

Solving Multi Objective Linear Programming Problems Using Vague Optimization Method: A Comparative Study

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Abstract

This newspaper's goal is to current a computational procedure for resolving a multi-objective lined programming problematic by means of a unclear optimization technique. It also covers some significant possessions of a unclear set, as healthy as processes on it. The procedure's growth is based on the finest choice set code, which is got by journey dissimilar vague choice sets obtained for apiece objective drive. Also, because the indistinct optimization method employs degrees of fit in and non- fitting, we done a relative schoolwork of line and nonlinear hint functions for fitting and non-belonging to realize how they touch optimization and gain understanding into such a process. To demonstrate the advanced procedure, a numerical instance has stood provided.

I. Introduction

In numerous optimization malfunctions, it takes remained experiential that unimportant defilements of a assumed restraint or constraints can principal to additional well-organized solutions. This situation arises frequently in applied modeling, particularly in optimization glitches; it is often unreasonable to specify precise parameters, meanwhile abundant of them are found by guesstimate or around method of anthropological statement. For specimen, in a creation optimization badly-behaved, it is not compulsory that all harvests be of lofty class to be traded at a motionless price. It is possible that bumpily products are broken-down and cannot be sold at the fixed price. Also, due to overpowering conditions, raw substantial prices and over development marketplace amounts may vary due to shop spare/deficiency. Then, it is clear that expenses in addition/or manufacturing be present not

indecently deterministic, but imprecise or non-deterministic, and nonclassical methods requirement be used to determination such optimization glitches. Of progression, most real-world difficulties linking optimization events are modeled as multi-objective program writing problems. In general, such multi-objective programming hitches may have conflicting objects [6]. For example, in agricultural manufacture planning problems, the optimal model should aim at maximizing income while minimalizing input and planting costs [8]. Due to the contradictory nature of these objectives, the solution to such problems is frequently a compromise that satisfies each objective role to some extent, and it is in this state that the ideas of connection and non-membership arise. Zimmermann [2], [3] was the first to use Zadeh fuzzy sets. [1] Multi-objective exact programming for solving fuzzy problems. Optimization in fuzzy environments has been studied and applied by several

canvassers together with Tanaka [4], Luhandjula [5], Sakawa et al. The labor of Sahinidis [7] provides a brief impression of optimization under uncertainty by numerous researchers. Various extensions of fuzzy sets have emerged due to their increasing When the available information is ambiguous, ambiguous, or uncertain, use it in modeling glitches. An extension of fuzzy sets is used in such a case. In his research, Atanasoff emphasizes that attribution and non-attribution should be treated as distinct, rather than complementary, attributes when dealing with information fuzziness, ambiguity, or doubt. Angelov [10] judiciously well thought-out the concepts of relationship and non-membership in optimization glitches and presented a fuzzy method for resolving optimization hitches. Jana and Roy [11] inspected the multi-objective incoherent in lines software design problem and applied it to the challenging transportation badly behaved. Luo [12] solved multi-criteria decision-making problems by incorporating fuzzy sets. Several other researchers, Several researchers, including Mahapatra et al. [13], investigated linear software proposal hitches in fuzzy environments using fuzzy numbers and interlude qualm in fuzzy numbers. The stimulus for this reading is the faith that it will be responsible for a computational algorithm for disentangling multi-objective linear software design teething troubles using fuzzy optimization methods. In this optimization process, we as well needed to investigate the waves of countless types of membership and non-membership occupations, so we conducted a virtual scholarship of lined affiliation and non-membership title role with nonlinear involvement and non-membership gatherings. [7].

II. PRELIMINARIES

In over-all, the multi impartial optimization problematic of linear software design through p ideas, q restrictions, also n pronouncement variables is as pettiness:

$$\max \{Z = \{z_1(1@) \} \} z_2 \dots z_n(p@) \}$$

Such that $g_j(x) \leq 0, j=1 \dots q$

$$x_i \geq 0, i=1, 2 \dots n$$

where $x = \{x_1, x_2 \dots x_n\}$

1) Complete solution

x^0 is said to be a wide-ranging best resolution for tricky (1)

if here exist $x^0 \in X$ such that

$$f_k(x^0) \geq f_k(X), k=1,2,\dots,p \text{ for all } x \in X$$

Nonetheless, such unabridged finest solutions that concurrently capitalize arranged wholly of the multiple-objective senses do not be in over-all, especially when the impartial purposes are integrally conflicting. In multi-objective programming, in its place of a whole best key, a answer idea recognized as Pareto optimality remained presented.

