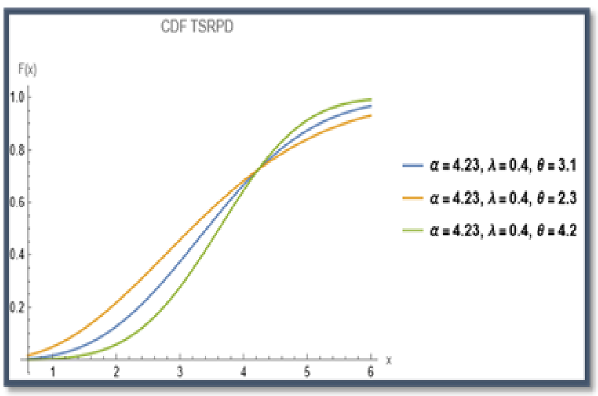
$$f(x,\theta,\alpha,\gamma)=\frac{\theta}{\alpha^{\theta}}x^{\theta-1}e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\left[1+\gamma-2\gamma\left(1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\right)\right]$$

The reasonable shapes of PDF Transmuted Rayleigh- Pareto distribution

The CDF of this distribution is

$$F(x,\theta,\alpha,\gamma)=(1\ +\ \lambda)1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}-\ \lambda\left[1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\right]^{2}\quad(5)$$

The reasonable shapes of CDF Transmuted Rayleigh- Pareto distribution

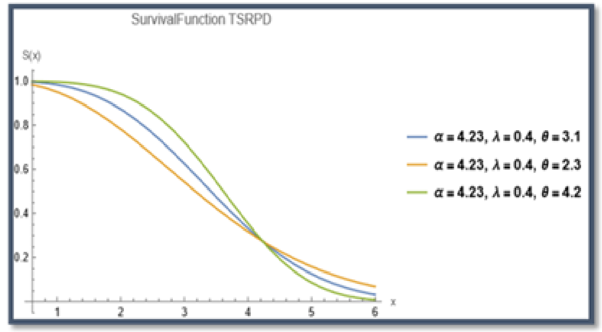


The Survival function of the new distribution is:

$$S(x,\theta,\alpha,\gamma)\ =1-F(x,\theta,\alpha,\gamma)$$

$$S(x,\theta,\alpha,\gamma)\ =1-(1\ +\ \lambda)1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}-\ \lambda\left[1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\right]^{2}\quad(6)$$

The reasonable shapes Survival Transmuted Rayleigh- Pareto distribution



The hazard function of the new distribution is

For any random variable X which follows Transmuted Rayleigh- Pareto distribution, its hazard function is given as:

$$h(x,\theta,\alpha,\gamma)=\frac{f(x,\theta,\alpha,\gamma)}{s(x,\theta,\alpha,\gamma)}$$

$$h(x,\theta,\alpha,\gamma)\frac{\frac{\theta}{\alpha^{\theta}}x^{\theta-1}e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\left[1+\gamma-2\gamma\left(1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\right)\right]}{1-(1\ +\ \lambda)1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\ -\ \lambda\left[1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\right]^{2}}\quad(7)$$

3.Statistical Properties

In this section, some of the properties of the Transmuted Rayleigh- Pareto distribution are discussed:

3.1Quantile function

The quintile function or inverse cumulative distribution function. returns the value t such that:

$$t=Q(u)=F^{-1}(u),0<u<1$$

$$u=(1\ +\ \lambda)1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\ -\ \lambda\left[1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\right]^{2}$$

$$x=(\alpha^{\theta}\mathrm{Log}[\frac{-1+\gamma+\sqrt{1+2\gamma-4u\gamma+\gamma^{2}}}{2(-1+u)}])^{\frac{1}{\theta}}\quad(8)$$

