Application of Jacobi Polynomial and Incomplete H-Function inHeat condition in Non-Homogeneous Moving Rectangular Parallelepiped

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Abstract

The present paper deals with an application of Jacobi polynomial and incomplete H –Function to solve the differential equation of heat conduction in non-homogeneous moving rectangularparallelepiped. The temperature distribution in the parallelepiped, moving in a direction of length(x-axis) between the limits x=-1 and x=1has been considered. The conductivity and velocity havebeen assumed to be variable.

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INTRODUCTION

Partial differential equation of $v(x,y,z,t)$ with time t and boundary condition y=0 and y=b, z=0 and z=c with constant velocity V along x-axis where limit of x is between -1 and 1 then by Carslaw ad Jaeger[16]

$$\mu\left[\frac{\partial^2\upsilon}{\partial u^2}+\frac{\partial^2\upsilon}{\partial v^2}+\frac{\partial^2\upsilon}{\partial w^2}\right]-V\frac{\partial\upsilon}{\partial u}-\frac{\partial\upsilon}{\partial t}=0\ ;\ \mu=\frac{E}{\sigma\rho} \tag{1}$$

Where E = Heat conductivity

ρ =Density

μ =Diffusivity

σ =Specific Heat

Let a parallelepiped [1]-[10] (non-homogeneous) whose conductivity is $\mu^{'}(1-u^2)$ and velocity is $\mu_0\left[(c_1-c_2)+(c_1+c_2)u\right]$ where $\mu^{'},\mu_0,c_1,c_2$ are constants. So from equation (1) we get

$$\frac{\partial\upsilon}{\partial t}=\mu_0\left[(1-u^2)\frac{\partial^2\upsilon}{\partial u^2}+((c_2-c_1)-(c_1+c_2+2)u)\frac{\partial\upsilon}{\partial u}\right]+\mu\left[\frac{\partial^2\upsilon}{\partial v^2}+\frac{\partial^2\nu}{\partial w^2}\right]=0 \tag{2}$$

Where $\mu_0=\dfrac{\mu^{'}}{\sigma\rho}$,$\mathrm{Re}(c_1)>-1,\mathrm{Re}(c_2)>-1$

Solution of the problem

Let $v(u,v,w,t) = X(u)Y(v)Z(w)T(t)$ (3)

be the solution of partial differential equation (2) then the equation reduces to [11]-[15]:

$$\frac{1}{T}\frac{dT}{dt} = \frac{\mu_0}{X}\left[(1-u^2)\frac{d^2X}{du^2} + (-c_1 + c_2 - (c_1 + c_2 + 2)u)\frac{dX}{du}\right] + \frac{\mu}{Y}\frac{d^2Y}{dv^2} + \frac{\mu}{Z}\frac{d^2Z}{dw^2} \tag{4}$$

Suppose $\frac{\mu_0}{X}\left[(1-u^2)\frac{d^2X}{du^2} + (-c_1 + c_2 - (c_1 + c_2 + 2)u)\frac{dX}{du}\right] = -\mu_0 m(m + c_1 + c_2 + 1)$ (5)

$$\frac{\mu}{Y}\frac{d^2Y}{dv^2} = -\mu\omega^2 \text{ and } \frac{\mu}{Z}\frac{d^2Z}{dw^2} = -\mu\lambda^2 \tag{6}$$

Where ω, λ constants and 'm' is a positive integer .Then from equation (4), (5), (6) we get

$$\frac{1}{T}\frac{dT}{dt} = \left[-m(m + c_1 + c_2 + 1) - \mu(\omega^2 + \lambda^2)\right]T \tag{7}$$

$$(1-u^2)\frac{d^2X}{du^2} + (-c_1 + c_2 - (c_1 + c_2 + 2)u)\frac{dX}{du} + m(m + c_1 + c_2 + 1)X = 0 \tag{8}$$

$$\frac{d^2Y}{dv^2} + \omega^2 v = 0 \tag{9}$$

$$\frac{d^2Z}{dw^2} + \lambda^2 w = 0 \tag{10}$$

Solution of equations (7), (8), (9), (10) is respectively

$$T = Me^{\left[-(\mu_0 m(m+c_1+c_2+1)+\mu(\omega^2+\lambda^2))t\right]} \tag{11}$$

$$X = P_m^{(c_1,c_2)}(u) \tag{12}$$

$$Y = C_1 \cos\omega v + C_2 \sin\omega v \tag{13}$$

$$Z = C_3 \cos\lambda w + C_4 \sin\lambda w \tag{14}$$

Where C_1, C_2, C_3, C_4, M are constants.