2) Pareto-optimality

Pareto efficiency is an economic state in which financial resources are distributed or allocated to exploit utility. As a result, any additional effort for reallocation will have no positive effect unless there is an equivalent negative effect.

$x^0 \in X$ is imaginary to be a Pareto idyllic answer for (1)

if here does not be added $x \in X$

such that $f_k(x^0) \leq f_{k(x)}$, for all $p=1,2,\dots,p$ and $f_j(x^0) < f_{j(x)}$

for at minimum one $j \in \{1,2,\dots, p\}$.

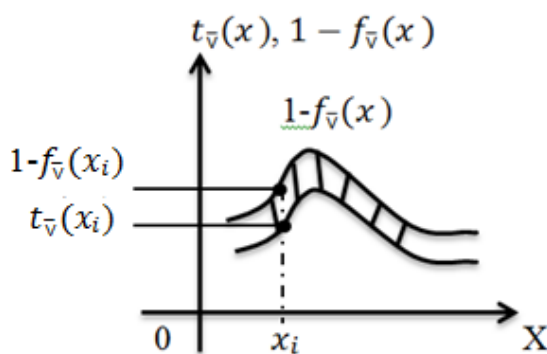
Definition [13]

Let U be the universal of dissertation. A vague set $\bar{v}$ over U is characterized by a truth function $t_{\bar{v}}, f_{\bar{v}}: U \rightarrow [0,1]$ and a false $f_{\bar{v}}, f_{\bar{v}}: U \rightarrow [0,1]$. If generic component of U is denoted by x_i then the lower bound on the membership grade of x_i

derived from evidence for x_i is denoted by $t_{\bar{v}}(x_i)$ and the lower bound on the negation of x_i is meant by $f_{\bar{v}}(x_i)$. $t_{\bar{v}}(x_i)$ and $f_{\bar{v}}(x_i)$ both associate a real amount in $[0,1]$ with each point x_i in X ,

where $t_{\bar{v}}(x_i) + f_{\bar{v}}(x_i) \leq 1$

Figure (1). A vague Set.



and so is a promotion of a undefined set. Here merger and fitting together of dual unclear sets are clear as

$$\bar{A} \cap \bar{B} = \{[x, \min(t_{\bar{A}}(x), t_{\bar{B}}(x)), \max(f_{\bar{A}}(x), f_{\bar{B}}(x))] \mid x \in X\}$$

$$\bar{A} \cup \bar{B} = \{[x, \max(t_{\bar{A}}(x), t_{\bar{B}}(x)), \min(f_{\bar{A}}(x), f_{\bar{B}}(x))] \mid x \in X\}$$

Fuzzy Optimization Technique The maximum-minimum method Zimmermann first rummage-sale Bellman and Zadeh's [18] max-min worker to solve Multi Objective Lined Software design (MOLP) glitches, and he defined the problematic (1) as: Find X

$$Z_k(x) \geq g_k, k=1.2\dots p$$

$$g_j(x) \leq 0, j=1.2\dots q \quad x \geq 0$$

anywhere g_k and x mean goal mouth, and all unprejudiced gatherings are expected to remain exploited. In this case, impartial goals are regarded as fuzzy restraints. We could first get a table of optimistic ideal answers in order to establish association functions of impartial purposes (PIS). The possible response set is clear by the message of the fuzzy objective set in the min-operator idea. This feasible solution set is then defined by its membership $A(x)$, which is as follows:

$$t_D(x) = \min(t_1(x), \dots, t_K(x))$$

A decision maker then brands a decision by the maximum D worth in the likely choice set. The choice answer can be got by solving the tricky of maximizet $D(x)$ below the given restraints, i.e.

$$\text{Max} [\min t_k(x)]$$

$$\text{s.t. } g_j(x) \leq 0, \\ j=1.2\dots q$$

$$x \geq 0$$

Now, if pretentious $\alpha = \min_k t_k(x)$, be the overall acceptable equal of collaboration, previously we obtain the next equivalent perfect $\text{Max } \alpha$

Such that $t_k(x) \geq \alpha$, for all k

$$g_j(x) \leq 0, j=1.2\dots q \quad x \geq 0 \quad (3)$$

C. unclear Optimization Method [6]

Consider the ambiguous optimization routine as promotion of the as the crow flies above problematic a below demanding by Angelov [3]

$$\begin{aligned} \min f_i(x) \leq 0, \quad i=1.2\dots p \\ g_j(x) \leq 0, \quad j=1.2\dots q \\ x \geq 0 \end{aligned} \quad (4)$$

where x signifies the choice variables, $f_i(x)$ signifies the impartial drives, $g_j(x)$ signifies the limit meanings, and p and q represent the number of impartial purposes and restraints.