3.2Moments

Let x denote the random variable follows Transmuted Rayleigh- Pareto distribution then r^th order moment about origin of μ_r is:

$$\mu_r^- = E(x^r) = \int_0^\infty x^r f(x,\theta,\alpha,\gamma).dx$$

$$= \frac{\theta(1+\gamma-2\gamma)}{\alpha^\theta}\int_0^\infty x^r x^{\theta-1} e^{-\left(\frac{x}{\alpha}\right)^\theta}.dx + \frac{2\gamma\theta}{\alpha^\theta}\int_0^\infty x^r x^{\theta-1} e^{-2\left(\frac{x}{\alpha}\right)^\theta}.dx$$

$$let\ u = \frac{x}{\alpha}, \qquad x = u\alpha, dx = \alpha du$$

$$= \frac{\theta(1+\gamma-2\gamma)}{\alpha^\theta}\int_0^\infty x^{\theta-1+r} e^{-\left(\frac{x}{\alpha}\right)^\theta}.dx + \frac{2\gamma\theta}{\alpha^\theta}\int_0^\infty x^{\theta-1+r} e^{-2\left(\frac{x}{\alpha}\right)^\theta}.dx$$

$$= \frac{\theta(1+\gamma-2\gamma)}{\alpha^\theta}\int_0^\infty (u\alpha)^{\theta-1+r} e^{-(u)^\theta}.\alpha du + \frac{2\gamma\theta}{\alpha^\theta}\int_0^\infty (u\alpha)^{\theta-1+r} e^{-2(u)^\theta}\alpha du$$

$$= \frac{\theta\alpha(1+\gamma-2\gamma)\alpha^{\theta-1+r}}{\alpha^\theta}\int_0^\infty (u)^{\theta-1+r} e^{-(u)^\theta}.du + \frac{\alpha\alpha^{\theta-1+r}2\gamma\theta}{\alpha^\theta}\int_0^\infty (u)^{\theta-1+r} e^{-2(u)^\theta}du$$

$$= \theta(1+\gamma-2\gamma)\alpha^r\int_0^\infty (u)^{\theta-1+r} e^{-(u)^\theta}.du + \alpha^r 2\gamma\theta\int_0^\infty (u)^{\theta-1+r} e^{-2(u)^\theta}du$$

$$= \theta(1+\gamma-2\gamma)\alpha^r\frac{\Gamma[\frac{r+\theta}{\theta}]}{\theta} + \alpha^r 2\gamma\theta\frac{2^{-\frac{r+\theta}{\theta}}\Gamma[\frac{r+\theta}{\theta}]}{\theta}$$

$$E(x^r) = (1+\gamma-2\gamma)\alpha^r\Gamma\left[\frac{r+\theta}{\theta}\right] + \alpha^r 2\gamma 2^{-\frac{r+\theta}{\theta}}\Gamma\left[\frac{r+\theta}{\theta}\right]; r = 1,2,3, \qquad (9)$$

Where r=1

$$E(x^{1`}) = (1+\gamma-2\gamma)\alpha^1\Gamma\left[\frac{1+\theta}{\theta}\right] + \alpha^1 2\gamma 2^{-\frac{1+\theta}{\theta}}\Gamma\left[\frac{1+\theta}{\theta}\right]$$

Where r=2

$$E(x^2) = (1+\gamma-2\gamma)\alpha^2\Gamma\left[\frac{2+\theta}{\theta}\right] + \alpha^2 2\gamma 2^{-\frac{2+\theta}{\theta}}\Gamma\left[\frac{2+\theta}{\theta}\right]$$

Where r=3

$$E(x^3) = (1+\gamma-2\gamma)\alpha^3\Gamma\left[\frac{3+\theta}{\theta}\right] + \alpha^3 2\gamma 2^{-\frac{3+\theta}{\theta}}\Gamma\left[\frac{3+\theta}{\theta}\right]$$

Where r=4

$$E(x^4) = (1+\gamma-2\gamma)\alpha^4\Gamma\left[\frac{4+\theta}{\theta}\right] + \alpha^4 2\gamma 2^{-\frac{4+\theta}{\theta}}\Gamma\left[\frac{4+\theta}{\theta}\right]$$