Hence the general solution of equation (2) is as follows:

$$\upsilon(u,v,w,t) = e^{\left[-(\mu_0 m(m+c_1+c_2+1)+\mu(\omega^2+\lambda^2))t\right]} P_m^{(c_1,c_2)}(u)$$

$$\times\left[C_1 \cos\omega v + C_2 \sin\omega v\right]\left[C_3 \cos\lambda w + C_4 \sin\lambda w\right] \tag{15}$$

In absence of heat $\left(\frac{\partial\upsilon}{\partial v}\right) = 0 \ at \ v = 0, v = b$

$$\left(\frac{\partial \upsilon}{\partial w}\right) = 0 \ at \ w = 0, w = b$$

This shows that $C_2 = C_4 = 0,\ \omega = \frac{m_1 \pi}{b},\ \lambda = \frac{l_1 \pi}{c}$

Then equation (15) reduces to

$$\upsilon(u,v,w,t) = \sum_{m,m_1,l_1}^{\infty} A_{mm_1l_1} e^{\left[-(\mu_0 m(m+c_1+c_2+1)+\mu(\omega^2+\lambda^2))t\right]} P_m^{(c_1,c_2)}(u) \cos\left(\frac{m_1\pi}{b}\right) v \cos\left(\frac{l_1\pi}{c}\right) w \quad (16)$$

On solving and applying result [17] we get

$$\sum_{m,m_1,l_1}^{\infty} A_{mm_1l_1} = \frac{m!(c_1+c_2+2m+1)\Gamma(c_1+c_2+m+1)}{2^{c_1+c_2+1}\Gamma(c_1+m+1)\Gamma(c_2+m+1)}$$

$$\times \int_0^c \int_0^b \int_{-1}^1 (1-u)^{c_1}(1+u)^{c_2} P_m^{(c_1,c_2)}(u) \cos\left(\frac{m_1\pi}{b}\right) v \cos\left(\frac{l_1\pi}{c}\right) v\, \phi(u,v,w)\, du\, dv\, dw \quad (17)$$

Hence the temperature distribution in moving rectangular parallelepiped is given by

$$\upsilon(u,v,w,t) = \sum_{m,m_1,l_1}^{\infty} \frac{m!(c_1+c_2+2m+1)\Gamma(c_1+c_2+m+1)}{2^{c_1+c_2+1}\Gamma(c_1+m+1)\Gamma(c_2+m+1)}$$

$$\times e^{\left[-\left(\mu_0 m(m+c_1+c_2+1)+\mu\pi^2\left(\frac{m_1^2}{b^2}+\frac{l_1^2}{c^2}\right)\right)t\right]} P_m^{(c_1,c_2)}(u) \cos\left(\frac{m_1\pi}{b}\right) v \cos\left(\frac{l_1\pi}{c}\right) w$$

$$\times \int_0^c \int_0^b \int_{-1}^1 (1-u)^{c_1}(1+u)^{c_2} P_m^{(c_1,c_2)}(u) \cos\left(\frac{m_1\pi}{b}\right) v \cos\left(\frac{l_1\pi}{c}\right) w\ \ \phi(u,v,w)\, du\, dv\, dw \quad (18)$$

with $\mathrm{Re}(c_1) > -1, \mathrm{Re}(c_2) > -1$

Incomplete H-function $\gamma_{p,q}^{n,k}(u)$ and $\Gamma_{p,q}^{n,k}(u)$ as follows:

The incomplete H-function and Gamma functions are defined below [18,19]:

$$\gamma_{p,q}^{n,k}(u) = \gamma_{p,q}^{n,k}\left[u \left| \begin{matrix} (a_1, A_1, h), (a_j, A_j)_{2,p} \\ (b_j, B_j)_{1,q} \end{matrix} \right.\right] = \gamma_{p,q}^{n,k}\left[u \left| \begin{matrix} (a_1, A_1, h), (a_2, A_2), \ldots, (a_p, A_p) \\ (b_1, B_1), (b_2, B_2), \ldots, (b_q, B_q) \end{matrix} \right.\right]$$