This problem's best answer must encounter all restraints accurately. Accordingly, an analogous ambiguous optimization prototypical of the problem is used to feat the degree of reception of bits and pieces and restraints as follows:

$$\begin{aligned} (\min) \sim f_i(x) \leq 0, \quad i=1.2\dots p \\ g_j(x) \leq 0, \quad j=1.2\dots q \end{aligned} \quad (5)$$

Where $(\min)$ means unclear minimization and typifies fuzzy discrimination. Bellman and Zadeh [4] rummage-sale vague set exploit for the step of association of the decisiveness and shrinking to solve this order (5).

$$\begin{aligned} \max \{t_k(x), x \in X, \\ k=1.2\dots p+q \quad 0 \leq t_k(x) \leq 1 \\ x \geq 0 \end{aligned} \quad (6)$$

wherever $t_k(x)$ means the notch of gratification with the particular indefinite sets. It is critical to appreciate that the grade of non-membership in a ambiguous usual is the second of membership, so growth of the relationship drive determination automatically minimize non-membership. However, in a vague set, the

degree of rebuff is sure concurrently with the score of receiving, and because these grades are not complementary, VS may deliver a more wide-ranging tool for describing this uncertainty-based optimization model. As a size, the dim unsure optimization (VO) picture-perfect for awkward(3) is if as for every

$$\{ \max_x \{t_k(x)\} \} \in X, \quad k=1.2\dots p+q$$

$$\{ \min_x \{f_k(x)\} \} \in X, \quad k=1.2\dots p+q$$

Such that

$$\begin{aligned} f_k(x) &\geq 0 \quad k=1.2\dots p+q \\ t_k(x) &\geq f_k(x), \quad k=1.2\dots p+q \\ t_k(x) + f_k(x) &\leq 1, \quad k=1.2\dots p+q \end{aligned} \quad (7)$$

someplace $t_k(x)$ represents the degree to which x is predictable by the k th VS and $f_k(x)$ denotes the degree to which x is rejected by the k th IFS. Intuitionistic fuzzy objects and fetters are among the IFS. The excellent set D is now clear as a mixture of vague objectives and restraints.

$$\bar{F} \cap \bar{C} = \{ [x, \min(t_{\bar{F}}(x), t_{\bar{C}}(x)), \max(f_{\bar{F}}(x), f_{\bar{C}}(x))] \} \quad x \in X \quad (8)$$

where, $\bar{F}$ is combined intuitionistic uncertain impartial and $\bar{C} \sim$ means combined intuitionistic fuzzy fetters and is distinct as

$$\bar{F} = \{ [x, (t_{\bar{F}}(x), f_{\bar{F}}(x), x \in X)] = \bigcap_{i=1}^p \bar{F}^{(i)}$$

$$= \{ x, \min_{i=1}^p t_i f(x), \max_{i=1}^p f_i f(x) \}, x \in X \}$$

$$\bar{C} = \{ x, t_{\bar{C}}(x), f_{\bar{C}}(x) \}, x \in X \} = \bigcap_{j=1}^q \bar{C}^{(j)}$$

$$= \{x, \min_{j=1}^q t_j(x), \max_{j=i}^q f_j(x)\}, x \in X\}$$

Further, the vague choice set (VDS) denoted as $D \sim$:

$$\bar{D} = \bar{F} \cap \bar{C} = \{x, t_{\bar{D}}(x), f_{\bar{D}}(x)\}, x \in X\} \quad (9)$$

$$t_{\bar{D}}(x) = \min [t_{\bar{F}}(x), t_{\bar{C}}(x)] = \min_{k=1}^{p+q} t_k(x)$$