3.3moments about the mean

Let x denote the random variable follows Transmuted Rayleigh- Pareto distribution then moments about the mean order moment about origin of μ_r is:

$$E(x-\mu)^r = \int_0^\infty (x-\mu)^r f(x,\theta,\alpha,\gamma).dx \qquad (10)$$

$$= \frac{\theta(1+\gamma-2\gamma)}{\alpha^\theta}\int_0^\infty (x-\mu)^r x^{\theta-1}e^{-\left(\frac{x}{\alpha}\right)^\theta}.dx + \frac{2\gamma\theta}{\alpha^\theta}\int_0^\infty (x-\mu)^r x^{\theta-1}e^{-2\left(\frac{x}{\alpha}\right)^\theta}.dx$$

$$E(x-\mu)^r = \sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}x^j$$

$$= \frac{\theta(1+\gamma-2\gamma)}{\alpha^\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\int_0^\infty x^j x^{\theta-1}e^{-\left(\frac{x}{\alpha}\right)^\theta}.dx + \frac{2\gamma\theta}{\alpha^\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\int_0^\infty x^j\, x^{\theta-1}e^{-2\left(\frac{x}{\alpha}\right)^\theta}.dx$$

$$= \frac{\theta(1+\gamma-2\gamma)}{\alpha^\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\int_0^\infty x^{\theta-1-j}e^{-\left(\frac{x}{\alpha}\right)^\theta}.dx + \frac{2\gamma\theta}{\alpha^\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\int_0^\infty x^{\theta-1-j}e^{-2\left(\frac{x}{\alpha}\right)^\theta}.dx$$

$$let\ u = \frac{x}{\alpha}, \qquad x = u\alpha, dx = \alpha du$$

$$= \frac{\theta(1+\gamma-2\gamma)}{\alpha^\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\int_0^\infty (u\alpha)^{\theta-1-j}e^{-(u)^\theta}.\alpha du + \frac{2\gamma\theta}{\alpha^\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\int_0^\infty (u\alpha)^{\theta-1-j}e^{-2(u)^\theta}.\alpha du$$

$$= \frac{\theta\alpha(1+\gamma-2\gamma)\alpha^{\theta-1-j}}{\alpha^\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\int_0^\infty (u)^{\theta-1-j}e^{-(u)^\theta}.du + \frac{2\alpha\gamma\theta\alpha^{\theta-1-j}}{\alpha^\theta}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\int_0^\infty (u)^{\theta-1-j}e^{-2(u)^\theta}.du$$

$$= \theta(1+\gamma-2\gamma)\alpha^j\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\int_0^\infty (u)^{\theta-1-j}e^{-(u)^\theta}.du + 2\gamma\theta\alpha^j\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\int_0^\infty (u)^{\theta-1-j}e^{-2(u)^\theta}.du$$

$$= \theta(1+\gamma-2\gamma)\alpha^j\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\frac{\Gamma[\frac{j+\theta}{\theta}]}{\theta} \qquad (11)$$
$$+ 2\gamma\theta\alpha^j\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\frac{2^{-\frac{j+\theta}{\theta}}\Gamma[\frac{j+\theta}{\theta}]}{\theta}$$

$$= \theta(1+\gamma-2\gamma)\alpha^j\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\frac{\Gamma[\frac{j+\theta}{\theta}]}{\theta} + 2\gamma\theta\alpha^j\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\frac{2^{-\frac{j+\theta}{\theta}}\Gamma[\frac{j+\theta}{\theta}]}{\theta}$$

$$= (1+\gamma-2\gamma)\alpha^j\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\Gamma[\frac{j+\theta}{\theta}] + 2\gamma\alpha^j\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}2^{-\frac{j+\theta}{\theta}}\Gamma[\frac{j+\theta}{\theta}]$$

$$E(x-\mu)^r = \sum_{j=0}^{r} \binom{r}{j} (-\mu)^{r-j} \Gamma\left[\frac{j+\theta}{\theta} \alpha^j\right] \left((1 + \gamma - 2\gamma) + 2\gamma 2^{-\frac{j+\theta}{\theta}}\right), r = 2,3,4 \qquad (12)$$