$$= \frac{1}{2\pi i} \int_L g(s,h) u^{-s} ds$$

where $g(s,h) = \dfrac{\gamma(1-a_1-A_1 s, h)\prod_{j=1}^{n}\Gamma(b_j+B_j s)\prod_{j=2}^{k}\Gamma(1-a_j+A_j s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j-B_j s)\prod_{j=n+2}^{p}\Gamma(a_j+A_j s)}$

and

$$\Gamma_{p,q}^{n,k}(u)=\Gamma_{p,q}^{n,k}\left[u\left|\begin{matrix}(a_1,A_1,h),(a_j,A_j)_{2,p}\\(b_j,B_j)_{1,q}\end{matrix}\right.\right]=\Gamma_{p,q}^{n,k}\left[u\left|\begin{matrix}(a_1,A_1,h),(a_2,A_2),...,(a_p,A_p)\\(b_1,B_1),(b_2,B_2),...,(b_q,B_q)\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_L g(s,h)u^{-s}ds$$

where $G(s,h)=\dfrac{\Gamma(1-a_1-A_1s,h)\prod_{j=1}^{n}\Gamma(b_j+B_js)\prod_{j=2}^{k}\Gamma(1-a_j+A_js)}{\prod_{j=m+1}^{q}\Gamma(1-b_j-B_js)\prod_{j=n+2}^{p}\Gamma(a_j+A_js)}$

$$A_{mm_1l_1}=\frac{n!(c_1+c_2+2m+1)\Gamma(c_1+c_2+m+1)}{2^{c_1+c_2+1}\Gamma(c_1+m+1)\Gamma(c_2+m+1)}\int_0^c\int_0^b\int_{-1}^1(1-u)^{c_1}(1+u)^{c_2}P_m^{(c_1,c_2)}(u)$$

$$\times\cos\left(\frac{m_1\pi}{b}\right)v\cos\left(\frac{l_1\pi}{c}\right)w.\phi(u,v,w)dw\,dv\,du$$

$$A_{mn_1l_1}=\frac{m!(c_1+c_2+2m+1)\Gamma(c_1+c_2+m+1)}{2^{c_1+c_2+1}\Gamma(c_1+m+1)\Gamma(c_2+m+1)}\int_0^c\int_0^b\int_{-1}^1(1-u)^{c_1}(1+u)^{c_2}P_m^{(c_1,c_2)}(u)\cos\left(\frac{m_1\pi}{b}\right)v\cos\left(\frac{l_1\pi}{c}\right)w$$

$$\times\phi_1(u)\phi_2(v)\phi_3(w)dw\,dv\,du$$

where $\phi_1(u)$ = incomplete H-Function in contour form

$\phi_2(v)=e^{-\eta v}$, $\phi_3(w)=e^{-\delta w}$

$$A_{mm_1l_1}=\frac{m!(c_1+c_2+2m+1)\Gamma(c_1+c_2+m+1)}{2^{c_1+c_2+1}\Gamma(c_1+m+1)\Gamma(c_2+m+1)}\int_0^c\int_0^b\int_{-1}^1(1-u)^{c_1}(1+u)^{c_2}P_m^{(c_1,c_2)}(u)\cos\left(\frac{m_1\pi}{b}\right)v\cos\left(\frac{l_1\pi}{c}\right)w$$

$$\times\gamma_{p,q}^{n,k}\left[(1-u)^{-r}(1+u)^{-t}\right]e^{-\eta v}e^{-\delta w}dw\,dv\,du$$

By the definition of incomplete H-function

$$A_{mm_1l_1}=\frac{m!(c_1+c_2+2m+1)\Gamma(c_1+c_2+m+1)}{2^{c_1+c_2+1}\Gamma(c_1+m+1)\Gamma(c_2+m+1)}\int_0^c\int_0^b\int_{-1}^1(1-u)^{c_1}(1+u)^{c_2}P_m^{(c_1,c_2)}(u)\cos\left(\frac{m_1\pi}{b}\right)v\cos\left(\frac{l_1\pi}{c}\right)w$$

$$\times\frac{1}{2\pi i}\left[\int_L g(s,h)(1-u)^{rs}(1+u)^{ts}ds\right]e^{-\eta v}e^{-\delta w}dw\,dv\,du$$

$$A_{mm_1l_1} = \frac{m!(c_1+c_2+2m+1)\Gamma(c_1+c_2+m+1)}{2^{c_1+c_2+1}\Gamma(c_1+m+1)\Gamma(c_2+m+1)}\int_{-1}^{1}\Big\{(1-u)^{c_1}(1+u)^{c_2}P_m^{(c_1,c_2)}(u)$$

$$\times\left[\frac{1}{2\pi i}\int_L g(s,h)(1-u)^{rs}(1+u)^{ts}\,ds\right]\times\left[\int_0^b e^{-\eta v}\cos\left(\frac{m_1\pi}{b}\right)v\,dv\right]\left[\int_0^c e^{-\delta w}\cos\left(\frac{l_1\pi}{c}\right)w\,dw\right]\Big\}dz \tag{19}$$