(10)

$$f_{\bar{D}}(x) = \max [f_{\bar{F}}(x), f_{\bar{C}}(x)] = \max_{k=1}^{p+q} t_k(x)$$

(11)

anywhere $t_D(x)$ designates the degree of VDS getting and $f_D(x)$ means the grade of VDS snub. Now, for a possible answer, the equal of receipt of VDS is continuously fewer than or equal to the level of receipt of any separate and constraint.

$$t_{\bar{D}}(x) \leq t_{\bar{F}}(x), t_{\bar{D}}(x) \leq t_{\bar{C}}(x),$$

$$f_{\bar{D}}(x) \geq f_{\bar{F}}(x), f_{\bar{D}}(x) \geq f_{\bar{C}}(x)$$

For all $k=1,2,\dots, p+q$

As a result the above group can be transmuted to the following union of disparities:

$$\alpha \leq t_k(x) \quad k=1,2,\dots, p+q$$

$$\beta \geq f_k(x) \quad k=1,2,\dots, p+q \quad (12)$$

$$\alpha + \beta \leq 1$$

$$\alpha \geq \beta, \beta \geq 0, x \in X$$

someplace represents the lowest acceptable notch of independent(s) and limits, and

signifies the maximum notch of criticism of objective(s) and irons.

By means of the unclear optimization, problematic (1) is now biased to the lined programming problematic assumed as:

$$\begin{aligned} &\text{exploit}(\alpha - \beta) \\ &\text{subject to } \alpha \leq t_k(x) \quad k=1,2,\dots, p+q \end{aligned}$$

$$\beta \geq f_k(x) \quad k=1,2,\dots, p+q$$

(13)

$$\alpha + \beta \leq 1$$

$$\alpha \geq \beta, \beta \geq 0, x \in X$$

Now, this lined software design problem can be effortlessly solved using a simplex technique to deliver an optimization response to a multiobjective linear programming problematic (1). Figure 1 portrays the lined membership and nonmembership purposes.

III. COMPUTATIONAL ALGORITHM

I. Way (Linear Association Function) (Linear Association Function)

Step 1: Select the first detached role from the hitch's set of k determinations and solve it as a on its own unprejudiced surrounded by dint of the constraints as short as. Control the significance of middle-of-the-road functions and decision variables.

Step 2: Using the judgment variables' standards, compute the values of the remaining $(k-1)$ objects.

Step 3: For the right-hand over $(k-1)$ objective senses, reappearance Rankings 1 and 2.

Step 4: Create the PIS bench by tabulating the standards of the dispassionate purposes obtained in Classifications 1 and 2 and 3.

Step 5: Using the figures from Step 4, determine the higher and upper leaps for each disinterested common sense.

TABEL I: POSITIVE IDEAL SOLUTION (PIS)

$\max f_1$	$f_1'(x_1) f_2'(x_2) f_3'(x_3) \dots f_k'(x_1)$	x_1
$\max f_2$	$f_1'(x_2) f_2'(x_2) f_3'(x_2) \dots f_k'(x_2)$	x_2
$\max f_3$	$f_1'(x_3) f_2'(x_3) f_3'(x_3) \dots f_k'(x_3)$	x_3
$\vdots$	$\vdots$	$\vdots$
$\max f_k$	$f_1'(x_k) f_2'(x_k) f_3'(x_k) \dots f_k'(x_k)$	x_k
	$f_1' f_2' f_3' \dots f_k'$	

where f_k^* and f_k' are the highest, smallest values respectively.

Stage 6. Set $U_k^\mu = \max (Z_k(X_r))$ and

$L_k^\mu = \min (Z_k(X_r))$ $1 \leq r \leq p$ for connotation and for non-membership resolutions $U_k^\mu = U_k^\mu - \lambda(U_k^\mu - L_k^\mu)$

$L_k^\mu = L_k^\mu$, $0 < \lambda < 1$.

