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Reliability Analysis of Topologies Derived from Ring Topology

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Abstract

With social development, people rely more and more on the computer network, and also have higher requirements for the reliability of network communications, and optimizing the reliability of computer network communications is a topological Structure optimization process, it involves network spending, medium delay and reliability, is a multi-objective optimization process. The main contributions to the analysis of topology constructed by adding new edges in ring topology are made in this paper. By determining the reliability using the Wiener Index and data transfer using Cisco packet simulation, the effectiveness of the novel topologies is evaluated. The results demonstrate that the new topologies at each stage are more reliable than the ring topology and take less time for data transfer, less hops and less risk factor.

Keywords—Reliability Wiener index, Wiener Polynomial, data transfer, networks**.**

1. Introduction

Operations Research (OR) performance analysis topics associated with Computer Network Topology tend to be concerned with logical topology rather than purely physical. This paper will review several performance analysis study examples with the intent of demonstrating the importance of topological considerations in network design. Since the fundamental Network Physical Topology is built upon by the Logical Network Topology. The set of standard computer physical topologies will be reviewed first as background to the performance analysis discussion. The physical topology of a network refers to the configuration of cables, computers, and other peripherals. The term "topology" refers to the layout of connected devices on a network. Ring Network Topology has each node in a network connected to two other nodes in the network in conjunction with the first and last nodes being connected. Messages from one node to another then travel from originator to destination via the set of intermediate nodes. The intermediate nodes serve as active repeaters for messages intended for other nodes. Ring Network Topology is typically employed in networks where inter node traffic volume is small and failure of the cabling between any two nodes has a broader impact on network communication as a whole, possibly leaving no path from message originator to recipient. Mesh Network Topologies capitalize on ring redundancy. This Topology is preferred when traffic volume between nodes is large. A proportion of nodes in this type of topologies have multiple paths to another destination node. With the exception of the Bi-directional Ring (and this was only when a failure was detected) each of the topologies discussed so far had only one path from message source to message destination. Thus the probability of single point network failure is greatly minimized with Mesh Network Topology.

A major advantage of the Mesh Network Topology is that source nodes determine the best route from sender to destination based upon such factors connectivity, speed, and pending node tasks. This paper derived mesh topology from ring topology. At every stage analysis is done by reducing the distance between the nodes of distance from two to one by adding new edges in ring topology and in finite number of times to obtain Mesh topology. Newman (2003) studied about complex networks in terms of structure and function. Mou Dasgupta and Biswas (2012) proposed technique to enhance reliability of communication networks, which identify the node-pairs having lower reliability, insert communication links (edges) in them and calculate the increase in reliability on insertion iteratively until the satisfactory reliability is achieved.

Definition 1.1: Wiener polynomial of a connected graph G is defined as $W(G;q)=\sum_{\{u,v\}\subset V(G)} q^{d_G(u,v)}$ where $d_G(u,v)$ denotes the distance between two vertices u and v in G and $W(G)=\sum_{\{u,v\}\subset V(G)} d_G(u,v)=\left.\frac{dW(G;q)}{dq}\right|_{q=1}$

2. Systematization of Mesh Topologies

By including a new edge, it is possible to decrease the distance between the nodes of the ring topology from two to one. The interesting case will be reducing the ring topology to mesh topology in finite number of steps. By this process ring topology is reduced to $G_2^k(C_n)$ thereby the reliability Wiener index of the resultant graph gradually decreases.

Definition 2.1: The topology $G_2^k(C_n)$ is obtained from $G_2^{k-1}(C_n)$ by introducing an edge between nodes of distance two.

Consider a ring topology C_n with n nodes. The topology $G_2^1(C_n)$ is obtained from C_n by connecting nodes of distance two by an edge. Consider a ring topology C8 then $G_2^1(C_8)\,\&\,G_2^2(C_8)$ are in Fig. 2.1 and Fig.2.2.