Let $I_1 = \left[\int_0^b e^{-\eta v}\cos\left(\frac{m_1\pi}{b}\right)v\,dv\right] = \frac{b^2\eta(1-e^{-\eta b}\cos m_1\pi)}{(b^2\eta^2+m_1^2\pi^2)}$

and $I_2 = \left[\int_0^c e^{-\delta z}\cos\left(\frac{l_1\pi}{c}\right)w\,dw\right] = \frac{c^2\delta(1-e^{-\delta c}\cos l_1\pi)}{(c^2\delta^2+l_1^2\pi^2)}$

On putting the value of I_1 and I_2 in equation (19) we get,

$$A_{mm_1l_1} = \frac{m!(c_1+c_2+2m+1)\Gamma(c_1+c_2+m+1)}{2^{c_1+c_2+1}\Gamma(c_1+m+1)\Gamma(c_2+m+1)}\frac{b^2c^2\eta\delta(1-e^{-\eta b}\cos m_1\pi)}{(b^2\eta^2+m_1^2\pi^2)}\frac{(1-e^{-\delta c}\cos l_1\pi)}{(c^2\delta^2+l_1^2\pi^2)}$$

$$\times\int_{-1}^{1}(1-u)^{c_1}(1+u)^{c_2}P_m^{(c_1,c_2)}(u)\left[\frac{1}{2\pi i}\int_L g(s,h)(1-u)^{rs}(1+u)^{ts}\,ds\right]du$$

$$A_{mm_1l_1} = \frac{m!(c_1+c_2+2m+1)\Gamma(c_1+c_2+m+1)}{2^{c_1+c_2+1}\Gamma(c_1+m+1)\Gamma(c_2+m+1)}\frac{b^2c^2\eta\delta(1-e^{-\eta b}\cos m_1\pi)}{(b^2\eta^2+m_1^2\pi^2)}\frac{(1-e^{-\delta c}\cos l_1\pi)}{(c^2\delta^2+l_1^2\pi^2)}$$

$$\times\frac{1}{2\pi i}\int_L g(s,h)\left[\int_{-1}^{1}(1-u)^{c_1+rs}(1+u)^{c_2+ts}P_m^{(c_1,c_2)}(u)\,du\right]ds$$

On applying result [17]

$$A_{mm_1l_1} = \frac{m!(c_1+c_2+2m+1)\Gamma(c_1+c_2+m+1)}{2^{c_1+c_2+1}\Gamma(c_1+m+1)\Gamma(c_2+m+1)}\frac{b^2c^2\eta\delta(1-e^{-\eta b}\cos m_1\pi)}{(b^2\eta^2+m_1^2\pi^2)}\frac{(1-e^{-\delta c}\cos l_1\pi)}{(c^2\delta^2+l_1^2\pi^2)}$$

$$\times\frac{1}{2\pi i}\int_L g(s,h)\,2^{c_1+c_2+rs+ts+1}\frac{\Gamma(c_1+rs+1)\Gamma(c_2+ts+1)}{\Gamma(c_1+c_2+rs+ts+2)}$$

$$\times{}_3F_2(-n,c_1+c_2+n+1,c_1+rs+1,c_1+1,c_1+c_2+rs+ts+2;1)ds$$

$$A_{mm_1l_1} = \frac{m!(c_1+c_2+2m+1)\Gamma(c_1+c_2+m+1)}{2^{c_1+c_2+1}\Gamma(c_1+m+1)\Gamma(c_2+m+1)}\frac{b^2c^2\eta\delta(1-e^{-\eta b}\cos m_1\pi)}{(b^2\eta^2+m_1^2\pi^2)}$$

$$\times\frac{(1-e^{-\delta c}\cos l_1\pi)}{(c^2\delta^2+l_1^2\pi^2)}\frac{\Gamma(c_1+rs+1)\Gamma(c_2+ts+1)}{\Gamma(c_1+c_2+rs+ts+2)}$$