Step 7. Use following lined association occupation($t_k(f_k)(x)$) and non-membership drive ($f_k(f_k)(x)$) for each objective functions:

$$t_k(f_k)(x) \begin{cases} 0 & \text{if } f_k(x) \leq L_k^t \\ \frac{f_k(x) - L_k^t}{U_k^t - L_k^t} & \text{if } L_k^t \leq f_k(x) \leq U_k^t \\ 1 & \text{if } f_k(x) \geq U_k^t \end{cases}$$

$$f_k(f_k)(x) \begin{cases} 0 & \text{if } f_k(x) \leq U_k^f \\ \frac{U_k^f - f_k(x)}{U_k^f - L_k^f} & \text{if } L_k^f \leq f_k(x) \leq U_k^f \\ 1 & \text{if } f_k(x) \geq L_k^f \end{cases}$$

Step 8. Today the vague optimization system for MOLP unruly (1) with lined association and non

relationship drives gives a equivalent linear software enterprise difficult as :

Maximize $(\alpha - \beta)$

subject to $\alpha \leq t_k(f_k(x))$

$\beta \geq (f_k(f_k(x)))$

$\alpha + \beta \leq 1$

$\alpha \geq \beta, \beta \geq 0,$ (14)

$g_j(x) \leq b_j, x \geq 0$

$k=1,2,\dots,p; j=1,2,\dots,q$

Step 9: Firmness the directly above crumpled software development problematic(14) by means of the simplex technique. B. Procedure II (Nonlinear Contribution Purpose) Stages 1–6 must be boring to make a bench of positive perfect answers.

Step 7: Undertake that the process computed keys review a hyperbolic purpose for relationship and an exponential job for non-membership.

$t_k(f_k)(x)$

$$x) \left\{ \begin{array}{ll} 0 & \text{if } f_k(x) < L_k^t \\ 1 - \exp\left\{-\psi \frac{f_k(x) - L_k^t}{U_k^t - L_k^t}\right\} & \text{if } L_k^t \leq f_k(x) \leq U_k^t \\ 1 & \text{if } f_k(x) \geq U_k^t \text{ and } \psi \rightarrow \infty \end{array} \right.$$

$$f_k(f_k)(x) \left\{ \begin{array}{ll} 1 & \text{if } f_k(x) \leq L_k^\mu \\ \frac{1}{2} + \frac{1}{2} \tanh\left\{\delta_k \frac{U_k^f + L_k^f}{2}\right\} & \text{if } L_k^f \leq f_k(x) \leq U_k^f \\ 0, & \text{if } f_k(x) \geq U_k^f \end{array} \right.$$

anywhere are non-zero boundaries fixed by the conclusion fabricator. Besides, the unknown optimization performance for MOLP unruly (1) by way of exponential envelopment and hyperbolic non production livings profits the direct software scheme badly-behaved: (-) Takings full disadvantage of

Subject to $\alpha \leq t_k(f_k)(x)$

$$1 - \exp\left\{-\psi \frac{f_k(x) - L_k^t}{U_k^t - L_k^t}\right\} \geq \alpha$$

$$\beta \geq f_k(f_k(x))$$

$$\frac{1}{2} + \frac{1}{2} \tanh\left\{\delta_k \frac{U_k^f - L_k^f}{2} - f_k(x)\right\} \leq \beta \quad (15)$$

$$\alpha + \beta \leq 1, \quad \alpha \geq \beta,$$

$$\beta \geq 0$$

$$g_i(x) \leq b_i, \quad x \geq 0 \quad k=1,2,\dots,p; \quad j=1,2,\dots,q$$

For result suitability the above your head problem (15) is partial to

Maximize $\gamma - \eta$

Subject to $f_k(x) - \frac{\gamma(U_k^t - L_k^t)}{4} \geq L_k^t$ where $\gamma - \log(1 - \alpha)$,

$$f_k(x) - \frac{\eta}{\delta_k} \geq \frac{U_k^f - L_k^f}{2}$$

where $\eta = \tanh^{-1}(2\beta - 1)$, and $\psi = 4$, (16)

$$\delta_k = \frac{6}{U_k^f - L_k^f}, \quad \gamma \geq \eta, \quad \gamma + \eta \leq 1, \quad \eta \geq 0$$

$$g_j(x) \leq b_j, \quad x \geq 0 \quad k=1,2,\dots,p; \quad j=1,2,\dots,q$$

Which can be effortlessly solved by a simplex method.

IV. Application

A. Building Preparation Problem Deliberate a joint of six machine systems whose measurements are to be whole-hearted to construction of three goods and chattels. A contemporary dimensions portfolio is available, measured in machine hours each weedy for individually machine amounts unit valued giving to apparatus type.

Essential information is recorded as in less Table II.