Where r=2

$$E(x-\mu)^2 = \sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} \Gamma\left[\frac{j+\theta}{\theta} \alpha^j\right] \left((1 + \gamma - 2\gamma) + 2\gamma 2^{-\frac{j+\theta}{\theta}}\right)$$

$$\sigma^2 = \sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} \Gamma\left[\frac{j+\theta}{\theta} \alpha^j\right] \left((1 + \gamma - 2\gamma) + 2\gamma 2^{-\frac{j+\theta}{\theta}}\right)$$

$$\sigma = \sqrt{\sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} \Gamma\left[\frac{j+\theta}{\theta} \alpha^j\right] \left((1 + \gamma - 2\gamma) + 2\gamma 2^{-\frac{j+\theta}{\theta}}\right)} \qquad (13)$$

Where r=3

$$E(x-\mu)^3 = \sum_{j=0}^{3} \binom{3}{j} (-\mu)^{3-j} \Gamma\left[\frac{j+\theta}{\theta} \alpha^j\right] \left((1 + \gamma - 2\gamma) + 2\gamma 2^{-\frac{j+\theta}{\theta}}\right)$$

Where r=4

$$E(x-\mu)^4 = \sum_{j=0}^{4} \binom{4}{j} (-\mu)^{4-j} \Gamma\left[\frac{j+\theta}{\theta} \alpha^j\right] \left((1 + \gamma - 2\gamma) + 2\gamma 2^{-\frac{j+\theta}{\theta}}\right)$$

3.4 Coefficient of Variation

The Coefficient of Variation for Transmuted Rayleigh- Pareto distribution is given by:

$$C \cdot V = \frac{\sigma}{\mu'_1} \times 100\%$$

$$C.V = \frac{\sqrt{\sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} \Gamma\left[\frac{j+\theta}{\theta} \alpha^j\right] \left((1 + \gamma - 2\gamma) + 2\gamma 2^{-\frac{j+\theta}{\theta}}\right)}}{(1+\gamma-2\gamma)\alpha^1 \Gamma\left[\frac{1+\theta}{\theta}\right] + \alpha^1 2\gamma 2^{-\frac{1+\theta}{\theta}} \Gamma\left[\frac{1+\theta}{\theta}\right]} \qquad (14)$$

3.5 Coefficient of Skewness

Coefficient of Skewness for Transmuted Rayleigh- Pareto distribution is given by:

$$S.K = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}}$$

$$S_k =. K = \frac{\sum_{j=0}^{3} \binom{3}{j} (-\mu)^{3-j} \Gamma\left[\frac{j+\theta}{\theta} \alpha^j\right] \left((1 + \gamma - 2\gamma) + 2\gamma 2^{-\frac{j+\theta}{\theta}}\right)}{\left(\sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} \Gamma\left[\frac{j+\theta}{\theta} \alpha^j\right] \left((1 + \gamma - 2\gamma) + 2\gamma 2^{-\frac{j+\theta}{\theta}}\right)\right)^{\frac{3}{2}}} \qquad (15)$$

3.6 Coefficient of Kurtosis

The Coefficient of Kurtosis of for Transmuted Rayleigh- Pareto Distribution is given by:

$$C.K = \frac{E(t-\mu)^4}{\sigma^4}$$

C.K

$$= \frac{\sum_{j=0}^{4}\binom{4}{j}(-\mu)^{4-j}\Gamma\left[\frac{j+\theta}{\theta}\alpha^{j}\right]\left((1+\gamma-2\gamma)+2\gamma 2^{-\frac{j+\theta}{\theta}}\right)}{\left(\sum_{j=0}^{2}\binom{2}{j}(-\mu)^{2-j}\Gamma\left[\frac{j+\theta}{\theta}\alpha^{j}\right]\left((1+\gamma-2\gamma)+2\gamma 2^{-\frac{j+\theta}{\theta}}\right)\right)^{2}}$$

(16)