$$\times 2^{s(r+t)}\frac{1}{2\pi i}\int_L g(s,h)\frac{\Gamma(-m+N)}{\Gamma(-m)}\frac{\Gamma(c_1+c_2+m+N+1)}{\Gamma(c_1+c_2+n+1)}\frac{\Gamma(c_1+rs+N+1)}{\Gamma(c_1+rs+1)}$$

$$\times \frac{\Gamma(c_1+1)}{\Gamma(c_1+N+1)} \frac{\Gamma(c_1+c_2+rs+ts+2)}{\Gamma(c_1+c_2+rs+ts+N+2)} \frac{z^N}{N!} ds$$

$$A_{mm_1l_1} = \frac{m!(c_1+c_2+2m+1)\Gamma(c_1+1)2^{s(r+t)}}{2\pi i\,\Gamma(c_1+m+1)\Gamma(c_2+m+1)} \frac{\eta\delta[1-(-1)^{m_1}e^{-\eta b}]}{\left(\eta^2+\frac{m_1^2\pi^2}{b^2}\right)} \frac{[1-(-1)^{l_1}e^{-\delta c}]}{\left(\delta^2+\frac{l_1^2\pi^2}{c^2}\right)}$$

$$\times \frac{\Gamma(c_1+rs+1+N)\Gamma(c_1+c_2+m+1+N)\Gamma(c_2+ts+1)}{\Gamma(c_1+c_2+rs+ts+2+N)\Gamma(c_1+1+N)} \frac{\Gamma(-m+N)}{\Gamma(-m)} \int_L g(s,h) \frac{z^N}{N!} ds$$

$$A_{mm_1l_1} = \frac{m!(c_1+c_2+2n+1)\Gamma(c_1+1)}{\Gamma(c_1+m+1)\Gamma(c_2+m+1)} \frac{\eta\delta[1-(-1)^{m_1}e^{-\eta b}]}{\left(\eta^2+\frac{m_1^2\pi^2}{b^2}\right)} \frac{[1-(-1)^{l}\ e^{-\delta c}]}{\left(\delta^2+\frac{l_1^2\pi^2}{c^2}\right)}$$

$$\times \frac{\Gamma(c_1+c_2+m+1+N)}{\Gamma(c_1+1+N)} \frac{\Gamma(-m+N)}{\Gamma(-m)} \left[\frac{\Gamma(c_2+ts+1)\Gamma(c_1+rs+1+N)}{\Gamma(c_1+c_2+rs+ts+2+N)} \frac{1}{2\pi i} \int_L 2^{s(r+t)} g(s,h) \frac{z^N}{N!} ds\right]$$

On putting the value in equation (16) we get

$$\upsilon(u,v,w,t) = \frac{1}{2^{c\ +c_2-1}bc} \sum_{m,m_1,l_1=0}^{\infty} \frac{m!(c_1+c_2+2n+1)\Gamma(c_1+1)}{\Gamma(c_1+m+1)\Gamma(c_2+m+1)} \frac{\eta\delta[1-(-1)^{m_1}e^{-\eta b}]}{\left(\eta^2+\frac{m_1^2\pi^2}{b^2}\right)} \frac{[1-(-1)^{l_1}e^{-\delta c}]}{\left(\delta^2+\frac{l_1^2\pi^2}{c^2}\right)}$$

$$\times \exp\left[-\left\{\mu_0(m+c_1+c_2+1)+\mu\pi^2\left(\frac{m_1^2}{b^2}+\frac{l_1^2}{c^2}\right)\right\}t\right] P_m^{(c_1,c_2)}(u)\cos\left(\frac{m_1\pi}{b}\right)v\cos\left(\frac{l_1\pi}{c}\right)w$$

$$\times \frac{\Gamma(c_1+c_2+m+1+N)}{N!\Gamma(c_1+1+N)} \frac{\Gamma(-m+N)}{\Gamma(-m)} \gamma_{p+1,q+2}^{n+2,k}\left[\left.\begin{matrix}(a,A_1,x)\ (a_j,A_j)_{2,p}\ (\alpha+\beta+N+2,r+t)\\ (\beta+1,t)\ (\alpha+N+1,r)(b_{j,}\ \beta_j)_{1,q}\end{matrix}\right| 2^{(r+t)}p\right]$$