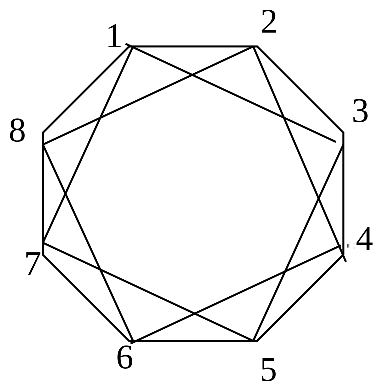

Fig. 2.1 First stage topology $G_2^1(C_8)$ Fig. 2.2 Second stage topology $G_2^2(C_8)$

Theorem 1.1: The Wiener polynomial of $G_2^1(C_n)$ is

$$W[G_2^1(C_{4m-2}),q]=2n(q+q^2+\ldots+q^{m-1})+\frac{n}{2}q^m$$

$$W[G_2^1(C_{4m-1}),q]=2n(q+q^2+\ldots+q^{m-1})+nq^m$$

$$W[G_2^1(C_{4m}),q]=2n(q+q^2+\ldots+q^{m-1})+\frac{3n}{2}q^m$$

$$W[G_2^1(C_{4m+1}),q]=2n(q+q^2+\ldots+q^{m}) \quad ,m=2,3,4,\ldots\,\&\,n=6,7,\ldots$$

Proof: Since C_n is a 2-regular graph $G_2^1(C_n)$ is a 4-regular graph. Graphs $G_2^1(C_n)$ with up to five vertices will be complete graphs and hence $n\geq 6$

The distance graphs of $G_2^1(C_6) \,\&\, G_2^1(C_{10})$ are as follows

$M(G_2^1(C_6)) =$

	1	**2**	**3**	**4**	**5**	**6**
1	0	1	1	2	1	1
2	-	0	1	1	2	1
3	-	-	0	1	1	2
4	-	-	-	0	1	1
5	-	-	-	-	0	1
6	-	-	-	-	-	0

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	0	1	1	2	2	3	2	2	1	1
2	-	0	1	1	2	2	3	2	2	1
3	-	-	0	1	1	2	2	3	2	2
4	-	-	-	0	1	1	2	2	3	2
5	-	-	-	-	0	1	1	2	2	3
6	-	-	-	-	-	0	1	1	2	2
7	-	-	-	-	-	-	0	1	1	2
8	-	-	-	-	-	-	-	0	1	1
9	-	-	-	-	-	-	-	-	0	1
10	-	-	-	-	-	-	-	-	-	0

Wiener polynomials of $G_2^1(C_n)$

$$W(G_2^1(C_6);q) = (5+4+1+2)q + 3q^2 = 12q + 3q^2$$

$$W(G_2^1(C_7);q) = (6+5+1+2)q + (4+3)q^2 = 14q + 7q^2$$

$$W(G_2^1(C_8);q) = (7+6+1+2)q + (5+4+3)q^2 = 16q + 12q^2$$

$$W(G_2^1(C_9);q) = (8+7+1+2)q + (6+5+4+3)q^2 = 18q + 18q^2$$

$$W(G_2^1(C_{10});q) = (9+8+1+2)q + (7+6+4+3)q^2 + 5q^3 = 20q + 20q^2 + 5q^3$$

$$W(G_2^1(C_{11});q) = (10+9+1+2)q + (8+7+4+3)q^2 + (5+6)q^3$$
$$= 22q + 22q^2 + 11q^3$$

$$W(G_2^1(C_{12});q) = (11+10+1+2)q + (9+8+4+3)q^2 + (7+6+5)q^3$$
$$= 24q + 24q^2 + 18q^3$$

$$W(G_2^1(C_{13});q) = (12+11+1+2)q + (10+9+4+3)q^2 + (8+7+6+5)q^3$$
$$= 26q + 26q^2 + 26q^3$$

Any number $n \geq 6$ can be written as 4m-2 or 4m-1 or 4m or 4m+1 for m = 2,3,4,…. Hence

$$W[G_2^1(C_{4m-2}),q] = 2n(q + q^2 + \ldots + q^{m-1}) + \frac{n}{2}q^m$$

$$W[G_2^1(C_{4m-1}),q] = 2n(q + q^2 + \ldots + q^{m-1}) + nq^m$$

$$W[G_2^1(C_{4m}),q] = 2n(q + q^2 + \ldots + q^{m-1}) + \frac{3n}{2}q^m$$

$$W[G_2^1(C_{4m+1}),q] = 2n(q + q^2 + \ldots + q^m), \qquad m = 2,3,4,\ldots \,\&\, n = 6,7,\ldots$$

Result 1.1:

$$W[G_2^1(C_{4m-2})) = nm^2 - \frac{nm}{2}, W[G_2^1(C_{4m-1})) = nm^2$$

$$W[G_2^1(C_{4m})) = nm^2 + \frac{nm}{2}, W[G_2^1(C_{4m})) = nm^2 + nm$$

3. Reliability Analysis

In mesh topology every node is connected to every other node and is more reliable than the ring topology. The systematic way of construction of mesh topology from ring topology derives different topologies in each stage which are physical topology which emphasizes the physical layout of nodes in different stages. The reliability of the new series of topologies is calculated using above reliability Wiener index formula. The calculation depends on the number of nodes in the ring topology. The sum of distance between all pair of nodes is derived in the above theorem in each stage which defines reliability of the network. The reliability of the new topology is proved by comparing its reliability with ring topology.