4. Moment Generating Function

Let X be random variable follows Transmuted Rayleigh- Pareto distribution, then the moment generating function (mfg) of x is obtained as:

$$M_x(t) = E(e^{tX}) \qquad \ldots 13$$

$$M_X(x) = E(e^{tx}) = \int_0^\infty e^{tx} f(x)\,dx$$

$$M_X(t) = \int_0^\infty \left(1 + tx + \frac{(tx)^2}{2!} + \cdots + \frac{(tx)^r}{r!}\right) f(x).\,dx$$

$$M_X(t) = \sum_{r=0}^{\infty}\frac{t^r}{r!}\,(1+\gamma-2\gamma)\alpha^r\Gamma\left[\frac{r+\theta}{\theta}\right] + \alpha^r 2\gamma 2^{-\frac{r+\theta}{\theta}}\Gamma\left[\frac{r+\theta}{\theta}\right] \qquad (17)$$

Similarly, the characteristic function of Transmuted Rayleigh- Pareto distribution, can be obtained as:

$$M_{Xi}(t) = \sum_{r=0}^{\infty}\frac{ti^r}{r!}\,(1+\gamma-2\gamma)\alpha^r\Gamma\left[\frac{r+\theta}{\theta}\right] + \alpha^r 2\gamma 2^{-\frac{r+\theta}{\theta}}\Gamma\left[\frac{r+\theta}{\theta}\right] \quad (18)$$

5. Parameter estimation:

the Method of Maximum Likelihood Estimate is Used for Estimating The Parameters of The Newly Proposed Distribution Known as of Transmuted Rayleigh-Pareto Distribution. Let x1, x2,…,xn be a Random Sample of Ssize n From of The Transmuted Rayleigh-Pareto Distribution, Fhen the Corresponding likelihood Function is Given By:

Let x1 ,x2,x3, x4,........xn be a random sample of size n from Transmuted Rayleigh-Pareto The likelihood function, L Transmuted Rayleigh-Pareto is given by:

$$f(x,\theta,\alpha,\gamma) = \frac{\theta}{\alpha^\theta}x^{\theta-1}e^{-\left(\frac{x}{\alpha}\right)^\theta}\left[1+\gamma - 2\gamma\left(1 - e^{-\left(\frac{x}{\alpha}\right)^\theta}\right)\right] \quad ;x>0,\alpha,\theta\geq 0,-1<|\gamma|<1 \quad (19)$$

$$Lf(x_1, x_2, \ldots \ldots x_n, \alpha, \theta, \gamma) = \prod_{i=1}^{n} f(x,\alpha,\theta,\gamma)$$

$$Lf(xi, \alpha,\theta,\gamma)=\prod_{i=1}^{n}\left[\frac{\theta}{\alpha^\theta}x^{\theta-1}e^{-\left(\frac{x}{\alpha}\right)^\theta}\left[1+\gamma - 2\gamma\left(1-e^{-\left(\frac{x}{\alpha}\right)^\theta}\right)\right]\right]$$

$$Lf(xi, \alpha,\theta,\gamma)=\frac{\theta^n}{\alpha^{\theta n}}\prod_{i=1}^{n}\left[x^{\theta-1}e^{-\left(\frac{x}{\alpha}\right)^\theta}\left[1+\gamma - 2\gamma\left(1-e^{-\left(\frac{x}{\alpha}\right)^\theta}\right)\right]\right]$$

The log-likelihood function for the vector of parameters can be printed as

$$\mathrm{LnLf}(xi,\alpha,\theta,\gamma)=n\mathrm{Ln}\theta-n\theta\mathrm{Ln}\alpha+(\theta-1)\sum_{i=1}^{n}\mathrm{Ln}(xi)-\sum_{i=1}^{n}\left(\frac{xi}{\alpha}\right)^{\theta}+\sum_{i=1}^{n}\mathrm{Ln}\left[1+\gamma-2\gamma\left(1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\right)\right] \quad (20)$$