Similarly,

$$\upsilon(u,v,w,t) = \frac{1}{2^{c\ +c_2-1}bc} \sum_{m,m_1,l_1=0}^{\infty} \frac{m!(c_1+c_2+2n+1)\Gamma(c_1+1)}{\Gamma(c_1+m+1)\Gamma(c_2+m+1)} \frac{\eta\delta[1-(-1)^{m_1}e^{-\eta b}]}{\left(\eta^2+\frac{m_1^2\pi^2}{b^2}\right)} \frac{[1-(-1)^{l_1}e^{-\delta c}]}{\left(\delta^2+\frac{l_1^2\pi^2}{c^2}\right)}$$

$$\times \exp\left[-\left\{\mu_0(m+c_1+c_2+1)+\mu\pi^2\left(\frac{m_1^2}{b^2}+\frac{l_1^2}{c^2}\right)\right\}t\right] P_m^{(c_1,c_2)}(u)\cos\left(\frac{m_1\pi}{b}\right)v\cos\left(\frac{l_1\pi}{c}\right)w$$

$$\times \frac{\Gamma(c_1+c_2+m+1+N)}{N!\Gamma(c_1+1+N)} \frac{\Gamma(-m+N)}{\Gamma(-m)} \Gamma_{p+1,q+2}^{n+2,k}\left[\left.\begin{matrix}(a,A_1,x)\ (a_j,A_j)_{2,p}\ (\alpha+\beta+N+2,r+t)\\ (\beta+1,t)\ (\alpha+N+1,r)(b_{j,}\ \beta_j)_{1,q}\end{matrix}\right| 2^{(r+t)}p\right]$$

Particular cases

(a) If $c_1 + c_2 = 0$ then we have $\frac{\partial v}{\partial t} = \mu_0 \left((1-u^2)\frac{\partial^2 v}{\partial u^2} + (c_2 - c_1 - 2u)\frac{\partial v}{\partial u} \right) + \mu \left(\frac{\partial^2 \upsilon}{\partial v^2} + \frac{\partial^2 \upsilon}{\partial w^2} \right) = 0$

$$v(u,v,w,t) = \frac{2}{bc} \sum_{m,m_1,l_1=0}^{\infty} \frac{m!(2m+1)\Gamma(c_1+1)}{\Gamma(c_1+m+1)\Gamma(c_2+m+1)} \frac{\eta\delta[1-(-1)^{m_1} e^{-\eta b}]}{\left(\eta^2 + \frac{m_1^2\pi^2}{b^2}\right)} \frac{[1-(-1)^{l_1} e^{-\delta c}]}{\left(\delta^2 + \frac{l_1^2\pi^2}{c^2}\right)}$$

$$\times \exp\left[-\left\{ \mu_0 m(m+1) + \mu\pi^2 \left(\frac{m_1^2}{b^2} + \frac{l_1^2}{c^2} \right) \right\} t \right] P_m^{(c_1,c_2)}(u) \cos\left(\frac{m_1\pi}{b}\right) v \cos\left(\frac{l_1\pi}{c}\right) w$$

$$\times \frac{\Gamma(m+1+N)}{N!\Gamma(c_1+1+N)} \frac{\Gamma(-m+N)}{\Gamma(-m)} \gamma_{p+1,q+2}^{n+2,k} \left[\left. \begin{matrix} (a,A_1,u)\ (a_j,A_j)_{2,p}\ (N+2,r+t) \\ (c_2+1,t)\ (c_1+N+1,r)(b_j,(c_2)_j)_{1,q} \end{matrix} \right| 2^{(r+t)} p \right]$$

Similarly

$$v(u,v,w,t) = \frac{2}{bc} \sum_{m,m_1,l_1=0}^{\infty} \frac{m!(2m+1)\Gamma(c_1+1)}{\Gamma(c_1+m+1)\Gamma(c_2+m+1)} \frac{\eta\delta[1-(-1)^{m_1} e^{-\eta b}]}{\left(\eta^2 + \frac{m_1^2\pi^2}{b^2}\right)} \frac{[1-(-1)^{l_1} e^{-\delta c}]}{\left(\delta^2 + \frac{l_1^2\pi^2}{c^2}\right)}$$

$$\times \exp\left[-\left\{ \mu_0 m(m+1) + \mu\pi^2 \left(\frac{m_1^2}{b^2} + \frac{l_1^2}{c^2} \right) \right\} t \right] P_m^{(c_1,c_2)}(u) \cos\left(\frac{m_1\pi}{b}\right) v \cos\left(\frac{l_1\pi}{c}\right) w$$

$$\times \frac{\Gamma(m+1+N)}{N!\Gamma(c_1+1+N)} \frac{\Gamma(-m+N)}{\Gamma(-m)} \Gamma_{p+1,q+2}^{n+2,k} \left[\left. \begin{matrix} (a,A_1,u)\ (a_j,A_j)_{2,p}\ (N+2,r+t) \\ (c_2+1,t)\ (c_1+N+1,r)(b_j,(c_2)_j)_{1,q} \end{matrix} \right| 2^{(r+t)} p \right]$$