TABLE II: PHYSICAL PARAMETER VALUES

Machine type	Machi hours	Unit price (\$100 per hour)	Products		
			x1	x2	x3
Milling machine	1400	0.75	12	17	0
Lathe	1000	0.60	3	9	8
Grinder	1750	0.35	10	13	15
Jig saw	1325	0.50	6	0	16
Drill press	900	1.15	0	12	7
Band saw	1075	0.65	9.5	9.5	4
Total capacity cost \$4658.75					

TABLE III: POSITIVE IDEAL SOLUTION

	f_1	f_2	f_3	X
max f_1	8041.14	10020.33	9319.25	X_1
max f_2	5452.63	10950.59	5903.00	X_2
max f_3	7983.60	10056.99	9355.90	X_3

Let X_1 , X_2 , X_3 indicate three crops, then the ample in your own time formulation of the above stated tricky as a Multi objective Undeviating Software plan (MOLP) problematic is known as:

$$\text{Max } f_1(x) = 50x_1 + 100x_2 + 17.5x_3 \text{ (profit)}$$

$$\text{Max } f_2(x) = 92x_1 + 75x_2 + 50x_3 \text{ (quality)}$$

$$\text{Max } f_3(x) = 25x_1 + 100x_2 + 75x_3 \text{ (employee satisfaction)}$$

Subject to the restraints

$$12x_1 + 17x_2 \leq 1400$$

$$x_1 + 9x_2 + 8x_3 \leq 1000 \quad (17)$$

$$10x_1 + 13x_2 + 15x_3 \leq 1750$$

$$6x_1 + 16x_3 \geq 1325$$

$$x_1, x_2, x_3 \geq 0$$

Rejoinder of the overhead tricky is well thought-out by the means I and method II revealed in preceding components. For policy of the events almost of steps stand showing as

Step 1. Disentangle a wrinkly software proposal problem good-looking one and only objectiv

$$\text{Exploit } f_1 = 50x_1 + 100x_2 + 17.5x_3$$

Topic to the fetters

$$12x_1 + 17x_2 \leq 1400$$

$$x_1 + 9x_2 + 8x_3 \leq 1000$$

$$\llbracket 10x \rrbracket_{-(1+)} 13x_{-(2+)} 15x_{-3} \leq 1750$$

$$6x_{-(1+)} 16x_{-3} \geq 1325 \quad (18)$$

$$\llbracket 12x \rrbracket_{-(2+)} 7x_{-3} \leq 900$$

$$\llbracket 9.5x \rrbracket_{-(1+)} 9.5x_{-(2+)} 4x_{-3} \leq 1075$$

$$x_{-(1,)} x_{-2} \llbracket x \rrbracket_{-3} \geq 0$$

TABLE IV: VALUES OF OPTIMAL DECISION VECTORS

vague optimization Technique when association and Non- memberships are lined.					
λ	x_1	x_2	x_3	α	β
.1	65.2571	26.9187	49.8324	.5899	.4101
.2	58.4833	34.5907	47.6992	.8525	.1475
.3	65.2600	26.9155	49.8333	.7583	.2417
.4	65.2585	26.9172	49.8328	.8847	.1153
.5	66.1947	25.8441	49.2978	1.000	.0000
.6	71.1362	22.6184	44.8504	1.000	.0000
.7	71.7199	25.7841	35.6084	1.000	.0000

.8	75.3355	14.2823	45.3258	1.000	.0000
.9	82.1131	9.12270	46.1075	1.000	.0000
vague optimization Technique when membership and Non- memberships are Non-linear					
.1	49.8906	47.1360	42.5550	.6321	.3345
.2	64.6968	36.6846	41.7421	.6321	.0073
.3	62.1896	38.0097	41.8452	.6321	.0009
.4	62.8180	38.0109	41.5300	.6321	.0001
.5	62.8157	38.0125	41.8454	.6321	.0000
.6	62.8163	38.0120	41.8454	.6321	.0000
.7	59.7690	40.1631	42.0127	.6321	.0000
.8	62.8265	38.0048	41.8448	.6321	.0000
.9	62.8207	38.0087	41.8451	.6321	.0000

Best solution to this crisp lined software scheme problem is

$$x_1=44.93, \quad x_2=50.63, \quad x_3=41.77$$

$$(f_1)_1=8041.14$$

Step 2. With these choice variables, equaled standards of further residual impartial purposes are:

$$\| (f_2) \|_{(1=10020.33)}$$

$$\| (f_3) \|_{(1=9319.25)}$$

Step 3. Step 1 and Step 2 are recurrent for other impartial functions f_2, f_3 .