By taking the first partial derivatives of the log-likelihood function with respect to the four parameters (α,θ,γ) as follows:

$$\frac{d\mathrm{LnLf}(xi,\alpha,\theta,\gamma)}{d\theta}=\sum_{i=1}^{n}\left(\frac{1}{\theta}+\mathrm{Log}[x]-\mathrm{Log}[\alpha]+\left(\frac{x}{\alpha}\right)^{\theta}\mathrm{Log}\left[\frac{x}{\alpha}\right](-1+e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\mathrm{Log}[1-\gamma])\right)=0 \quad (21)$$

$$\frac{d\mathrm{LnLf}(xi,\theta,\alpha,\gamma)}{d\alpha}=\frac{\sum_{i=1}^{n}\theta\left(-1+\left(\frac{x}{\alpha}\right)^{\theta}-e^{-\left(\frac{x}{\alpha}\right)^{\theta}}\left(\frac{x}{\alpha}\right)^{\theta}\mathrm{Log}[1-\gamma]\right)}{\alpha}=0 \quad (22)$$

$$\frac{d\mathrm{LnLf}(xi,\alpha,\theta,\gamma)}{d\gamma}=-\sum_{i=1}^{n}\frac{(1-e^{-\left(\frac{x}{\alpha}\right)^{\theta}})n}{1-\gamma}=0 \quad (23)$$

The maximum likelihood estimates $(\hat{\gamma},\hat{\theta},\hat{\alpha})$ equations $\frac{dLogL}{d\alpha}=0$, $\frac{dLogL}{d\theta}=0$, $\frac{dLogL}{d\gamma}=0$,The Equation (21),(22) and Equation (23) cannot be solved as they both are in closed forms. So we compute the parameters of the Transmuted Rayleigh-Pareto Distribution

6. Application of Transmuted Rayleigh-Pareto distribution.

The elasticity and performance of the Transmuted Rayleigh-Pareto(TRP) are evaluated on competing models such as the Pareto distribution (PD), the Rayleigh distribution(RD), and the Rayleigh-Pareto distribution. In this study, a new probability distribution is identified.. Here, the distribution is fit to the data set of the number of weeks cirrhosis patients were in the hospital before death for Al-Hussein Teaching Hospital in Karbala, for the sample size (n = 105) (see Table 1), the performance from the distribution was a Rayleigh, Pareto, and Rayleigh distribution Pareto dataset using the aka information criterion (AIC), (BIC), Akaike information criterion correction (AICC). The distribution with the lowest AIC, the AICC is considered the most flexible and superior distribution for a given data set. The results are shown in tables (1)

Table 1. Data set for the number of hours patients were in hospital before death

xi	xi	xi	xi	xi	xi	xi	xi	xi	xi	xi
0.5	1.7	2.3	2.6	2.9	3.1	3.5	3.9	4.3	5.1	7
0.5	1.7	2.3	2.6	2.9	3.1	3.7	3.9	4.3	5.1	7
0.7	1.7	2.3	2.7	2.9	3.2	3.7	3.9	4.3	5.1	7.1
1.2	1.8	2.4	2.7	2.9	3.4	3.7	4	4.4	5.2	7.2
1.2	1.8	2.4	2.7	2.9	3.4	3.8	4	4.4	5.3	7.5
1.3	1.9	2.5	2.7	3.1	3.4	3.8	4	4.6	5.4	
1.5	1.9	2.5	2.7	3.1	3.5	3.8	4.1	4.6	5.6	
1.5	1.9	2.5	2.7	3.1	3.5	3.9	4.1	4.7	6	
1.5	2	2.6	2.7	3.1	3.5	3.9	4.1	4.8	6.1	
1.6	2.1	2.6	2.7	3.1	3.5	3.9	4.2	4.9	6.6	

To choose the best model within the set of models that was compared with the new distribution, the best is the model corresponding to the lowest value for Akaike Information Criterion (AIC) and Akaike Information Correct (AICc) (see tabul 2.) , the general formula for (AIC) ,(AICc) and (BIC) are:

Information Correct (AIC_c) (see tabul 2.) , the general formula for (AIC) ,(AIC_c) and (BIC) are:

$$\mathrm{AIC} = -2\log\left(\frac{\hat{\theta}_{\mathrm{MLE}}}{\mathrm{x}}\right) + 2\mathrm{K} \qquad (24)$$

Where:
$\log\left(\frac{\hat{\theta}_{\mathrm{MLE}}}{\mathrm{x}}\right)$: value of the logarithm maximum likelihood function.