(b) If $c_1 = c_2 = 0$ then we have $\frac{\partial \upsilon}{\partial t} = \mu_0 \left((1-u^2)\frac{\partial^2 \upsilon}{\partial u^2} - 2u\frac{\partial \upsilon}{\partial u} \right) + \mu \left(\frac{\partial^2 \upsilon}{\partial v^2} + \frac{\partial^2 \upsilon}{\partial w^2} \right) = 0$

$$v(u,v,w,t) = \frac{2}{bc} \sum_{m,m_1,l_1=0}^{\infty} \frac{m!(2m+1)}{(\Gamma(m+1))^2} \frac{\eta\delta[1-(-1)^{m_1} e^{-\eta b}]}{\left(\eta^2 + \frac{m_1^2\pi^2}{b^2}\right)} \frac{[1-(-1)^{l_1} e^{-\delta c}]}{\left(\delta^2 + \frac{l_1^2\pi^2}{c^2}\right)}$$

$$\times \exp\left[-\left\{ \mu_0 (m+1) + \mu\pi^2 \left(\frac{m_1^2}{b^2} + \frac{l_1^2}{c^2} \right) \right\} t \right] P_m(u) \cos\left(\frac{m_1\pi}{b}\right) v \cos\left(\frac{l_1\pi}{c}\right) w$$

$$\times \frac{\Gamma(m+1+N)}{N!\Gamma(1+N)} \frac{\Gamma(-m+N)}{\Gamma(-m)} \gamma_{p+1,q+2}^{n+2,k} \left[\left. \begin{matrix} (a,A_1,u)\ (a_j,A_j)_{2,p}\ (N+2,r+t) \\ (1,\ t)\ \ (N+1,r)(b_j,(c_2)_j)_{1,q} \end{matrix} \right| 2^{(r+t)} p \right]$$

$$v(u,v,w,t)=\frac{2}{bc}\sum_{m,m_1,l_1=0}^{\infty}\frac{m!(2m+1)}{(\Gamma(m+1))^2}\frac{\eta\delta[1-(-1)^{m_1}e^{-\eta b}]}{\left(\eta^2+\frac{m_1{}^2\pi^2}{b^2}\right)}\frac{[1-(-1)^{l_1}e^{-\delta c}]}{\left(\delta^2+\frac{l_1{}^2\pi^2}{c^2}\right)}$$

$$\times\exp\left[-\left\{\mu_0(m+1)+\mu\pi^2\left(\frac{m_1{}^2}{b^2}+\frac{l_1{}^2}{c^2}\right)\right\}t\right]P_m(u)\cos\left(\frac{m_1\pi}{b}\right)v\cos\left(\frac{l_1\pi}{c}\right)w$$

$$\times\ \frac{\Gamma(m+1+N)}{N!\Gamma(1+N)}\frac{\Gamma(-m+N)}{\Gamma(-m)}\Gamma_{p+1,q+2}^{n+2,k}\left[\left.\begin{matrix}(a,A_1,u)\ (a_j,A_j)_{2,p}\ (N+2,r+t)\\(1,\,t)\ \ (N+1,r)(b_{j,}\,(c_2)_j)_{1,q}\end{matrix}\right|2^{(r+t)}\,p\right]$$

CONCLUSIONS

In this paper, we have successfully used the Jacobi polynomial and incomplete H-function to solve the heat equation. Apart with this, we have also generated some particular cases to validate our method and results. For further study, we can take some new approach with new functions. We can also compare their results as well in our future work.

Authors Contribution

Shalini Shekhawat led the study and designed the study map. Akanksha Shukla did all the calculations and made the manuscript. Ravi Shanker Dubey arranged all the study literature. Kanak Modi formatted the manuscript. All the authors read and approved the final manuscript.

Conflict of Interest

The authors declare no conflict of interest.

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