A graphical comparison is made in Fig. 3.1 by taking number of nodes in X-axis and Wiener index in Y-axis using above Table 3.1. The two curves are plotted based on Calculation of Wiener Index. One curve include the plot made between Number of nodes and its equivalent Wiener Index of ring Topology and other include Number of nodes and its equivalent Wiener Index of $G_2^k(C_n)$ topology. We can see that new topology has lower Wiener Index compared to same number of nodes in ring topology.

Table 3.1 Results of Ring Topology and $G_2^k(C_n)$

N	W(C_n)	W($G_2^k(C_n)$)
5	15	10
6	27	15
7	42	21
8	64	28
9	90	36
10	125	45

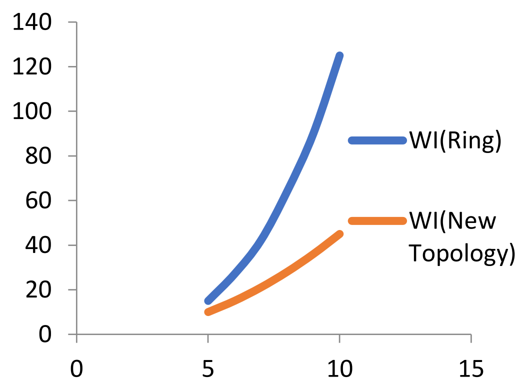

Fig. 3.1 Graphical Representation

Logical topology focuses on the pattern of the data transfer between network nodes and is studied by analyzing the different parameters like data transfer, packet loss, fault tolerance and reliability of network. Data transfer from source to destination takes 2ms in new topology in particular stage and 15ms in ring topology. Packet loss occurs when one or more packets of data travelling across a network fail to reach the destination, 0% packet loss is identified in 2ms in new topology whereas 0% packet loss is identified in 15ms in ring topology and number of hops is reduced from 8 in ring topology to 3 in new topology. The ability of the new topology to continue operating without interruption even one or more components fail is more than the ability of the ring topology. The sum of distance between nodes decreases in the new topology and hence reliability increases.

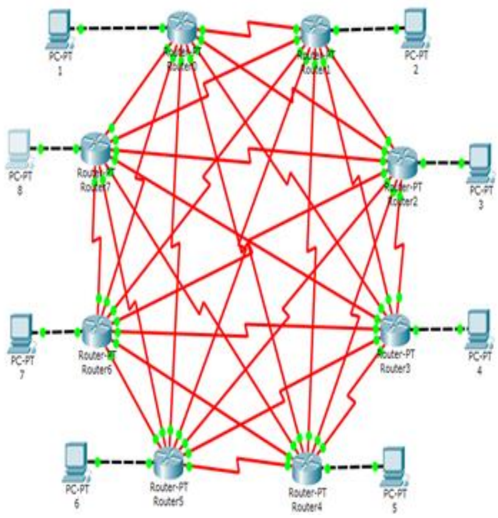

Fig. 3.2 Fourth stage topology $G_2^4(C_8)$

Table 3.2 Comparison of Ring Topology and $G_2^k(C_n)$

Parameter	Ring Topology	$G_2^k(C_n)$
Data Transfer	15ms	2ms
Packet Loss	0% packet loss data reaches destination in 15ms	0% packet loss data reaches destination in 2ms
No of hops	8	3
Wiener Index	High	Low
Reliability	Low	High

Hence, from Fig. 3.2 and Table 3.2, it is concluded that $G_2^k(C_n)$ topologies, derived from ring topology by introducing edge between nodes of distance two are more reliable than the ring topology as the $G_2^k(C_n)$ topologies take less time for data transfer, less number of hops, packet loss is less and the data still reaches the destination by travelling through the most reliable path in every stage.

4. Conclusion

The reliability Wiener index for new topologies constructed by introducing edges in a systematic way is calculated. The regular topology with every vertex of degree four is more efficient in terms of reliability, data transfer, number of hops and packet loss. At each stage in the process of obtaining mesh topology from ring topologies take less time for data transfer, more reliable and less packet loss and data reaches the destination through the reliable path.

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