K: Estimated number of parameters.
And

$$\mathrm{AIC_c} = \mathrm{AIC} + \frac{2\mathrm{K}(\mathrm{K}+1)}{\mathrm{N}-\mathrm{K}-1} \qquad (25)$$

Where

AIC: Akaike Information Criterion.

K: Estimated number of parameters.

N: sample size

$$\mathrm{BIC} = -2\log\left(\hat{\theta}_{\mathrm{MLE}}\right) + \mathrm{K}\log(\mathrm{N}) \qquad (26)$$

Where

BIC: Bayesian Information Criterion.

K: Estimated number of parameters.

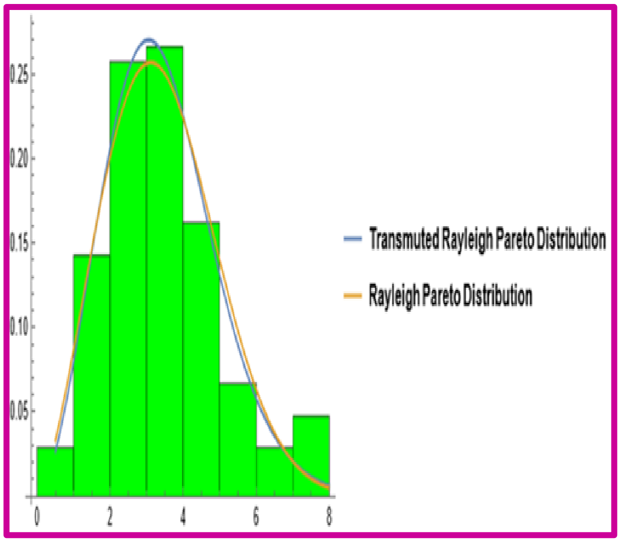
N: sample size

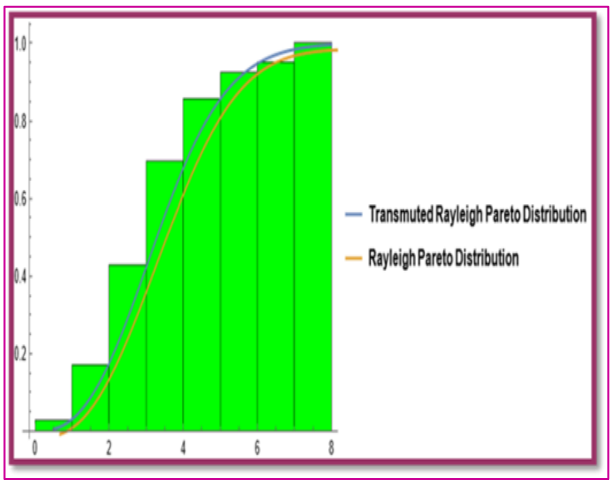
Table. 2. Parameters Estimates and Goodness – of – Fits by akaike information criterion (AIC), akaike

Distributions	Parameter			Logl	AIC	AICc	BIC
	$\hat{\alpha}$	$\hat{\theta}$	$\hat{\gamma}$				
Transmuted Rayleigh Pareto Distribution	4.45290	2.6332	0.59412	−144.1286	294.257	294.49492	294.3207
Rayleigh Pareto Distribution	3.8581	2.4437	-------	−156.6282	317.256	317.37404	317.2987

7. Conclusion

In this paper we dealt with the transformed Rayleigh-Pareto distribution, some properties were derived and discussed such as moments, reliability analysis and risk rate. The maximum likelihood estimation method is used to determine the parameters. The performance of a new model is determined by its fit to real-life data using quality of fit criteria such as AIC, AICC, and BIC. , the fit of the real data to the probability distributions under study, it was found that the transformed Rayleigh-Pareto (TRPD) gives a better fit to the data set such as the comparative Pareto distribution (PD), the Rayleigh distribution (RD) and the Rayleigh-Pareto distribution (RPD). In this study, a new probability distribution is identified.





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