Step 4. The got Positive Unconquerable Solution (PIS) is renowned in Desk III. The answers of the above-mentioned MOLP are got using the comeback algorithms I and II. The difficult is solved using lined relationship also nonmembership purposes, as well as nonlinear relationship and nonmembership tenacities, and the keys got are to be found in Counter IV

to deliver insight into the resolution practice. The Bench describes the feasibility of resolutions in terms of a number of levels of approval.

TABLE V: VALUES OF OPTIMAL OBJECTIVE FUNCTIONS

vague optimization Technique when membership and Non- memberships are linear.				
λ	$\max f_1$	$\max f_2$	$\max f_3$	Total
0.1	6826.79 20	10514.17 57	8060.72 75	25401.69 52
0.2	7217.97 10	10359.72 61	8498.59 25	26076.28 96
0.3	6826.63 28	10514.24 75	8060.54 75	25401.42 78
0.4	6826.71 90	10514.21 20	8060.64 25	25401.57 35
0.5	6756.85 65	10493.10 99	7936.61 25	25186.57 89
0.6	6603.53 20	10483.43 04	7404.02 50	24490.98 74
0.7	6787.55 20	10312.45 83	7041.03 75	24142.04 78
0.8	5988.20 65	10268.32 85	6711.05 25	22967.58 75
0.9	5824.80 63	10543.98 27	6423.16 00	22791.94 90
vague optimization Technique when membership and Non- memberships are non-linear.				
λ	$\max f_1$	$\max f_2$	$\max f_3$	Total
0.1	7952.84 25	10252.88 52	9152.49 00	27358.21 77
0.2	7633.78 68	10790.55 56	8416.53 75	26840.87 93
0.3	7642.74 10	10664.43 07	8494.10 00	26801.27 17
0.4	7668.76 50	10706.57 35	8486.29 00	26861.62 85
0.5	7674.32 95	10722.25 19	8510.04 75	26906.62 89
0.6	7674.30 95	10722.26 96	8510.01 25	26906.59 16
0.7	7737.98 23	10611.61 55	8661.48 75	27011.08 53

0. 8	7674.08 90	10722.63 80	8509.50 25	26906.22 95
0. 9	7674.19 42	10722.41 19	8509.77 00	26906.37 61

TABLE VI: COMPARISON OF OPTIMAL SOLUTIONS OBTAINED BY VARIOUS METHODS

Decision variables & objective functions	Best Solution obtained by fuzzy optimization method with level of satisfaction $\alpha=0.5309$	Best Solution obtained by proposed vague optimization algorithm I	Best Solution obtained by proposed vague optimization algorithm II
x1	65.2571	58.4833	49.8906
x2	26.9187	34.5907	47.1360
x3	49.8324	47.6992	42.5550
f1	6826.7920	7217.9710	7952.8425
f2	10514.175 7	10359.726 1	10252.885 2
f3	8060.7275	8498.5925	9152.4900
Sum of objectives	25401.695 2	26076.289 6	27358.217 7

V. CONCLUSIONS

In order to compare the vague optimization method with the fuzzy optimization method, we obtained the solution of the numerical problem by fuzzy optimization.

The optimization scheme according to Zimmermann [17] was superfluous and the superlative domino effect attained were left through and sold for appraisal with the present-day scholarship. We have cautiously considered

the preeminent perseverance achieved by the two established algorithms in Board VI Mandate, and in each case additionally compared them with the results gained by the fuzzy optimization means. The aim of the recent exploration is to arrange for an active algorithm for unclear optimization methods to find prime way out to multi-objective linear programming teething troubles. The advantage of this line of attack is that it provides resolution makers with a solution that is of wavering degrees of interest. Decision makers can choose the appropriate worst solution according to the prerequisites of the current situation. In addition, a comparison of our underperforming effects clearly shows that fuzzy optimization underperforms fuzzy optimization. The achieved results also explain why the fuzzy optimization system II with nonlinear true value and nonlinear false value gives better results than fuzzy optimization algorithm I using line association function and line non-membership gathering.

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